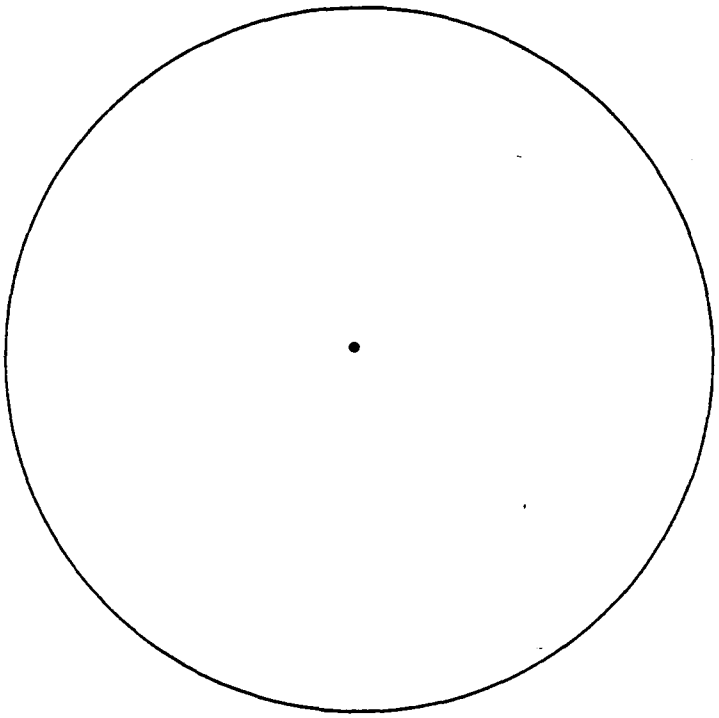
- Ask students what things Rita does at her job. (clean, cashier, make hamburgers and make french fries) Point to each section and the symbol as students answer. How many hours does she clean? (1) How many hours does she make hamburgers? (3) How many hours is she a cashier? (2) How many hours does she make french fries? (2)
- Give each student a copy of the Rita's Job activity sheet. Have students complete the activity sheet on their own.

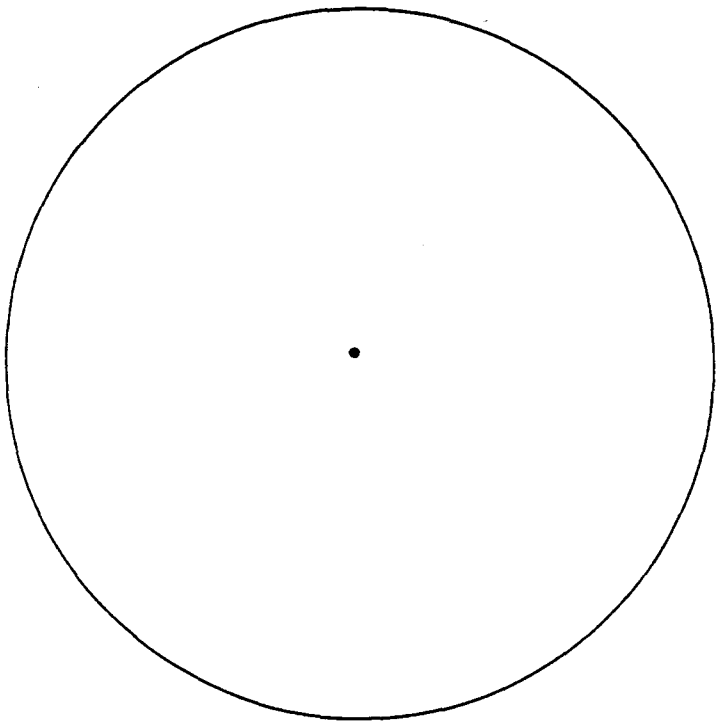
- Put up the At the Pet Store transparency. Let students look at it for a minute. Ask them what this graph is about. (pet sales)
- Ask students how much each part of the graph represents. (4 pets sold) How do they know? (Cats, birds, and snakes are one part and they represent 4 pets sold.) If there are 3 parts fish, how many fish were sold? (12)
- Ask students:
  * How many snakes did the pet store sell? (4)
  * How many bunnies did the pet store sell? (8)
  * How many birds did the pet store sell? (4)
  * What did the pet store sell the most of? (fish)
  * What fraction of the pets sold was cats? (one-eighth)
  * What fraction of the pets sold was fish? (three-eighths)
  * How many pets did the store sell in one week? (32)

4. Construct a circle graph.
   - Pass out the Tutti-Fruitti activity sheet for the students to complete. Tell students they are going to construct a circle graph on their own. Have them make up their own title for the graph.

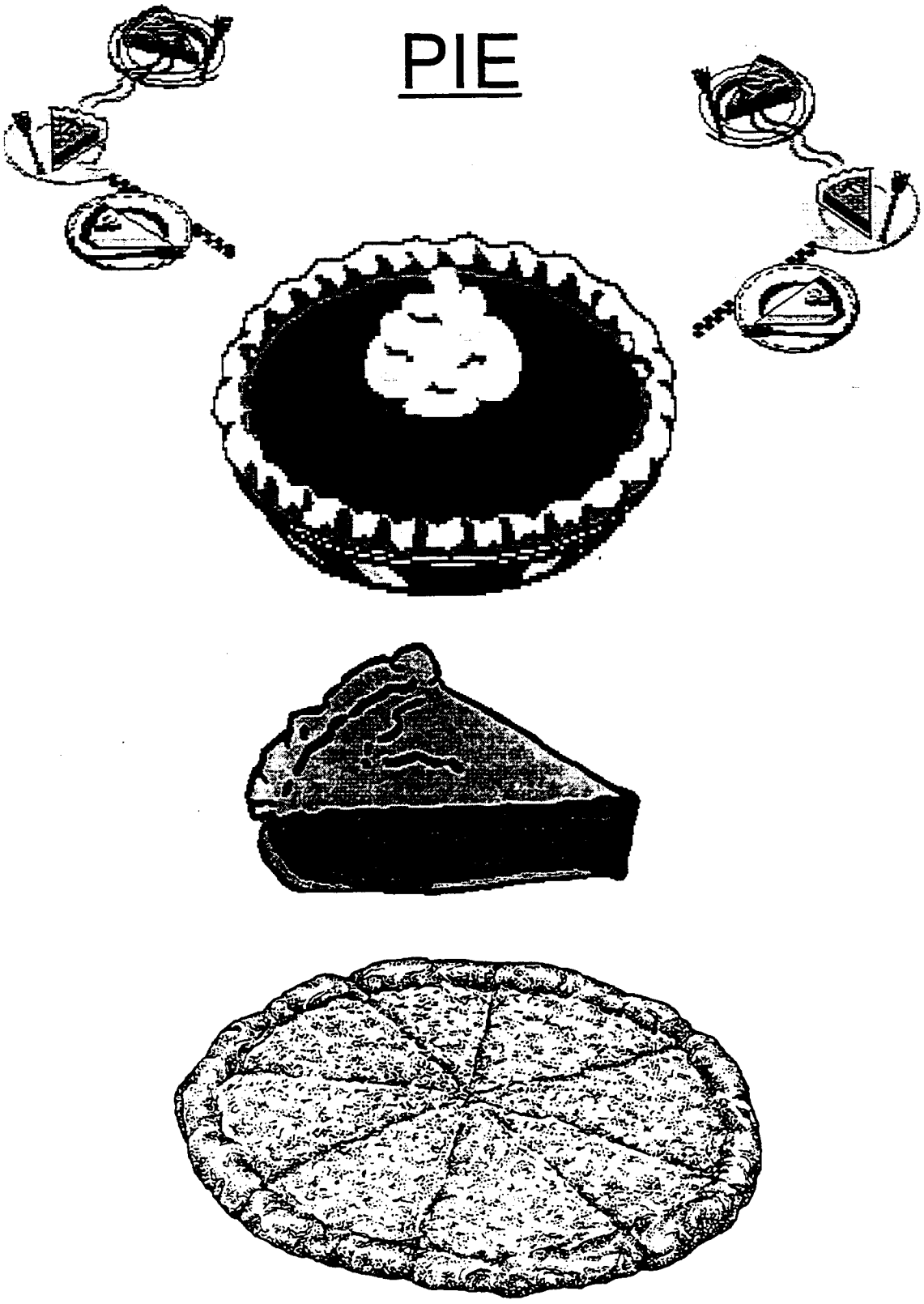
5. Extension Activity
   - Ask students how they spend a weekday (24 hours).
   - Have them make a list of all their activities i.e., sleeping, eating, studying, playing soccer, school, etc. and the number of hours a day they spend doing each (rounded to the nearest whole hour). Remind students that their hours must total 24.
   - Pass out Hours in a Day Graph and ask students to complete it. Color the sections that are the same i.e., sleeping = red, eating = blue, etc. Have them write five sentences about their day. These can be simple, such as "I spend two hours a day eating" or more difficult such as, "I spend more time sleeping than playing" depending on the English level of the students.

CIRCLES



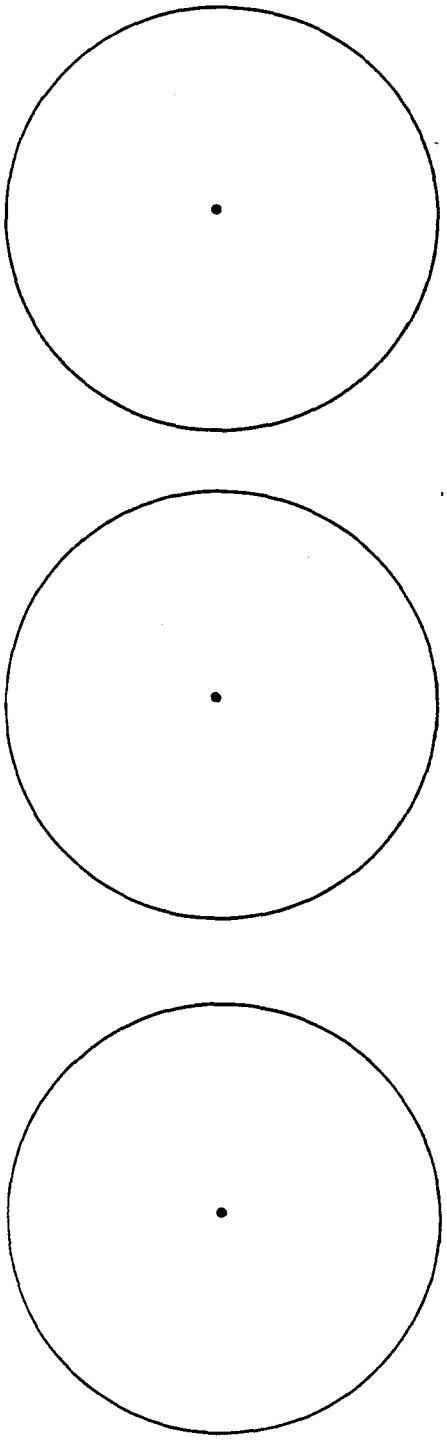


PIE



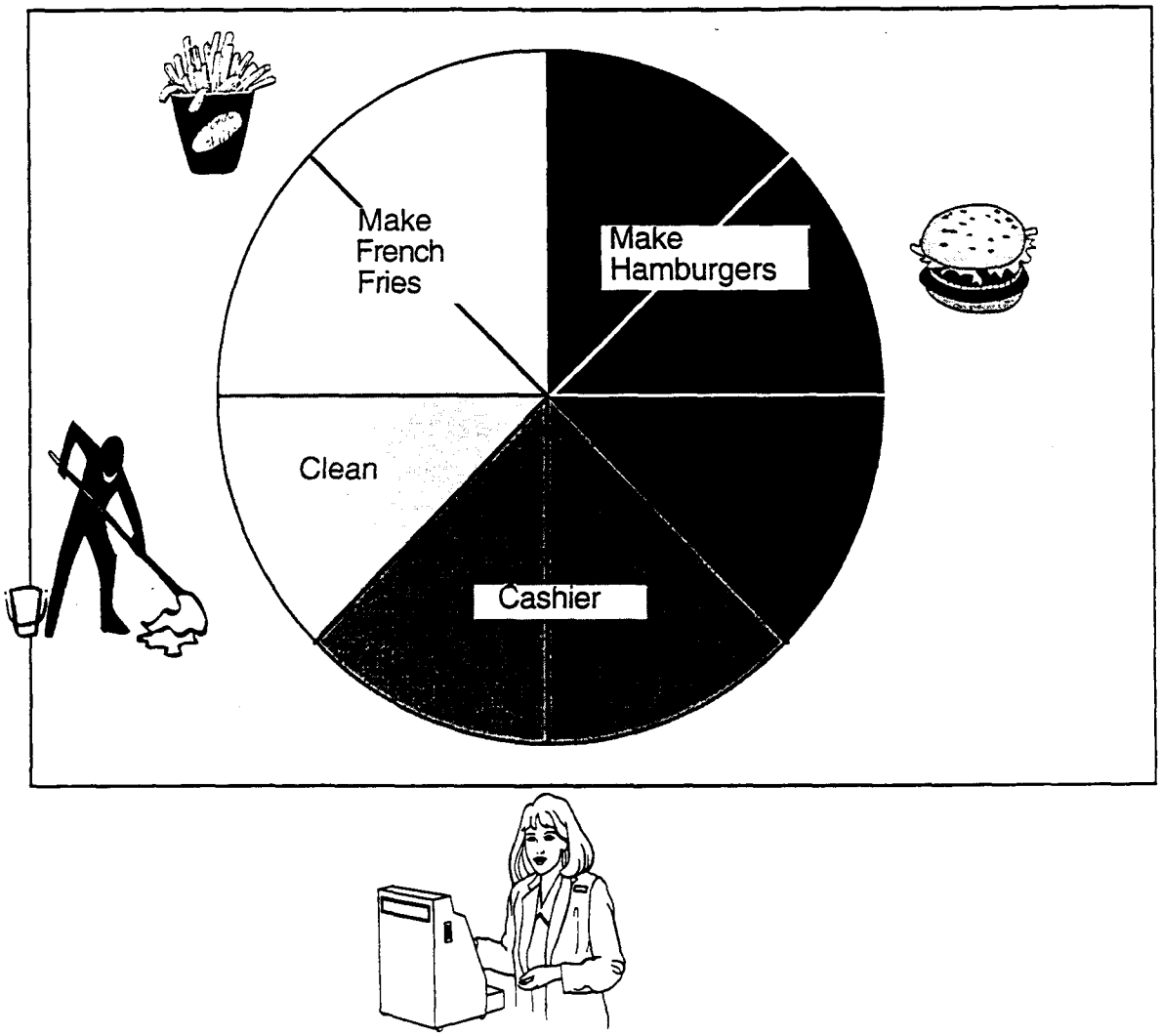
Circles Activity Sheet

Name _____



Rita's Job

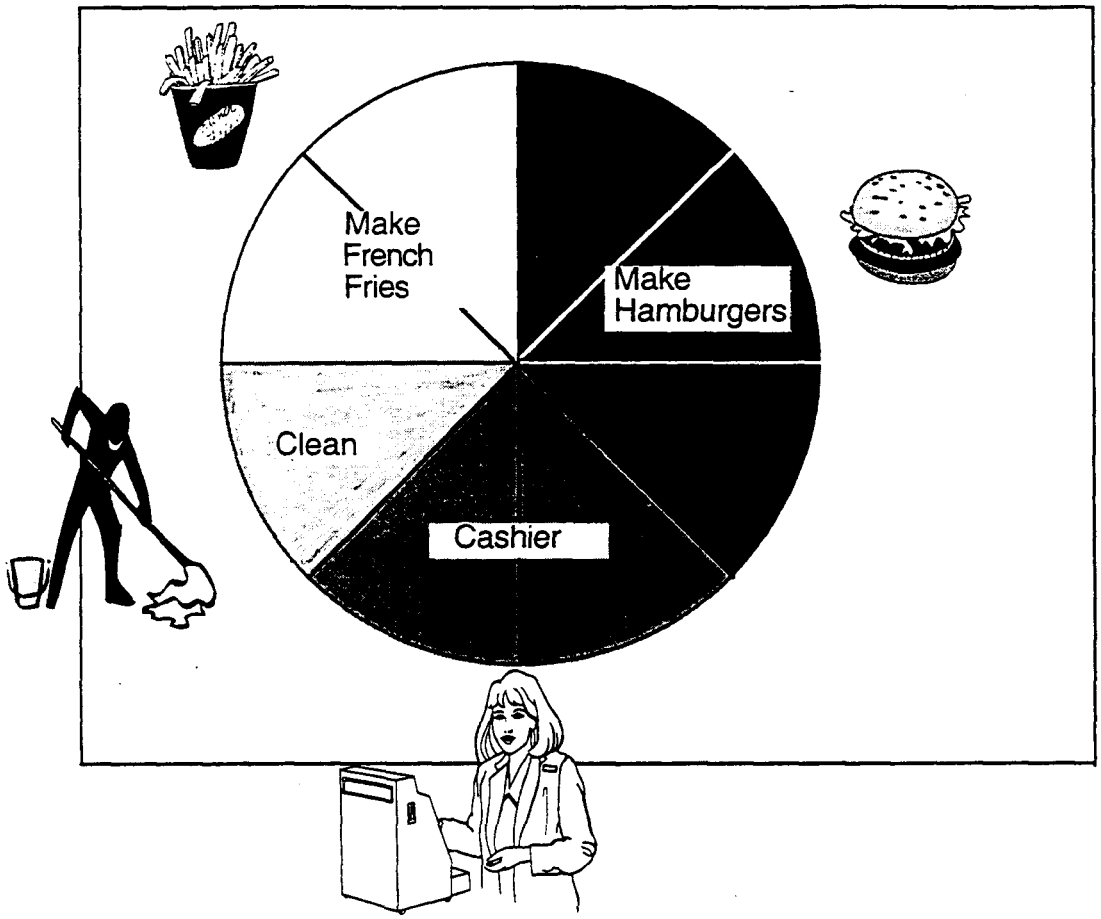
Rita has a job working in a fast food restaurant. She works 8 hours a day.



Name _____

Rita's Job

Rita has a job working in a fast food restaurant. She works 8 hours a day. Use the circle graph below to answer the questions.

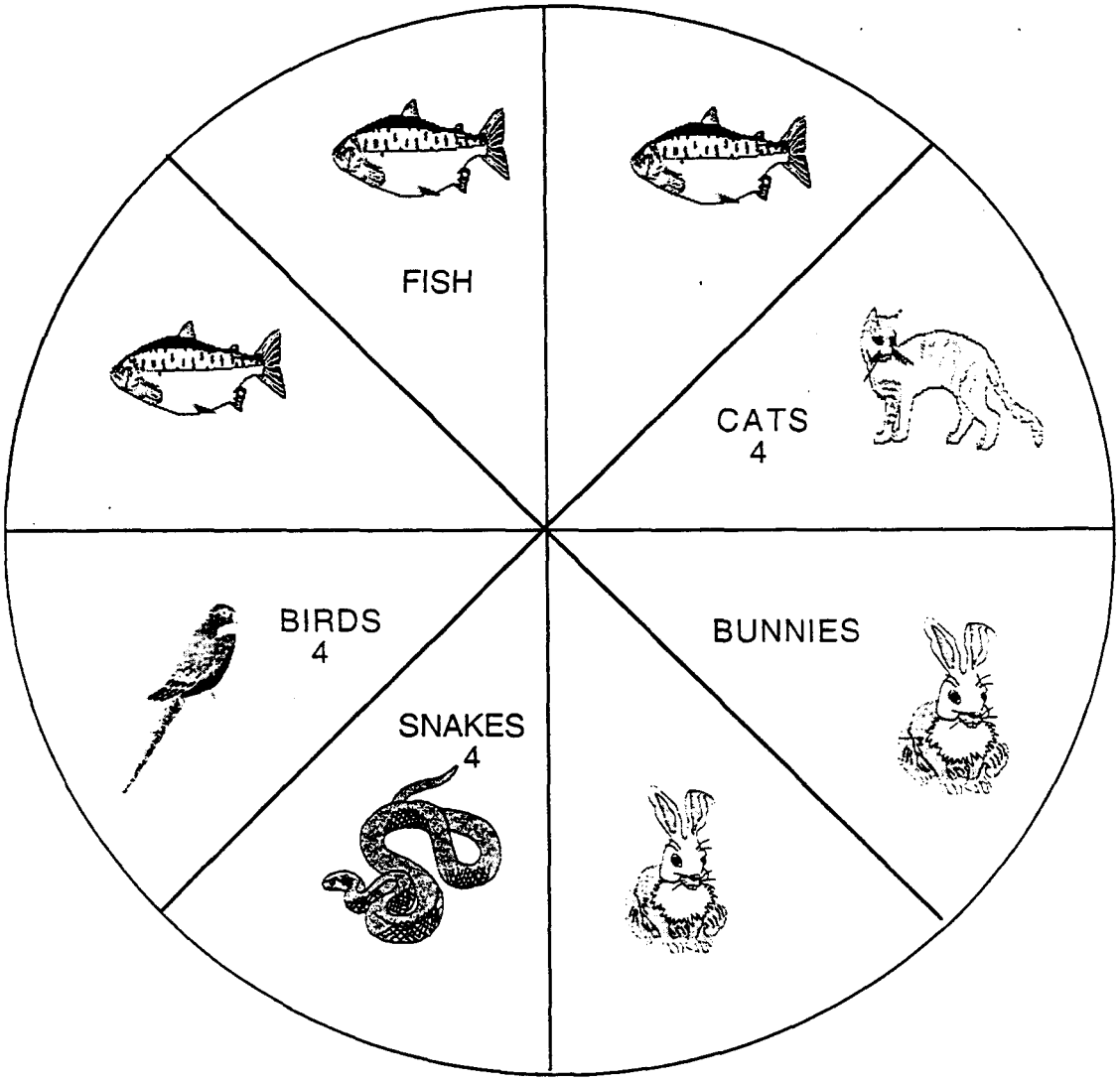


1. Which activity takes the most time? _____
2. Which activity takes the least time? _____
3. How many hours is Rita a cashier? _____
4. How many hours does Rita clean? _____
5. Which activities does Rita do for two hours? _____
6. How many more hours does Rita make hamburgers than clean? _____
7. Do you think Rita's job is the same every day? _____
   Why or why not? _____
   _____

AT THE PET STORE

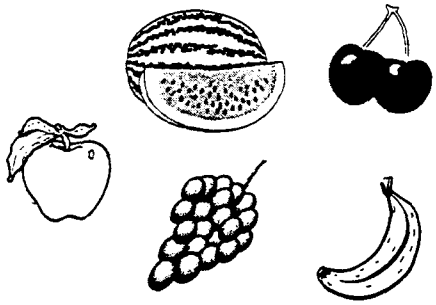
Look at the circle graph below to see how many pets the pet store sold in one week.

NUMBER OF PETS SOLD



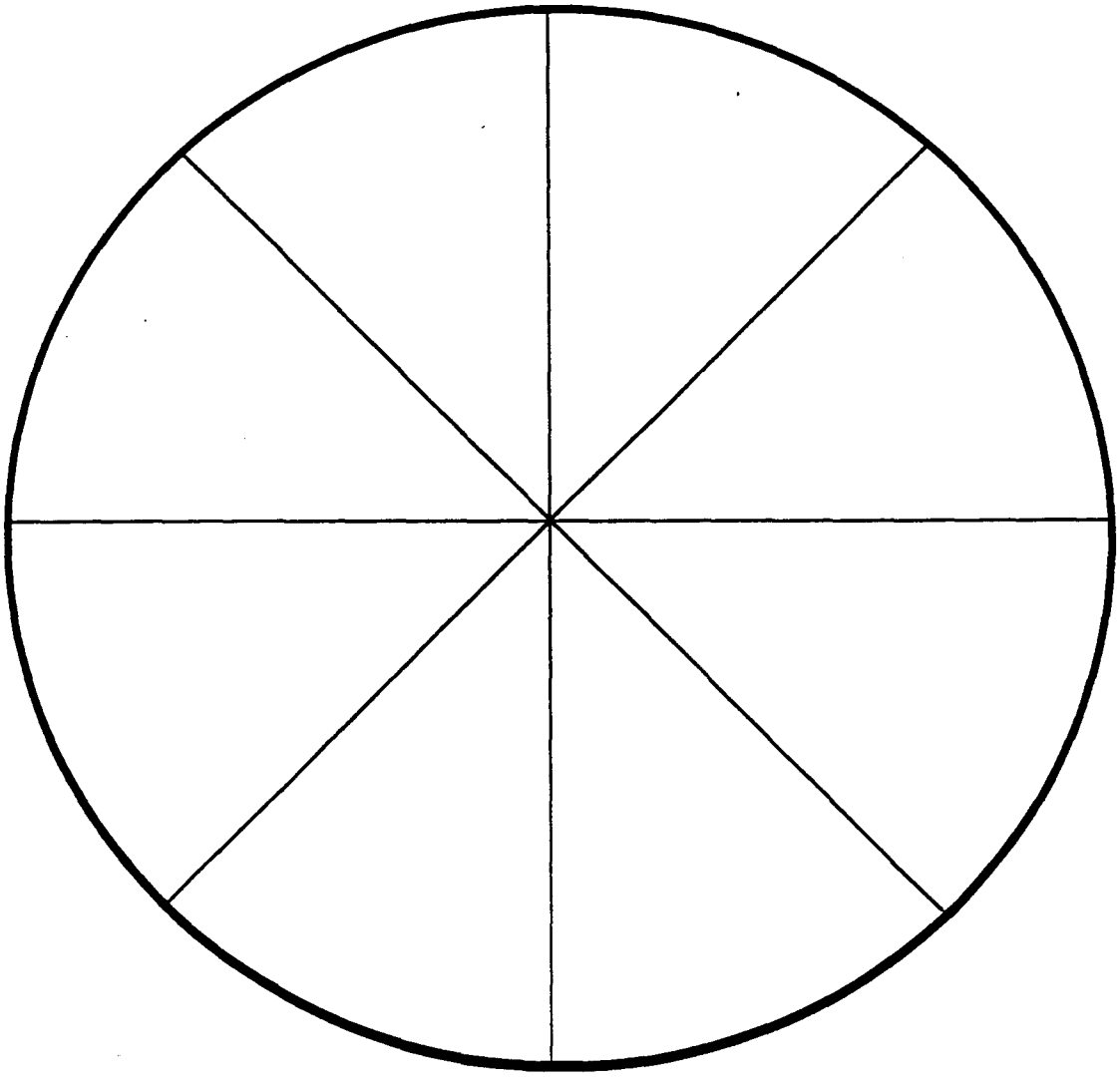
Name _____

TUTTI-FRUITI



Forty sixth graders were asked to name their favorite fruit. Use the data below to make a circle graph.

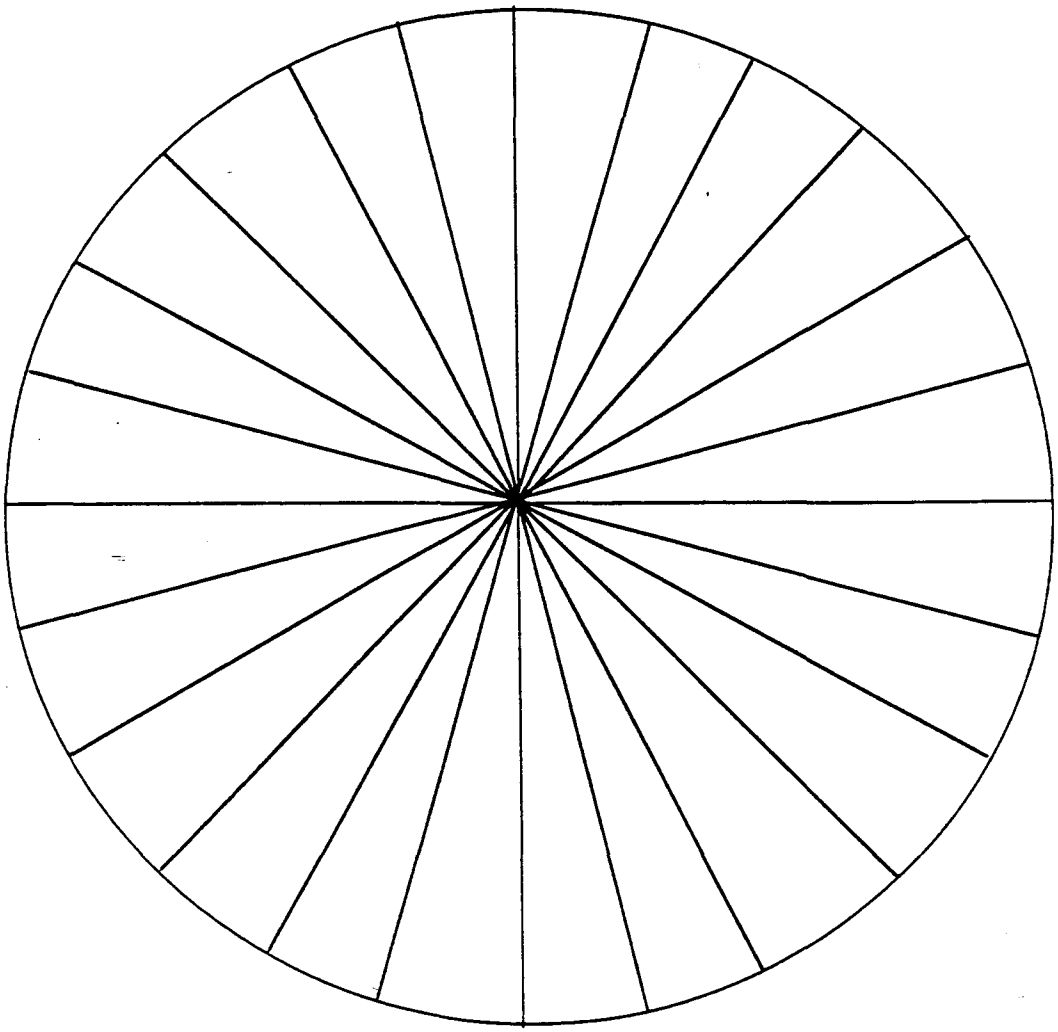
Cherries	15
Apples	5
Grapes	5
Watermelon	5
Bananas	10



Name _____

Hours in a Day Graph

The circle is divided into 24 parts. Each part is one hour. Use your list of what you do in one day to complete the graph. Be sure to label and color the graph. Give the graph a new title.



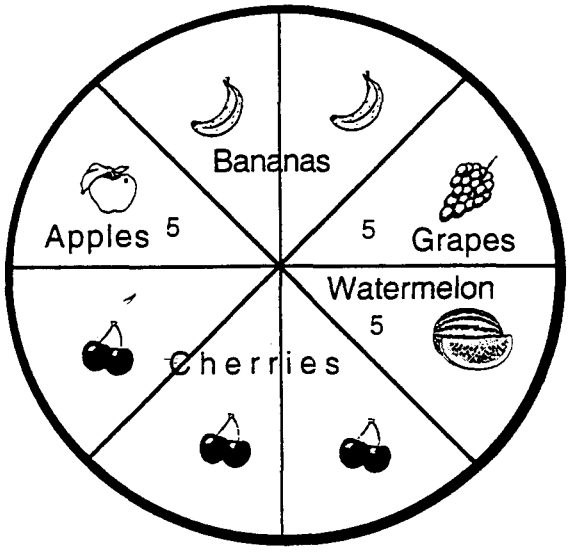
Answer Key
Obj. 6

Rita's Job

1. make hamburgers
2. clean
3. two hours
4. 1 hour
5. make french fries, cashier
6. two hours
7. Answers will vary. Possible answers are:
   No, because some days the restaurant will be busier than others. She might spend more hours cooking. If someone is sick, she might have to do their job.

Tutti-Fruiti

SIXTH GRADERS' FAVORITE FRUITS



Hours in a Day

Answers will vary. All graphs should have a title, labels, and be colored.

Objective 7: Determine and describe the mean, median, and mode as measures of central tendency and determine their meaning for a given set of data. Find the range.

Vocabulary

mean
average
mode
median
range

Language Foundation

1. Students should be exposed to both the words mean and average although you may prefer to use one or the other in your class. Some students may know the word mean as the opposite of kind or to signify as in word meaning. You can explain to them that many words in English have more than one "meaning" or definition.

2. The word median is very similar to the word "mediano" in Spanish which means middle size and "medio" which means middle. You can explain to students that the prefix "med" often refers to things in the middle such as medium size or mediator.

Materials

multilink cubes
candy or other small objects
calculators

Transparencies:

Marbles, Marbles
Finding the Mean

Student Copies:

Marbles, Marbles
Finding the Mean
Finding the Mean
Which Would You Choose?
Mean, Median, Mode and Range
Finding the Mean, Median, Mode and Range
Which Would You Choose?

Mathematics Component

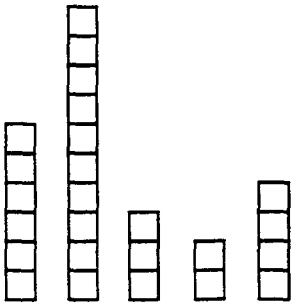
1. Determine and describe the mean.
   - Call three students up to the front of the room. Give the first student 6 candies, the second student 2 candies, and the third student 7 candies. (If you prefer not to use candy, any small object can be used.) Ask students to figure out how many candies each needs for each student to have an equal number of candies. (Encourage them to move the candies around until each has 5.) On an overhead write Candies. Underneath it write 6,2,7. Write Equal Groups Of. Underneath it write 5. Put an = sign between the 7 and the 5.

Candies		Equal Groups of
6, 2, 7	=	5

   - Call up another three students. Give the first student 3 candies, the second student 7 candies and the third student 8 candies. Ask them to make each group of candies equal. (Encourage them to move the candies around until each has 6.) On the overhead write 3,7,8 under Candies. Under Equal Groups of write 6. Put an = sign between the 8 and the 6. (Save the data on the overhead *for later use.*)

Candies		Equal Groups of
6, 2, 7	=	5
3, 7, 8	=	6

   - Divide the students into small groups and give each group stacks of 6 multilink cubes, 10 multilink cubes, 3 multilink cubes, 2 multilink cubes, and 4 multilink cubes each.



   - Write 6, 10, 3, 2, 4 on the board or overhead. Tell students each stack needs to be equal. (Encourage them to move the cubes around until each stack is the same height.)
   - When students are done, ask them how many cubes are in each stack. (5)
   - On the overhead write:

Cubes		Equal Groups of
6, 10, 3, 2, 4	=	5

- Put the Marbles, Marbles transparency on the overhead. Give each student a copy. Read the problem aloud to the students. Ask them to find the answer. (11)
- When students are done, ask them to explain how they got their answers. Then tell them you are going to show them an easy way to solve the problem. Point to the transparency and ask, "How many marbles does Jose have?" (12) Write 12 next to Jose's marbles. Ask, "How many marbles does Juan have?" (14) Write 14 next to Juan's marbles. Ask, "How many marbles does Anna have?" (7) Write 7 next to Anna's marbles. Point to the last circle. Ask students, "How many marbles are there in all?" (33) Write 12 + 14 + 7 = 33 next to the last circle. Have students copy this on their sheets. Ask students how many friends were sharing the marbles. (3) Write 33 ÷ 3 = 11. Each student gets 11 marbles. Underneath the previous examples, write:

Marbles		Equal Groups of
12, 14, 7	=	11

- Tell students that 11 is called the **mean** or the **average**. (They need to know both words.) 11 is the average or the mean of 12, 14, and 7. The mean is the number you get when groups of different sizes are made the same or equal.
- Ask students what the rule is for finding the mean of a group of numbers. (If students have trouble with this, encourage them to go back and look at what you just did.)
- Put up the Finding the Mean transparency/wall poster. (This can be enlarged and posted in the classroom or copied for students to keep in their binders.) Go over the rule with students.
- Go back and point to the first candy problem. Ask students, "How many candies were there altogether?" (15) Write 6 + 2 + 7 = 15. Ask students, "How many groups of candy did we have?" (3) Write15 ÷ 3. Ask students how many candies were in each equal group. (5) Write = 5 after 15 ÷ 3. Ask students why it was necessary to divide by 3. (There were 3 students or groups.) Ask students what 5 is called. (the mean) Have them repeat, "The mean of 6, 2, and 7 is 5."
- Go to the second candy problem. Ask students what to do first to find the mean. (Add up all the numbers.) Have them do it at their desks. Ask students for their answers and write 3 + 7 + 8 = 18 on the overhead. Ask them what the next step is. (To divide by the number of numbers) Have them do this at their desks. Check to see that everyone is dividing by 3. Ask students why they divided by 3. (There were 3 students or groups.) Write 18 ÷ 3 = 6. Ask students what the mean is of 3, 7 and 8. (6) Have them repeat, "The mean of 3, 7, and 8 is 6."
- Go back to the multilink problem. Repeat the process above. Have students repeat, "The mean of 6, 10, 3, 2, and 4 is 5. Ask students if using the rule is easier than moving the cubes around. (Yes)
- Pass out the Finding the Mean student activity sheet. Do the first problem together as a class. Have students complete the rest of the sheet on their own. Remind them to refer to the Finding the Mean wall poster and/or student copy if they cannot remember the steps.

2. Determine and describe the mode.

2. Determine and describe the mode.
   - Put the following numbers on the board or overhead - 8, 15, 7, 6, 10, 7, 18, 1. Tell students to put the numbers in order from least to greatest horizontally on their papers. It will look like this - 1, 6, 7, 7, 8, 10, 15, 18. Ask students to find the mean. (9) (72 ÷ 8 = 9) (Let them use calculators, if necessary.) Next ask students which number appears the most often. (7) Tell them 7 is called the **mode**. Point out that mode and most often both begin with the letters "m" and "o". The number that appears the most in a group is called the mode.
   - Put the following numbers on the board - 2, 3, 5, 7, 8, 9, 11. Ask students what the mode is. (All the numbers appear only once, so there is no mode.) Explain to students that sometimes a group of numbers may not have a mode.
   - Add an 8 to the above group of numbers and ask students what the mode is. (8) Now add a 2 to the group and again ask students what the mode is. (2 and 8) Tell students that sometimes there can be more than one mode. This group of numbers has 2 modes. The first group of numbers had no mode. Lastly, add one more 8 to the group. The data will now look like this 2, 3, 5, 7, 8, 9, 11, 8, 2, 8. Ask students what the mode is now. If they say 2 and 8, remind them that the mode is the number that appears the most often. Ask students how many 2s there are. (2) How many 8s are there? (3) What number is the mode? (8) Why? (It appears the most times.)

3. Determine and describe the median.
   - Now tell students we are going to find the number that is called the **median**. Ask students if anyone knows what the median on a highway is. Tell them the median is the piece in the middle of the highway or street that people can stand on. (Show or draw them a picture if possible or refer to a street with a median in the area.) Explain that the median is the number in the middle of a group of numbers when the numbers are in order. Ask students to copy the numbers 2, 2, 5, 7, 8, 8, 8, 9, 11 on their papers. Have them put one finger on the 2 and one finger on the 11. Next have them move their fingers in by putting a finger on the next 2 and the 9. (Do this on the overhead while the students do it at their desks.) Have them move both fingers in one number at a time until only the 8 is left in the middle. Tell students the 8 is the median because it is the number in the middle. If you prefer, students can cross the numbers out instead of using their fingers. (If students cross out the numbers, make sure they do so lightly so that the numbers can still be read.)
   - Put the numbers 13, 4, 18, 37, 55, 8, 29 on the overhead. Ask students what the first step is to find the median. (Put the numbers in order.) Have them do this on their papers. (It will look like this - 4, 8, 13, 18, 29, 37, 55.) Ask students to find the median. (18) Encourage them to use their fingers to find the median if you see that they are having trouble. (Some students may be able to find the median without using their fingers.) Remind students the median is the number in the middle.

- On the board write 15, 17, 20, 23, 24. Ask students what the mode is (there is none.) Ask them what the median is (20). Now ask them, "Suppose there was no 20 (erase it as you say this) now what would the median be?" Since there is no number in the middle, tell students they have to find the mean of 17 and 23. 17 and 23 = 40. 40 ÷ 2 = 20. So the median is 20. Even though 20 is not one of the numbers, it is still the median.
- Put the numbers 64, 67, 90, 87 on the overhead. Ask students to find the median. (77)
- Ask students what the mean is. (77) Tell students that sometimes the mean and the median are the same number.
- Pass out Which Would You Choose? activity sheet for students to do. Allow students to use calculators. When students are done, discuss the reasoning behind their answers.

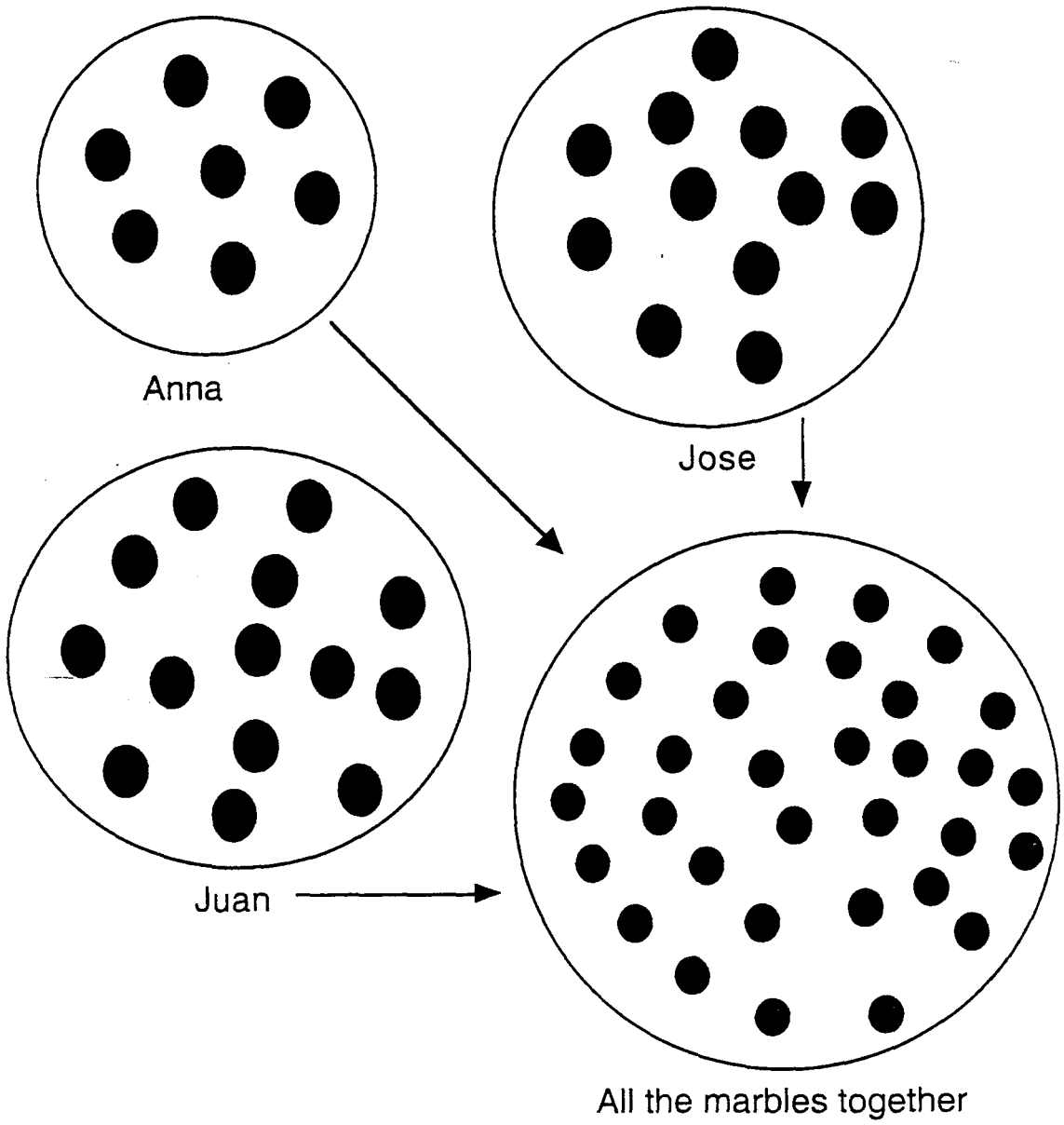
4. Find and determine the range.
   - Ask students to look at the 2, 2, 5, 7, 8, 8, 8, 9, 11 numbers again. Tell them we are going to find the range. Range is the difference between the smallest and largest numbers. Ask them what the smallest number is. (2) What is the largest number? (11) How do we find the difference between two numbers? (subtract) 11 - 2 = 9. The range of the numbers is 9.
   - Put 8, 23, 4, 13, 3, 19, 7 on the overhead. Ask students to copy these numbers and find the range. Ask them what they need to do first. (Put the numbers in order.) After they have done this ask them what the next step is. (Subtract the smallest number from the largest number i.e. 23 - 3.) What is the range? (20) Is there a mode? (no) What is the mean? (11) What is the median? (8)

5. Practice with mean, median, mode and range.
   - Pass out the Mean, Median, Mode and Range paper. This paper can be blown up and used as a poster or copied and given to the students to keep in their binders for reference. Go over the paper with students to make sure they understand the examples.
   - Ask students how long it takes them to get to school every day. Write the times on the board or overhead. Ask students to find the mean, median, mode and range. Encourage students to look at the Finding the Mean wall poster or the Mean, Median, Mode and Range paper if they are having trouble remembering the different terms.
   - Pass out the Finding the Mean, Median, Mode and Range student activity sheet. Have students complete individually. Let students use calculators if needed.
   - For additional practice, divide students into groups and have them collect their own data and find the mean, median, mode, and range. Brainstorm some questions with them that they could ask their classmates. Some possibilities include group's average shoe size, height, number of letters in their names, hours they watch TV per week, etc. Have them display their results in poster form.

Marbles, Marbles

Can you find the answer?

Jose, Juan, and Anna want to share their marbles equally. Jose has 12, Juan has 14 and Anna has 7. How many marbles should **each** child have?



FINDING THE MEAN

To find the mean of a group of numbers:

1. Add up all the numbers.
2. Count how many numbers there are.
3. Divide by that number.

Example: Find the mean of 2, 3, 6, 9, 15.

1. $2 + 3 + 6 + 9 + 15 = 35$
2. There are 5 numbers.
3. Divide by 5. $35 \div 5 = 7$

   The mean is 7.

Name _____

FINDING THE MEAN

1. 14, 9, 10, 3 The average or mean is _____.

2. 12, 5, 4, 3 The average or mean is _____.

3. 10, 18, 14 The average or mean is _____.

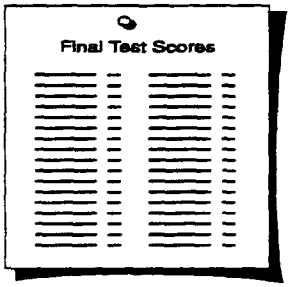
4. 17, 3, 6, 7, 2 The average or mean is _____.

5. 9, 6, 6, 7 The average or mean is _____.

6. 11, 8, 6, 7 The average or mean is _____.

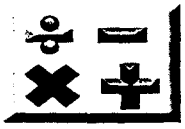
7. 10, 13, 8, 6, 8 The average or mean is _____.

Name _____



Which Would You Choose?

At the end of the quarter, your teacher tells you that for your report card, you may choose the mean or the median grade in each of your classes. Which would you choose? Why? Write the one you would choose in the oval.



1. Math 34, 82, 82, 86, 91
   mean _____
   median _____

2. Science 80, 80, 88, 97, 100
   mean _____
   median _____

3. English 65, 67, 80, 94, 98, 100
   mean _____
   median _____

4. Social Studies 55, 74, 78, 86, 95, 98
   mean _____
   median _____

5. P.E. 54, 60, 75, 88, 90, 95, 98
   mean _____
   median _____

MEAN, MEDIAN, MODE AND RANGE

MEAN
V
E
R
A
G
E

A mean teacher averages the grades.

MEDIAN
I
D
D
L
E

The median on the highway has many flowers.

MODE
O F
S T
T E
N

The mode occurs most often.

S
M
A
L
L
L A
RANGE
G S
E T
S
T

The difference between the largest and smallest number is the range.

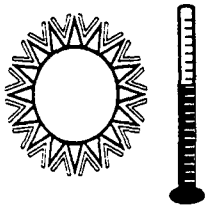
Name _____

Finding the Mean, Median, Mode and Range

Find the mean, median, mode and range for each of the following situations. Label your answers.

1. Time Sook-Yin spent watching television: Monday - 2 hours, Tuesday - 3 hours, Wednesday - 2 hours, Thursday - 1 hour, Friday - 1 hour, Saturday - 4 hours, Sunday -1 hour.

Mean _____
Median _____
Mode _____
Range _____

2. 

High temperature in Washington: 105° - July; 98° - August; 77° - June, 71° - May; 64° - April.

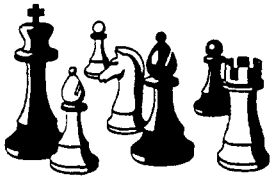
Mean _____
Median _____
Mode _____
Range _____

3. Heights of the sunflower plants in Mrs. Chin's garden: 68 inches, 62 inches, 50 inches, 45 inches, and 75 inches.

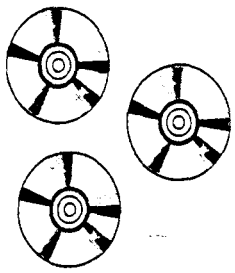
Mean _____
Median _____
Mode _____
Range _____

4. Number of students in after school activities: Math Help - 68, Spanish Club - 35, Art Club - 55, Chess Club - 70, Basketball - 55, Soccer - 53.

Mean _____
Median _____
Mode _____
Range _____

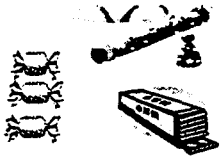


5. Number of CDs sold in one week: Monday - 25, Tuesday - 20, Wednesday -13, Thursday - 8, Friday - 45, Saturday - 98, Sunday - 50.



Mean _____
Median _____
Mode _____
Range _____

6. Amount Jose spent on candy: Monday - $3.50, Wednesday - $5.00, Saturday - $4.20, Sunday - $1.50.



Mean _____
Median _____
Mode _____
Range _____

7. Carlos's math test grades: 82, 87, 93, 92, 68, 79, 73.

Mean _____
Median _____
Mode _____
Range _____

Answer Key
Obj. 7

Finding the Mean

1. The average or mean is 9.
2. The average or mean is 6
3. The average or mean is 14.
4. The average or mean is 7.
5. The average or mean is 7.
6. The average or mean is 8.
7. The average or mean is 9.

Which Would You Choose?

1. Math Mean - 75; Math Median - 82 (median)
2. Science Mean - 89; Science Median - 88 (mean)
3. English Mean - 84; English Median - 87 (median)
4. Social Studies Mean - 81; Social Studies Median - 82 (median)
5. P.E. Mean - 80; P.E. Median - 88 (median)

Finding the Mean, Median, Mode and Range

1. Mean - 2 hours; Median - 2 hours; Mode - 1 hour; Range - 3 hours
2. Mean - 83°; Median - 77°; Mode - None; Range - 41°
3. Mean - 60 inches; Median - 62 inches; Mode - None; Range - 30 inches
4. Mean - 56 students; Median - 55 students; Mode - 55 students; Range - 35 students
5. Mean - 37 CDs; Median - 25 CDs; Mode - None; Range - 90 CDs
6. Mean - $3.55; Median - $3.85; Mode - None; Range - $3.50
7. Mean - 82; Median - 82; Mode - None; Range - 25

Objective 8: Construct and display data using a line plot. Collect, organize, display, and interpret data using a stem and leaf plot.

Vocabulary

stem and leaf
plot
leaves

Materials

calculators

Transparencies:

- My Flower!
- Students' Weights
- Stem and Leaf Plot
- How Many Times Can You Snap Your Fingers in One Minute?
- Steps to Make a Stem and Leaf Plot
- Fourth Graders' Weights
- Average Temperature

Student Copies:

- Line Plot
- Test Grades
- Graph Paper I

Language Foundation

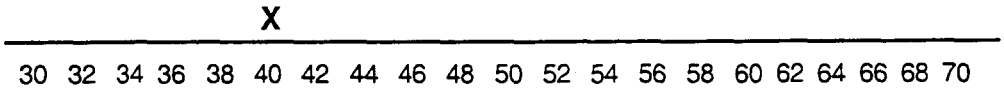
1. In this lesson students are learning about a stem and leaf plot; however, the individual pieces of data are called leaves. Explain to students that the plural of "leaf" is "leaves". Other examples of this kind of plural include scarf - scarves, knife - knives.

2. Some textbooks do not separate their leaves with commas. It is up to the individual teacher to decide what he/she would like the students to do. If students do not use commas, it is important that the data be equally spaced.

3. A piece of graph paper is included at the end of this lesson. Younger students may find it easier to line their numbers up if they use graph paper. Lined notebook paper works well also but may not ensure that the data is equally spaced. If the students use graph paper, it is helpful to have them draw the horizontal and vertical lines in a different color so that they stand out.

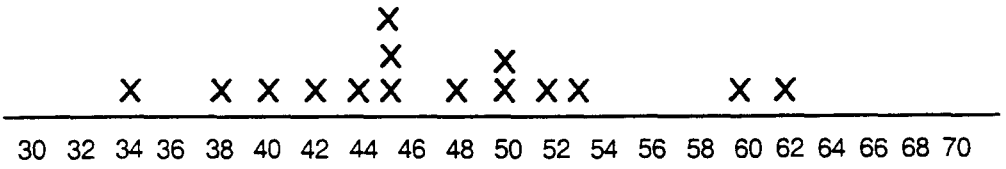
Mathematics Component

1. Construct a line plot.
   - Put up the Stamp Club transparency. Cover up the line plot. Ask students what the data represents. (number of stamps owned by students in the stamp club) Tell students they are going to use this data to make a line plot. Tell them the first step is to draw a line. Slide the paper down on the transparency so that the line is showing. Ask students if they remember what a scale and interval are. (If they don't, you can go back to obj. 3 and review.) Tell them in a line plot the scale is under the line and the data is above the line.
   - Ask students what the smallest piece of data is. (40) What is the largest piece of data? (62) Ask students what scale they think should be used. (Answers will vary but should be in the neighborhood of 30 to 70.) Next ask students what interval should be used. (Two is the best.) Have students draw a line on their papers and write the numbers from 30 to 70. (The numbers should be evenly spaced. There is graph paper at the end of this lesson that can be used.)
   - Uncover the line plot completely so the scale is showing. Tell students they are now ready to begin plotting. What is the first piece of data? (40) Find the 40 on the scale. Put an **X** above it.



   - Ask students what the next piece of data is. (53) Have a student come up to the overhead and make an **X** in the appropriate spot. Go through the rest of the data until all of the numbers have been plotted. (You may choose to do this yourself or continue having students come up to the overhead.) When students are done ask them to count the number of data and then count the Xs on their line plot to make sure they haven't missed any data. Ask students if the line plot reminds them of any other graph they have learned about. (bar graph) What is good about a line plot? (It's easy to make and each piece of data can be seen.) Make up a title.

Number of Stamps Collected by Stamp Club Members



   - Have students put these line plots in their binders or collect and save them for the next part of the lesson.
   - For additional practice have students do the Line Plot student activity sheet. Note: Boxes are provided for the students to write in their scale. Encurage students to think about what numbers should go in the boxes before writing in their numbers. (See answer key for possible solutions.)

2. Construct a stem and leaf plot.
   - Show students the My Flower! transparency (or bring in a real or plastic flower). Point to the stem and leaves. Have students repeat the words after you. Tell them they are going to be learning about a new graph called a stem and leaf plot. Ask them if they remember some of the other graphs. (pictograph, line plot, bar graph, scatterplot, line graph, circle graph)
   - Have students get out their Stamp Club line plots. Tell them they will be using this data to make their stem and leaf plots.
   - Put up the Stem and Leaf transparency. Give each student a copy. Note: (Some books do not use the horizontal line.) Point out the Stem column on the left and the Leaves column on the right. Tell students that in a stem and leaf plot the Stem column represents the tens digits and the Leaves column represents the ones digits. (when the data is only two digits)
   - Ask students what the first piece of data on their line plot is. (34) Which digit is the stem in 34? (3) Write a 3 in the Stem column (close to the vertical line). What is the last piece of data? (62) Which digit is the stem in 62? (6) Fill in all the stems from 3 to 6. Have students do the same on their papers.

Stem	Leaves
3	
4	
5	
6	

   - Now they are ready to put in the leaves. The first piece of data was 34. The stem is already written. Since the 4 is the leaf put that in the Leaves column opposite the 3.

Stem	Leaves
3	4
4	
5	
6	

   - Ask students what the next piece of data is. (38) Which digit is the stem? (3) Tell students that since the 38 has the same stem as the 34, they can share the same stem. Only the leaf needs to be put on the plot. Leave a space and write an 8 after the 4. Point out that the leaves are usually separated by commas.

Stem	Leaves
3	4, 8

- Go to the next piece of data on the line plot. (40) Ask students what the stem is. (4) Since the 4 is already on the plot, only the leaf or the 0 needs to be written in. Do the 42 and 44. Reiterate to students that the 42 and 44 share the same stem with the 40. Make sure students line their leaves up underneath each other and keep them equidistant apart.

Stem	Leaves
3	4, 8
4	0, 2, 4

- The next three pieces of data are 45, 45, and 45. Ask students where these will go. (after the 44) Tell students the 45 must appear three times because each 45 represents a different student even though they each had the same number of stamps.

Stem	Leaves
3	4, 8
4	0, 2, 4, 5 5, 5

- Have students finish plotting the data on their own. When students are done, tell them to give their stem and leaf plot a title.

Stamp Club Collections

Stem	Leaves
3	4, 8
4	0, 2, 4, 5, 5, 5, 8
5	0, 0, 2, 3
6	0, 2

- Ask students questions about the data such as, "What was the least number of stamps collected?" (34), "What was the most stamps collected?" (62), "Which stem has the most data?" (4), "What is the mode?" (45), "How many students collected more than 45 stamps?" (7) etc.
- Ask students how a stem and leaf plot is different from a pictograph, bar graph and a line graph. (A stem and leaf plot uses numbers rather than pictures, bars, or lines.) What other kinds of data could be graphed on a stem and leaf plot? (test grades, points scored by a team, ages, temperatures, etc.)
- Pass out the Test Grades student activity sheet. Have students answer the questions.

3. Collect data to construct a stem and leaf plot.
   - Put up How Many Times Can You Snap Your Fingers In One Minute? Have students estimate how many times they can snap in one minute. Record their estimates on the board. Have students snap fingers and count silently while you time them for half a minute. Have students multiply their count by 2 (for a full minute) and record the answers on the board. Reward the student with the most accurate estimate.
   - Tell students they are going to use this data to construct a stem and leaf plot. Put up Steps to Make a Stem and Leaf Plot transparency. Go over steps with students. (This transparency can also be enlarged and made into a wall poster for easy reference.)
   - Have students construct the stem and leaf plot on their own. (Decide whether you would like the students to construct a line plot or simply put the data in order on their papers.)
   - Note: Some students may snap more than l00 times. If this happens, make sure students understand that the stem is l0 and the leaf is the ones digit.
   - Have students find the mean, median, mode, and range of the data.

   - As an extension activity, let students come up with their own questions for gathering data . They can then survey their classmates and construct a stem and leaf plot. If you want, you can require them to find the mean, median, mode and range.

4. Construct a double stem and leaf plot (also called back-to-back stem and leaf)
   - Tell students that sometimes data needs to be compared. Put up the Fourth Graders' Weights transparency. Say to students, "Suppose the students in Mrs. Green's class want to compare their weights to the students in Mr. Ramirez's class. How could we do this and not mix up the data? To do this we use a double stem and leaf plot. "
   - Have students arrange the weights from Mrs. Green's and Mr. Ramirez's class in order from least to greatest making sure they keep the two classes separate. (Students can use a line plot or put data in order horizontally from least to greatest.) Data will now look like this:

     Mrs. Green's class - 34, 38, 40, 42, 44, 45, 45, 45, 48, 50, 50, 52, 54, 60, 62
     Mr. Ramirez's class - 28, 33, 33, 40, 41, 46, 46, 48, 49, 50, 51, 54, 58, 58, 59, 60, 63, 65

   - Record data on the transparency. Model counting both sets of data to make sure none have been missed or have a student do it out loud.
   - Draw two lines vertically on the overhead. Write Stem over the two lines. Write Mrs. Green's Class Leaves to the left of the stem and Mr. Ramirez's Class Leaves to the right of the stem.

Mrs. Green's Class Leaves	Stem	Mr. Ramirez's Class Leaves

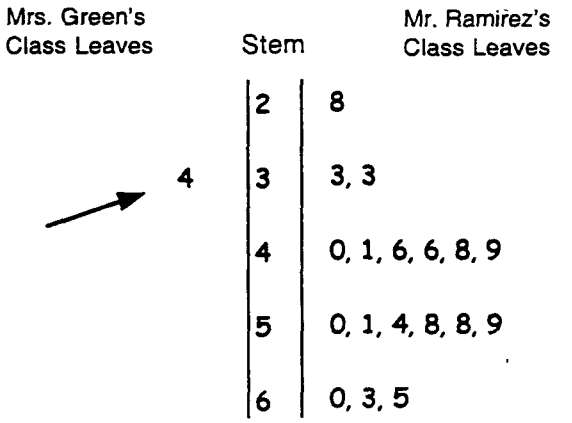
- Tell students this is a double stem leaf plot. Ask them what the next step is. (Put in the stems.) Ask them what stems our plot needs. (2 to 6) If they have trouble with this, go through the same procedure used in constructing the Stamp Club stem and leaf plot. (i.e. What is the smallest piece of data? What is the largest piece of data?)

Mrs. Green's Class Leaves	Stem	Mr. Ramirez's Class Leaves
	2	
	3	
	4	
	5	
	6	

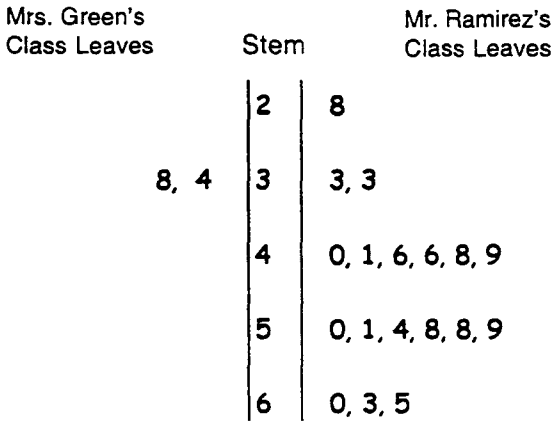
- Have students plot the data for Mr. Ramirez's class at at their desks.

Mrs. Green's Class Leaves	Stem	Mr. Ramirez's Class Leaves
	2	8
	3	3, 3
	4	0, 1, 6, 6, 8, 9
	5	0, 1, 4, 8, 8, 9
	6	0, 3, 5

- Explain to students that in a double stem and leaf plot, the leaves on the right go from left to right, but the leaves on the left go from right to left. To illustrate, ask students what the lowest weight is in Mrs. Green's class. (34) Ask students where the 34 should go. Since the 3 stem is already on the plot, only the 4 leaf needs to be plotted. Tell students the 4 leaf should go to the left of the stem since that is where the leaves for Mrs. Green's class go. Write in the 4 for Mrs. Green's class.



- What is the next weight in Mrs. Green's class? (38) The leaves on the left side of the plot will be plotted just like the leaves on the right side. Put the 8 to the left of the 4 and separate with a comma. Have students do the same on their papers.



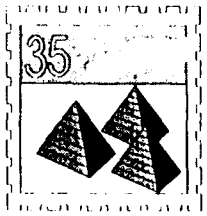
- Have students complete the data for Mrs. Green's class at their desks. (See diagram below.) Make sure they remember to title their plot.

Fourth Graders' Weights

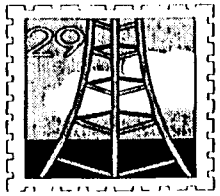
Mrs. Green's Class Leaves	Stem	Mr. Ramirez's Class Leaves
	2	8
8, 4	3	3, 3
8, 5, 5, 5, 4, 2, 0	4	0, 1, 6, 6, 8, 9
4, 2, 0, 0	5	0, 1, 4, 8, 8, 9
2, 0	6	0, 3, 5

- Ask students questions about the plot when they are finished. i.e. What weight appeared the most? (45) Which class had the heaviest student? (Mr. Ramirez's) Which class had the lowest weight? (Mr. Ramirez's) What was the median weight in Mrs. Green's class? (45) What was the median weight in Mr. Ramirez's class? (49.5) Which class do you think had the higher mean weight? Why? (Have students figure out the mean weight for each class.) (Mrs. Green's class - 47.2, Mr. Ramirez's class - 49) Let them use calculators for the mean.

- For additional practice, put up the Average Temperature transparency and have the students choose two cities and construct a double stem and leaf plot. Students can reference the Steps to Make a Stem andLeaf Plot transparency/wall poster if needed.



STAMP CLUB



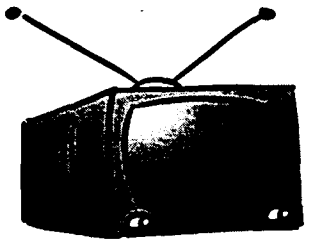
There are15 students in the Morgan Middle School Stamp Club. Listed below are the number of stamps in each collection.

40, 53, 45, 38, 44, 42, 45, 50, 62, 48, 50, 45, 52, 60 and 34

30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70

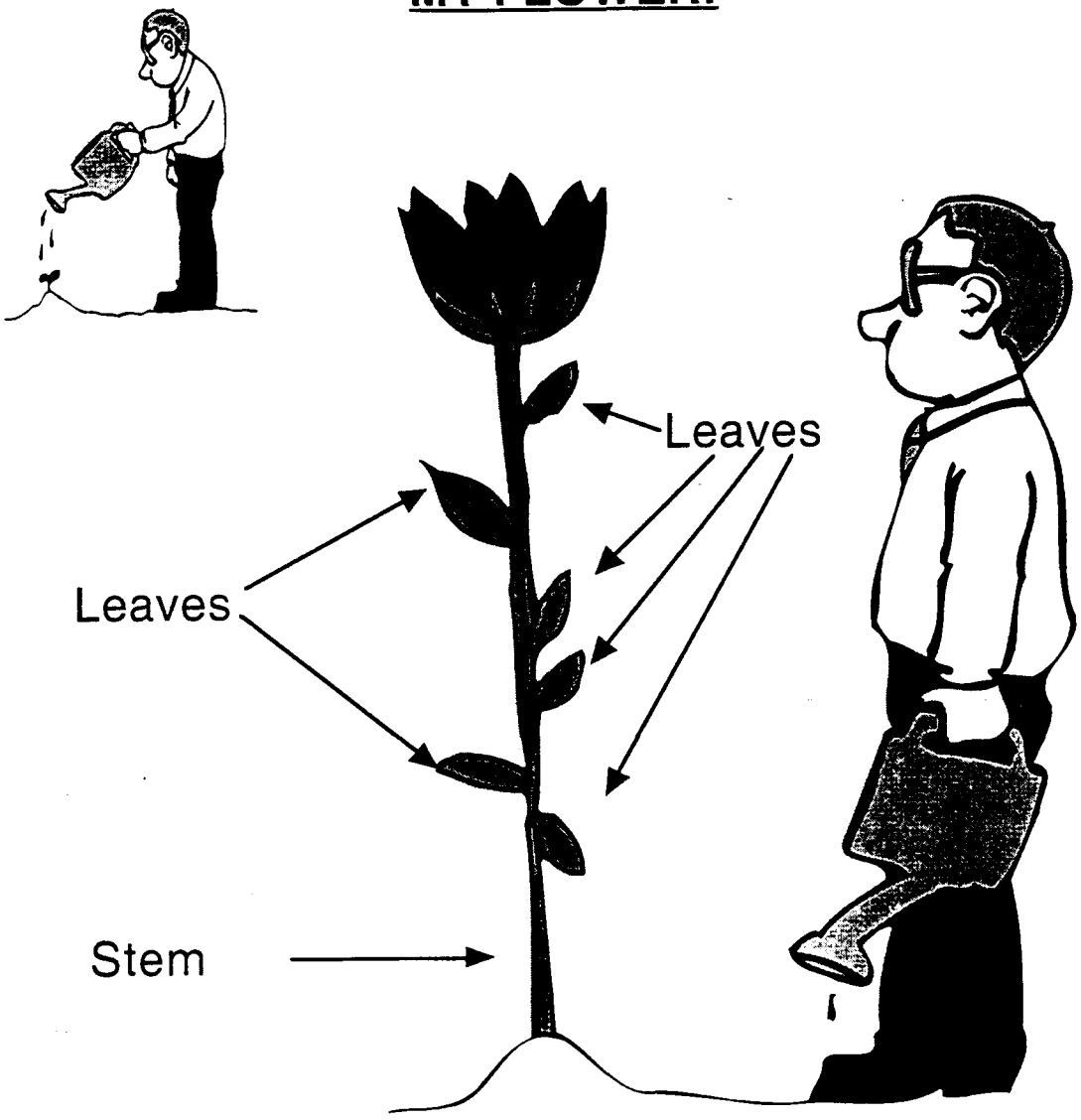
Name _____

LINE PLOT



The students in Ms. Cruz's class watched TV for the following number of hours in one week: 16, 25, 23, 10, 15, 8, 5, 10, 12, 18, 10, 16, 6, 10, 20, and 5. Make a line plot using this data. (Use the boxes to write in your scale.)

MY FLOWER!

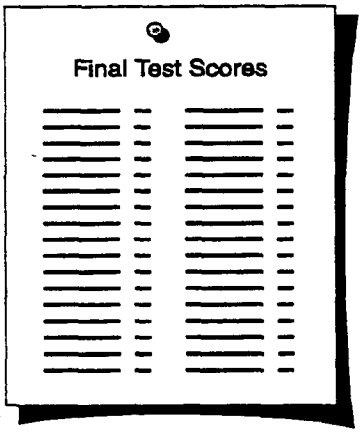


STEM AND LEAF PLOT

STEM	LEAVES

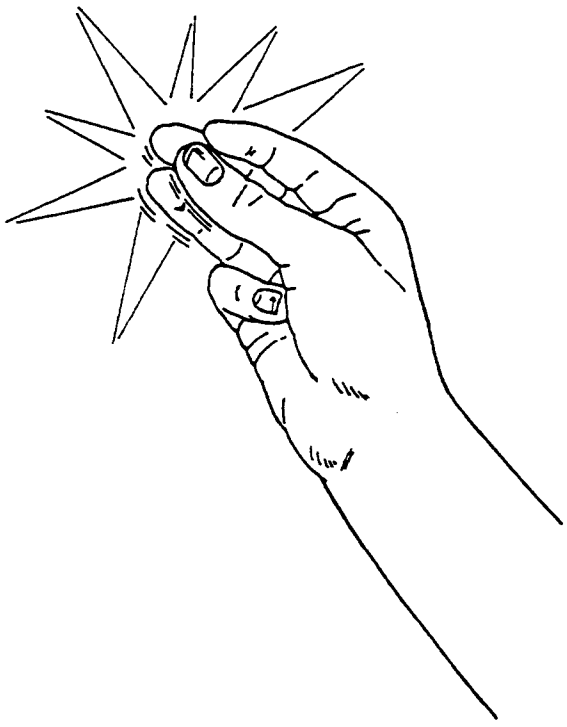
TEST GRADES

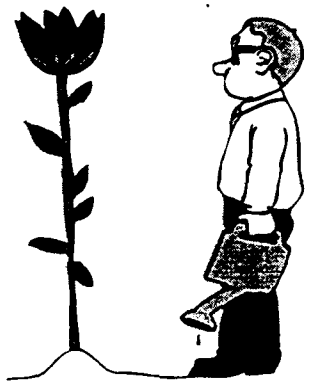
STEM	LEAVES
6	2
7	4 5 7 7 9
8	2 3 5 8 8 8
9	0 1 3 7 9



1. What was the lowest grade on the test? _____
2. What was the highest grade on the test? _____
3. Did any student get a 95 on the test? _____
4. What is the mode of the test grades? _____
5. What grades are shown by the second stem? _____
6. What grade is shown on the third stem, second leaf? _____
7. What is the mean of the test grades? _____
8. What is the range of the test grades? _____
9. What is the median of the test grades? _____
10. Which grades did two students get? _____
11. Did more students get an 88 or a 77? _____
12. Would you rather have the mean or the median as your grade? _____

How Many Times Can You Snap Your Fingers in One Minute?





Steps to Make a Stem and Leaf Plot

1. Put your data in order from least to greatest.

2. Draw a line(s) vertically and horizontally.

3. Label left side Stem and right side Leaves.

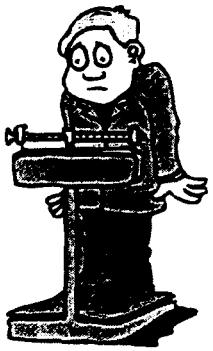
4. Find the least and greatest pieces of data.

5. Write the stems from top to bottom. Put the least stem first.

6. Write in the leaves.

7. Count your data to make sure every piece is there.

8. Title your stem and leaf plot.



Fourth Graders' Weights

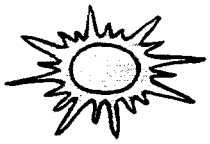
Make a double stem and leaf plot from the following information.

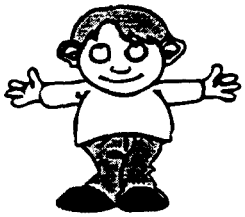
Mrs. Green's Class

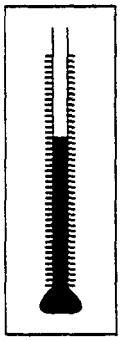
The weights of the students in Mrs. Green's fourth grade class are 40, 54, 45, 38, 44, 42, 45, 50, 62, 48, 50, 45, 52, 60, and 34 pounds.

Mr. Ramirez's Class

The weights of the students in Mr. Ramirez's fourth grade class are 33, 40, 65, 28, 33, 41, 60, 48, 63, 46, 49, 59, 46, 58, 50, 51, 58, 54 pounds.



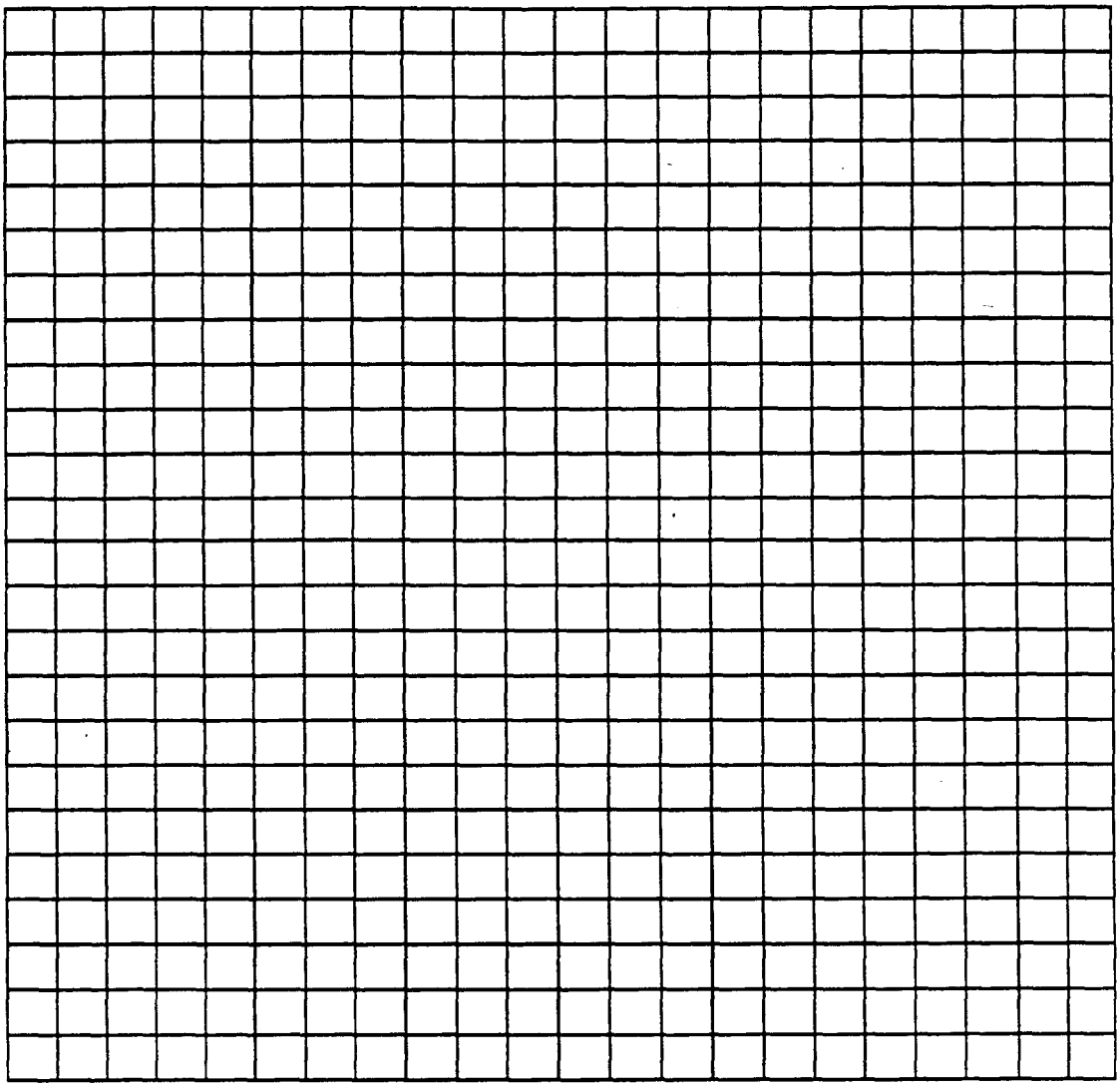




AVERAGE TEMPERATURE

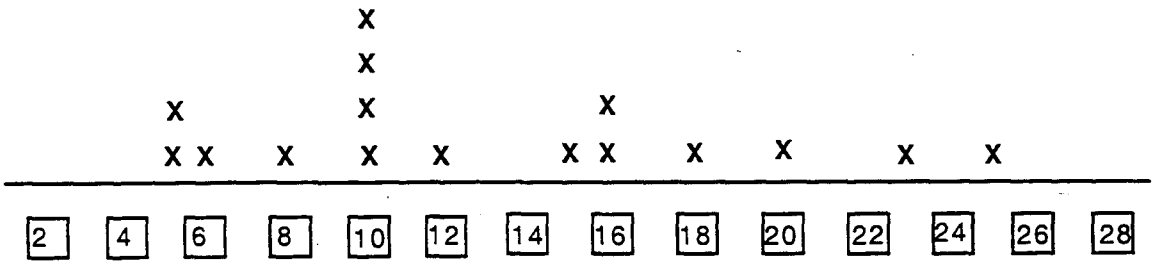
City	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Boston	30	31	38	49	59	68	74	72	65	55	45	34
Chicago	21	26	36	49	59	69	73	72	65	54	40	28
Honolulu	73	73	74	76	78	79	80	81	81	80	77	74
Houston	51	55	61	69	75	81	83	83	78	70	60	54
Juneau	22	28	31	39	46	53	56	55	49	42	33	27
L.A.	57	59	60	62	65	69	74	75	73	69	63	58
Miami	67	68	72	75	79	81	83	83	82	78	73	69
New York	32	33	41	53	62	71	77	75	68	58	47	36
Washington	31	34	42	53	62	71	76	74	67	55	45	35

Graph Paper 1



Answer Key
Data Analysis/Stat. & Prob. - Obj. 8

Line plot (possible solution)



Test Grades

1. What was the lowest grade on the test? 62
2. What was the highest grade on the test? 99
3. Did any student get a 95 on the test? no
4. What is the mode of the test grades? 88
5. What grades are shown by the second stem? 74, 75, 77, 77, 79
6. What grade is shown on the third stem, second leaf? 83
7. What is the mean of the test grades? 84
8. What is the range of the test grades? 37
9. What is the median of the test grades? 85
10. Which grades did two students get? 77
11. Did more students get an 88 or a 77? 88
12. Would you rather have the mean or the median as your grade? The median

Objective 9: Use concrete materials (coins, colored counters, spinners, number cubes, etc.) to estimate, interpret, and write the probability of a given simple event.

Vocabulary

impossible
possible
certain
event
probability
likely
experiment
outcome
favorable
chance

Materials

multilink cubes
overhead colored spinner
two sided counters (red/yellow)

Transparencies:

Probability
Probability 2
Marbles, Marbles

Student Copies:

Impossible?
What is the Probability?
Chance It!
Vocabulary Review

Language Foundation

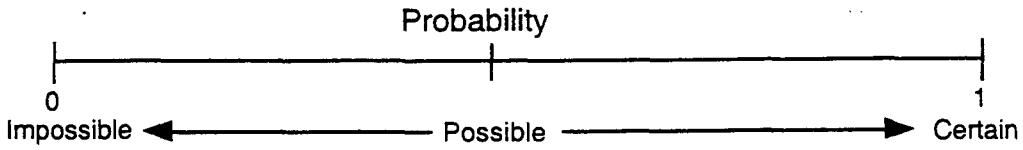
1. This lesson contains a lot of new vocabulary. It may be necessary to spend some extra time going over the vocabulary and having the students practice it. If students have a vocabulary notebook, they can write the words and their definitions for easy reference. There is also a vocabulary review page included at the end of the lesson. Students should be encouraged to use appropriate vocabulary as much as possible and held accountable for learning it.

2. Depending on the level of the students, the prefix "im" could be taught. Explain that "im" means "not." Examples include possible - impossible, perfect - imperfect, polite - impolite, personal - impersonal.

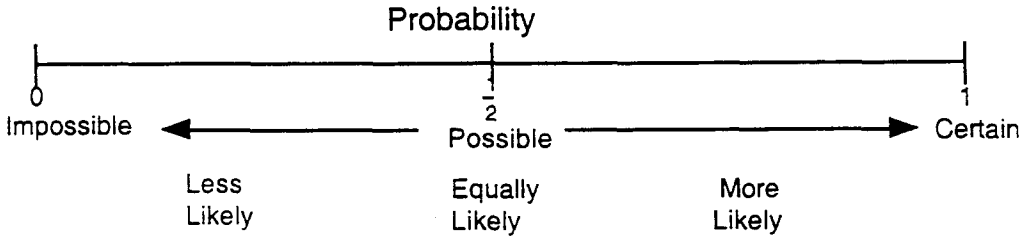
Mathematics Component

1. Introduce the words *event, impossible, possible, and certain.*
   - Write the word event on the board or overhead. Ask students if they know what this word means. (Some students may be familiar with current events.) Explain that an event is something that may or may not happen. Underneath event write the word impossible. Tell students impossible means something that cannot happen. Say to students, "It is impossible for me to kiss my elbow." Demonstrate. Write the following sentences: 1) Cars can fly. 2) 5 + 5 = 20. 3) Our desks can talk. Tell students these are all examples of events that are impossible. Ask students if they can think of other examples of impossible events. Write these down.
   - Write the word possible on the board. Tell students possible is the opposite of impossible. Possible means something can happen. Say to the students, "It is possible one of us has a penny." Take a couple of seconds to see if anyone does. Write the following sentences on the board or overhead: 1) It will rain tomorrow. 2) We will have a fire drill today. 3)_____(your name) will be absent next week. Tell students these are all examples of events that are possible. Ask them if they can think of other examples. Write these down.
   - Write the word certain on the board or overhead. Tell students certain means something will happen for sure. Say to the the students, "It is certain my pencil will fit in the pencil sharpener." Demonstrate. Write the following sentences on the board or overhead: 1) Tomorrow is _____ (put in the date). 2) The sun will come up tomorrow. 3) 3 + 5 = 8. Tell students these are all examples of events that are certain. Ask them for other examples. Write these down.
   - Put ten pink, one blue, and two red multilink cubes in a transparent container. Ask students, "Is it impossible, possible, or certain that I will pick out a pink cube?" (possible) Why? "Is it impossible, possible, or certain that I will pick out a blue cube?" (possible) Why? "Is it impossible, possible, or certain that I will pick out a red cube?" (possible) Why? "Is it impossible, possible, or certain that I will pick out a purple cube?" (impossible) Why?
   - Put an overhead colored spinner on the overhead. Ask students, "Is it impossible, possible, or certain that I will spin red?" (possible) "Is it impossible, possible, or certain that I will spin black?" (impossible) "Is it impossible, possible, or certain that I will spin green?" (possible) "Is it impossible, possible, or certain that I will spin yellow or blue?" (possible) "Is it impossible, possible, or certain that I will spin red, yellow, green, or blue?" (certain)
   - Do other examples with students if they are still having trouble with the words.
   - Give students the Impossible? student activity sheet to complete on their own.

2. Introduce the words probability, likely, more likely, and less likely.
   - Tell students that when we talk about whether or not an event can happen, it is called probability. Show them the Probability transparency. Point out the words Impossible, Possible, Certain, and Probability. Explain to them that when an event is impossible, the probability is 0. When an event is certain, the probability is 1. All other events are between 0 and 1.



- Put four red multilink cubes in a transparent container. Ask students, "What is the probability of picking a red cube?" (certain or 1) Point to Certain on the line. Why? (There are only red cubes in the container.) Ask students, "What is the probability of picking a green cube?" (impossible or 0) Point to Impossible on the line. Why? (There are no green cubes.)
- Exchange one of the red cubes for a green cube. Ask students where on the line they think the probability would be for picking a red cube. Explain to them that because the probability is very high but not certain, the probability would be on the line between Possible and Certain.
- Tell students we call this area More Likely. Write the words More Likely on the transparency between Possible and Certain. Explain that when an event is more likely, it means the probability is closer to certain or 1. Give other examples such as, "It is more likely to rain than snow tomorrow." (or vice versa depending on the weather) Ask students for examples.
- Tell students the probability of picking a green cube is not as good as for picking a red cube. It is less likely. Write Less Likely on the transparency between Impossible and Possible. Tell students that when an event is less likely it means it has a low probability, but it is not impossible. Give students other examples such as, "It is less likely to snow than rain/be sunny tomorrow." (depending on the weather) Ask students for additional examples.
- Show students two red and two green multilink cubes. Ask them what color cube is more likely to be picked. Let them speculate. Tell them since the number of red cubes is the same as the number of green cubes, the probability of picking a red or a green cube is the same or Equally Likely. Write this in on the transparency **under** the word Possible. Explain that when an event is equally likely, it means the event has the same probability of happening as not happening. Ask students what number they think should go **above** the word possible. (Hopefully they will remember their fractions and say $\frac{1}{2}$.) Write this in.



- Ask them if they can think of other events that are equally likely to occur.
- Put 4 red cubes, 2 blue cubes, 2 green cubes and 1 yellow cube in a transparent container. Ask students, "What is the probability of picking a pink cube?" (impossible) "What is the probability of picking a red cube?" (more likely) "What is the probability of picking a yellow cube?" (less likely)
- Pass out What is the Probability? student activity sheet.

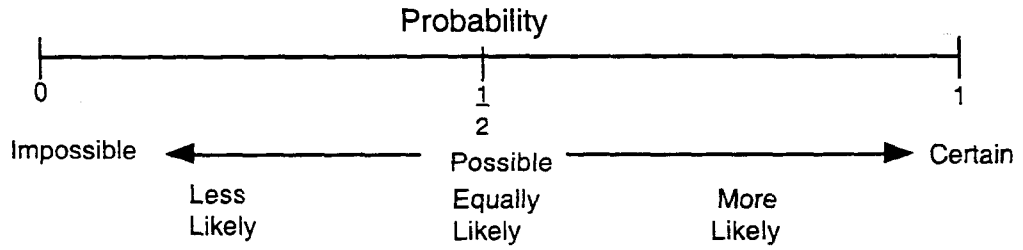
3. Explore the concept of probability.
   - Show students a two sided red-yellow counter. Tell them you are going to toss the counter. Ask them what color they think will come up. Toss the counter and note what color comes up. Ask students what color they think will come up if the counter is tossed again. Toss the counter again and note what color comes up.
   - Tell students they are going to do an <u>experiment</u> to see which color comes up the most. Tell them they are going to toss the counter 20 times and record their results using tally marks. (Remind students what tally marks look like if they have forgotten.) 𝍸 Before students begin, ask them to write down how many of each color they think they will get. i.e. red 15, yellow 5. Give each student or pair of students (depending on your class size) a two sided red-yellow counter. Have students toss their counters and record the results.
   - When everyone is done, compile the results and see how many red and yellow the entire class has. (The numbers should be fairly close.) Discuss the results with the class. Which color came up the most often? Do they think it is less likely, equally likely, or more likely for either color to come up? (equally likely) If the class threw the counters again, would the results be the same? (No, but the numbers for red and yellow should be close.)

4. Introduce the concept of a ratio or fraction to describe probability.
   - Explain to students that an <u>outcome</u> is another word for a possible event. Remind them there are impossible, possible and certain events. Ask students how many <u>outcomes</u> are possible when a red-yellow counter is tossed. (2) What are they? (Red can come up or yellow can come up.) Out of these two events, how many are red? (1) How many are yellow? (1) Tell students the probability of getting red or yellow is <u>one out of two</u>. Red is one <u>outcome</u> out of two possible outcomes so the probability of getting red is one out of two. Yellow is one outcome out of two possible outcomes so the probability of getting yellow is <u>one out of two</u>. On the overhead write "yellow - 1 out of 2, red - 1 out of 2." Tell students we can also write this as a ratio or fraction $\frac{1}{2}$ (which is a ratio). Write next to each color. Explain to students that the numerator is the <u>favorable</u> outcome or the outcome that we want. The denominator tells the possible <u>outcomes</u>.

     Yellow 1 out of 2 $\frac{1}{2}$

     Red 1 out of 2 $\frac{1}{2}$

   - Put up the <u>Probability</u> transparency again. Remind students that $\frac{1}{2}$ is the same as equally likely. Tossing a yellow is <u>equally likely</u> as tossing a red.



- Put up the Probability 2 transparency. Go over the example with students making sure they understand why the probability is $\frac{4}{5}$. Is this a high probability? (yes)
- Put four multilink cubes in a paper bag - one red, one blue, and two green. Show the cubes to the students before putting them in the bag. Tell them we are going to find the probability of picking each color. Ask students, "If I reach in the bag and pick out a cube, how many outcomes are there?" (4) How do you know? (There are 4 cubes in the bag.) How many outcomes are red? (1) What is the probability of picking a red cube out of the bag? (1 out of 4) On the overhead write:

  Red 1 out of 4 $\frac{1}{4}$

  Tell students the probability of picking a red cube is 1 out of 4. Another way to say this is, "There is a one out of four chance of picking a red cube." Have students repeat this. Ask students if they have heard the word chance before and where. (Some of them may know the word chance from games they have played.) Explain that chance means possibility. "There is a chance it will rain tomorrow." "There is a small chance you will not have homework today." "There is no chance you will not have homework for the whole week." Ask students to generate a sentence or two on their own.
- Now ask students what the probability is of picking a blue cube. (I out of 4) If they have trouble put up the Probability 2 transparency again and ask them the number of outcomes and the number of favorable outcomes. On the overhead write:

  Blue 1 out of 4 $\frac{1}{4}$

  Have students repeat, "There is a one out of four chance of picking a blue cube."
- Ask students what the probability is of picking a green cube. (2 out of 4) Write green on the overhead and next to it $\frac{2}{4}$. (Note: Some books reduce the fraction to the simplest form. You may or may not want to do this depending on the students in your class. It is important for students to remember two out of four (favorable outcomes out of possible outcomes) which they may not if the fraction is simplified.) Ask students what color has the greatest chance of being picked. (green) Why? (There are more green cubes than red or blue and the probability for green is the highest.)

  Red 1 out of 4 $\frac{1}{4}$

  Blue 1 out of 4 $\frac{1}{4}$

  Green 2 out of 4 $\frac{2}{4}$

- Ask students what the probability is of picking red **or** blue. (two out of four) If they have trouble with this, ask them how many cubes are red or blue. (2) Is this number the numerator or the denominator? (numerator) How many cubes are there in total? (4) Where does this number go? (in the denominator) On the overhead write:

  Red or Blue 2 out of 4 $\frac{2}{4}$

- Ask students what the probability is of picking blue **or** green. (three out of four) On the overhead write:

  Blue or Green 3 out of 4 $\frac{3}{4}$

- Ask students what the probability is of **not** picking green. How many cubes are **not** green? (2) How many cubes are there? (4) What is the probability? (2 out of 4) On the overhead write:

  Not Green 2 out of 4 $\frac{2}{4}$

- If students are still having difficulty, do other examples with the multilink cubes.
- Put up the Marbles, Marbles transparency. Ask students what the probability is of picking a white marble. (four out of seven) Why? What does the 4 represent? (favorable outcomes or white marbles) What does the 7 represent? (possible outcomes) On the line next to White, write $\frac{4}{7}$.

  White 4 out of 7 $\frac{4}{7}$

- Ask students what the probability is of picking a black marble. (two out of seven or $\frac{2}{7}$) Why? What does the 2 represent? (2 black marbles) What does the 7 represent? (possible outcomes) Write $\frac{2}{7}$ on the line next to Black. Continue through the list having students explain their answers.

Black	2 out of 7	$\frac{2}{7}$
Striped	1 out of 7	$\frac{1}{7}$
Not White	3 out of 7	$\frac{3}{7}$
Not Black	5 out of 7	$\frac{5}{7}$

- Ask students which type of marble has the greatest chance of being picked. (white) Why? (There are more white marbles than black or striped.) Which type of marble has the least chance of being picked? (striped) Why? (There are fewer striped marbles than black or white.)
- Assign the Chance It! student activity sheet.

Name _____

IMPOSSIBLE?

Read each event. Circle the correct answer.

1. My mother will be 5 years old.  Impossible Possible Certain

2. Friday comes after Thursday. Impossible Possible Certain

3. It will snow tomorrow.  Impossible Possible Certain

4. I will eat an apple today.  Impossible Possible Certain

5. 7 - 3 = 4. Impossible Possible Certain

6. I will have homework today. Impossible Possible Certain

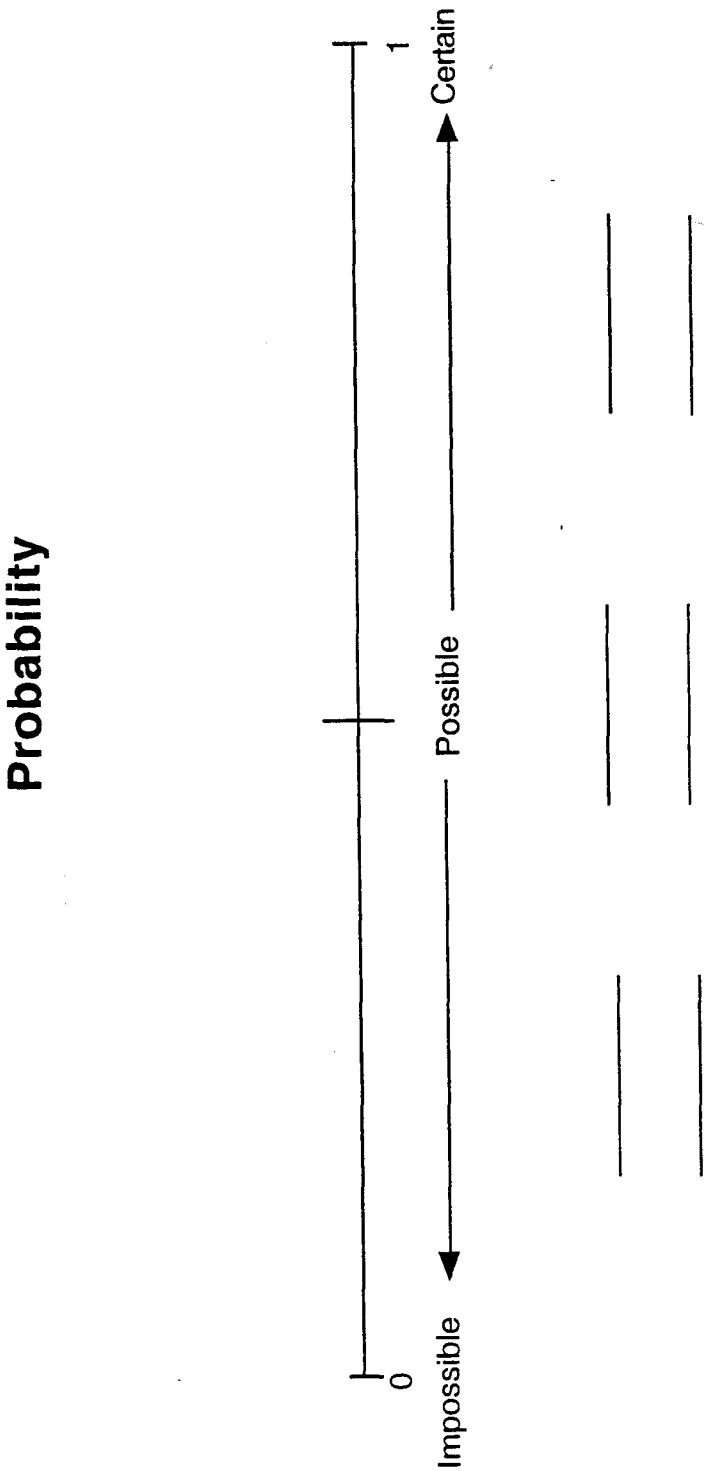
7. I will get an A in math. 

Impossible Possible Certain

8. My book will talk.  Impossible Possible Certain

9. The sun will come up tomorrow.  Impossible Possible Certain

10. I will talk to my friends today. Impossible Possible Certain


Probability
1
Certain
Possible
Impossible
0

Name _____

WHAT IS THE PROBABILITY?

Read the problem and circle the correct word. Give a reason for your answer.

Last night your mother made your lunch. There were 6 bananas, one apple and one bunch of grapes on your kitchen table.

1. What is the probability that she put a banana in your lunch? Why?

Impossible Less Likely More Likely Certain

_____

2. What is the probability that she put an apple in your lunch?

Impossible Less Likely More Likely Certain

_____

3. What is the probability that you she put an orange in your lunch?

Impossible Less Likely More Likely Certain

_____

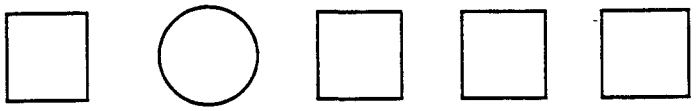
4. What is the probability that she put grapes in your lunch?

Impossible Less Likely More Likely Certain

_____

PROBABILITY 2

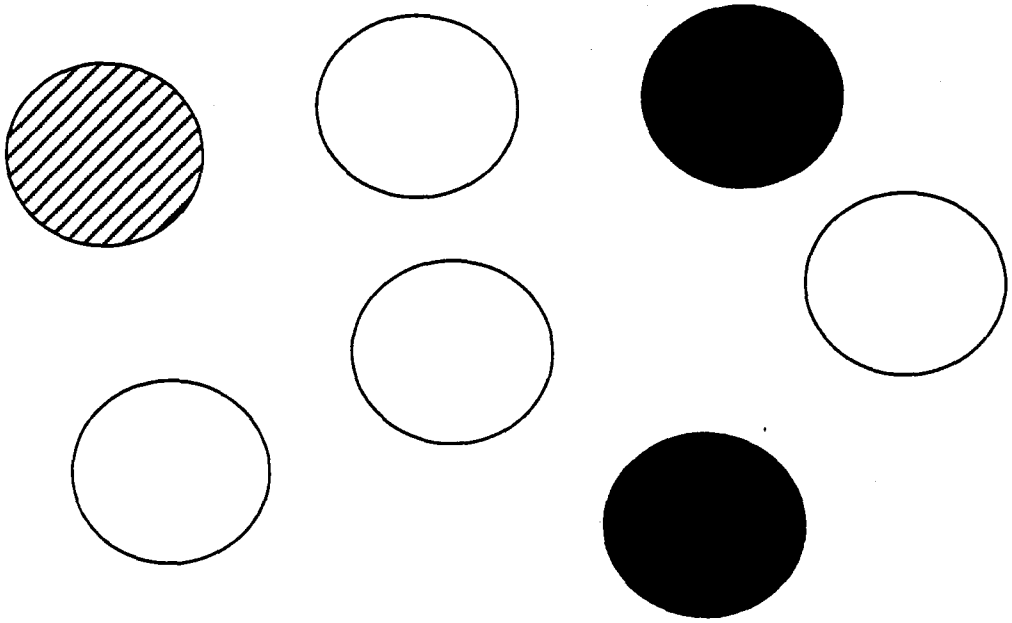
Probability can be written as a ratio or a fraction.



What is the probabilty of picking a square?

Favorable Outcome → $\frac{4}{5}$ ← Possible Outcomes

MARBLES, MARBLES



White _____

Black _____

Striped _____

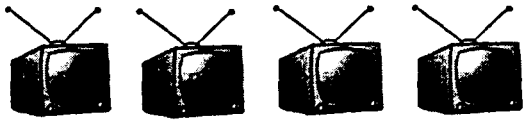
Not White _____

Not Black _____

Name _____

CHANCE IT!

Read the problem and write the probability on the line next to the problem.

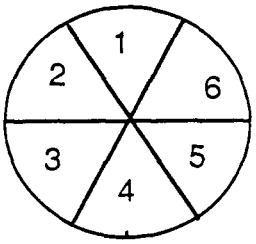


1. A customer walks into a television store. There are four televisions on the table. Three are color and one is black and white. What is the probability that the customer will turn on a black and white television? _____

2. Miguel has three green shirts, two blue shirts and four yellow shirts. What is the probability that he will wear a yellow shirt to school?

_____

3. Bekir spins the spinner at the right. What is the probability that he will get a 6? _____ an even number? _____ an odd number? _____ a number less than 3? _____



4. For her birthday, Mai's mother takes her to the ice cream store. The store sells chocolate, vanilla, strawberry, peach and bubble gum ice cream. What is the probabilty that Mai will pick peach or bubble gum? _____ What is the probability that she will **not** pick vanilla? _____

Name _____

VOCABULARY REVIEW

impossible	favorable	chance
possible	certain	event
probability	likely	experiment(s)
outcome(s)		

Use the words from the box above to fill in the blanks below.
Some of the words will need the plural form.

1. _____means something cannot happen.
2. An _____is something that may or may not happen.
3. What is the _____there will be french fries for lunch?
4. _____ means something has a good chance of happening.
5. When you toss a coin, there are two _____.
6. _____means an event **will** happen.
7. When an outcome is _____, it is the one you want.
8. _____means an event **can** happen.
9. The _____ is one out of four.
10. In science class we sometimes do _____.

Answer Key
Data Analysis/Stat. & Prob. - Obj. 9

Impossible?

1. impossible
2. certain
3. possible
4. possible
5. certain
6. possible
7. possible
8. impossible
9. certain
10. possible

What is the Probability?

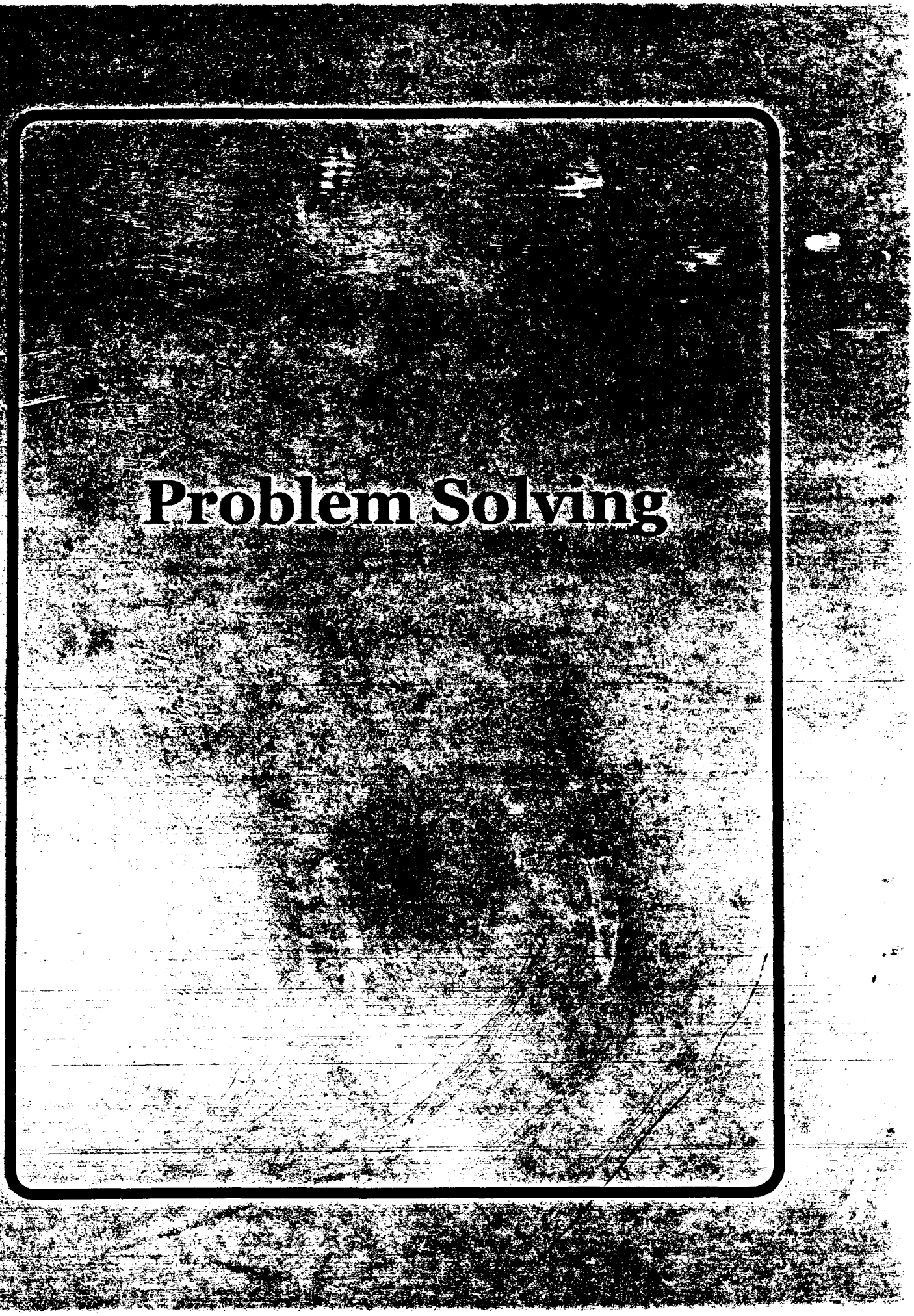
1. more likely
2. less likely
3. impossible
4. less likely

Chance It!

1. $\frac{1}{4}$ or one out of four
2. $\frac{4}{9}$ or four out of nine
3. $\frac{1}{6}$ or one out of six, $\frac{3}{6}$ or three out of six, $\frac{3}{6}$ or three out of six, $\frac{2}{6}$ or two out of six
4. $\frac{2}{5}$ or two out of five, $\frac{4}{5}$ or four out of five

Vocabulary Review

1. impossible
2. event
3. chance
4. likely
5. outcome
6. certain
7. favorable
8. possible
9. probability
10. experiments


Problem Solving

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Problem Solving

Introduction to Problem Solving

The objectives in this unit require students to apply the mathematics learned, both in routine and non-routine situations. Underlying all the objectives in this unit is the development of thinking skills, mathematical curiosity, and the willingness to explore options and take risks. In this unit, students are taught how to organize information and apply strategies to various situations. Although problem solving is often a great challenge to second language learners, if they master the strategies below, they can gain a sense of some control over the problems. They should read and talk about problems. They should evaluate their solutions and make changes when necessary. While some of the problems can be solved by computation, most are classified as non-routine. However, in either case alternative solutions and creative thinking are encouraged.

What is problem solving? According to Krulick and Rudnick, "Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The individual must synthesize what has been learned and apply it to the new situation." The process involves five steps:

1 - First the student must **isolate the question** being asked.
2 - Then the student **isolates the pertinent data** from the given situation.
3 - The student must then **choose a strategy** to use to solve the problem.
4 - Using the strategy, the student now **solves** the problem.
5 - Finally, the student **evaluates the solution** to see if it answers the question in step one. (If it does not, the process is repeated.)

There are two kinds of problems in mathematics. The first type, and by far the most common, is the one found at the bottom of the page in a math book that requires the student to apply the skill being practiced at the top of the page. The second type, the focus of this unit, is the non-routine problem. When solving these problems, students may find more than one strategy that works, more than one solution, or even no solution. Even students with a background in mathematics education in their native countries may not have had much experience with non-standard problems. Students accustomed to math classrooms which are highly teacher-directed may actually be resistant to problems which require multiple solutions.

In addition to teaching the problem solving process and how to use the strategies, help the students understand the problem by teaching the following:

1 - *Highlight the question.* Help the students isolate what is being asked.
2 - *Restate the problem in your own words.* Doing this may help to break the language into more meaningful pieces.
3 - *Get rid of extraneous material.* Help the students cross out any information that is not needed to focus on the pertinent information.

4 - Ask the students if they can *think of a similar problem* they solved in the past.

5 - *Remove all the numbers and then discuss the problem.* Help the student focus on the "action" involved in the problem. After they have decided what needs to be done, put the numbers back and discuss the problem again.

6 - *List the data.* Making a list of the data in the problem may help students see not only what information is there but how it is related.

The remainder of this unit is structured around ten problem solving strategies. Each one is a separate objective and the vocabulary, materials, and language foundation are presented for each. An introductory mathematics component is presented, followed by sample problems requiring use of the strategy. These problems increase in difficulty, not only in the level of thinking required, but also in the language used.

Using cooperative groups to solve the problems will allow all students to work despite differences in language development and computational skill levels. Working in groups will also facilitate math language development while building thinking skills.

The problems can be used as a mini-unit over several days to reinforce a particular strategy. Another method would be to teach each strategy for 2 or 3 days and then combine the remaining problems from all the strategies to allow students to practice choosing a strategy. Spread throughout the year, this method reinforces thinking skills with a variety of objectives.

Classroom Strategies for Teaching Problem Solving

1. Begin with problems at an appropriate level of difficulty.

2. Choose problems consistent with the students' experience, interest, knowledge, and understanding.

3. Incorporate problems from everyday life.

4. Begin with a problem solving plan that reflects the recursive nature of problem solving.

5. Model the process of "getting started" on difficult problems.

6. Teach students a variety of problem solving strategies.

7. Encourage students to consider different strategies to solve a problem.

8. Devote time to problem solving and provide an ample supply of problems for practice.

9. Help students become aware of similarities and differences among problems.

10. Allow students to work with each other and share their strategies with the class.

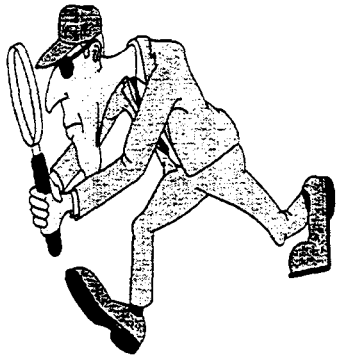
11. Incorporate problem solving throughoutthecurriculum.

12. Give equal importance to the "process" and "getting the answer."

13. Differentiate instruction by modifying problems, not the "process."

14. Include the use of calculators where appropriate.

Problem Solving Strategies



Act it Out

Draw a Picture

Find a Pattern

Make a List or Chart

Guess and Check

Work Backwards

Solve a Simpler Problem

Problem Solving Objectives

1 - Use the 5 step problem solving process to solve both routine and non-routine problems.

2 - Act out or use objects to represent and solve both routine and non-routine problems.

3 - Draw a picture to solve both routine and non-routine problems.

4 - Use guess and check to solve routine and non-routine problems.

5 - Find a pattern to solve routine and non-routine problems.

6 - Make a list to solve routine and non-routine problems.

7 - Make a table to solve routine and non-routine problems.

8 - Use logical reasoning to solve routine and non-routine problems.

9 - Solve routine and non-routine problems by solving a simpler problem.

10 - Solve routine and non-routine problems by working backwards.

11 - Determine whether there is just enough, not enough, or more than enough information to solve a problem.

Objective 1: Solve problems using a logical procedure (a plan)

Vocabulary

understand
plan
solve
evaluate
strategy
logical
reasonable

Materials

The Problem Solving Plan (TR)
transparency master

Problem Solving Plan A (TR)
Problem Solving Plan B (TR)
student copies

The Apple
The Soccer Game
student copies

Language Foundation

1. Ask students to think of a recent trip they took either in the U.S. or from their country to the U.S. Lead students to remember what was important to think about when planning for this trip. Let volunteers share their plans with the class. Tell students that they are going to use a **plan** as a process (or way) to solve word problems.

2. Certain vocabulary (in **bold** type) should be taught as each step of the plan is introduced. The vocabulary is defined in the lesson. This is an excellent opportunity to refer to *cognates* which students may be familiar with in their own languages, especially Spanish speakers.
   understand = comprehend
   answer = response
   look back = reflect, think, consider
   choose = select
   Using *synonyms* and short explanations may also be helpful. For example, organize = put together, arrange in order.

3. Help students understand the concepts of **logical** and **reasonable** by asking then the following choice questions: If there is an item on sale at one store, but not on sale at another store, from which store would you prefer to buy the item? If it is rainy and dark outside, would you prefer to go out to play or stay inside? Explain that these are logical choices which make good sense.

Mathematics Component

Note:

This lesson is designed to introduce students to the problem solving process. It is being presented early in the curriculum so that problem solving can become an integrated part of the curriculum and problems can be tied to the concepts students are learning. This first lesson focuses on the basic four-step problem solving process: understand, plan, solve, look back. A recording sheet will be used as a tool to help students learn to use a logical procedure to solve problems. Two different problem solving plans (A and B) are provided for students' use. Plan B requires more reading and writing. Teachers should choose the plan which seems **most appropriate** based on the students' English proficiency levels. An additional transparency master, The Problem Solving Plan, has been included for reinforcement/review. Making a poster of this plan, posting it in the room, and reviewing it frequently will help students recall the steps and strengthen their problem solving skills throughout the year. Specific problem solving strategies will be introduced throughout the curriculum and will emphasize use of the plan. It will also help students to post specific problem solving strategies as they are introduced.

1. Give each student a copy of The Apple Problem and a copy of one of the problem solving plans, A or B, which you have chosen according to the English proficiency level of **most** of the students in the class. (It is easier to introduce the plan if all students initially use the same version; however, students may use different versions at other times.)

   UNDERSTAND:

   a. Read the problem with the students using a transparency copy to point to the words as it is read. Display a transparency copy of the plan being used by the students and point to the word **understand**. Explain that in order to solve The Apple Problem, we must first be sure that we know all of the words in the problem. Lead students through each of the steps required to "understand" the problem.

Understand includes:

1) find the meaning of any words that are unfamiliar,
2) retell what is happening in your own words, and
3) tell what the problem is asking.

b. Look back at The Apple Problem and **clarify any words which might be unfamiliar** to students. Then ask, "What is happening in this problem?" Explain that **data** is the information which answers the question. Data is the numbers in the problem and information about the numbers.

c. The second part of the understand process may be confusing for students. Teaching them to "retell the problem in their own words" may best be accomplished by covering up the problem and then asking a student to tell what the problem said. If the student does not include all of the necessary information, go back to the problem and circle the numbers on the apple problem transparency. Tell students that just finding the numbers is not enough. Say, "We need to know what is happening to the numbers." Go back and underline "has 2", "gives him 3 more," and "eats 2." Make sure that the students understand what each phrase means. Model how to summarize this information in sentences on the transparency. (Example: A boy has 2 apples. He gets 3 more apples. He eats 2 apples.)

d. Read the sentences together and then ask, "What does the problem ask? What is the question?" (They are asking how many apples are left or how many remain.) Add this information to what was previously recorded and reread all of the information together, orally reviewing the steps included in understanding the problem. Ask students to copy this information below the word understand on their individual copies. An example of student work might be as follows; however, the ability to copy will depend on each individual student's English proficiency level:

Understand

A boy has 2 apples. He gets 3 more apples. He eats 2 apples. They are asking how many apples are left.

PLAN:

a. Point to the word, **plan**, and say "decide what to do."

Plan includes:

1) decide what to do with the data to answer the question,
2) choose a special strategy, if needed

Note: **Skip over** the second step where students choose a special strategy. A special strategy is **not** needed with this lesson. See the Problem Solving Unit for a list of the strategies that will be taught later.

b. Explain that sometimes our plan just requires us to add, subtract, multiply, or divide to solve the problem. Other times, we might have to do more.

c. Ask students what they think we should do to solve (get the answer for) this problem. Elicit add and subtract, model filling in these words below the word plan, and have students copy onto their individual papers.

SOLVE:

a. Tell students that we are now ready to solve the problem and point to the word **solve** on the transparency as you say "get the answer."

Solve includes:

1) write or show the math work to solve the problem and
2) write the answer to the problem.

b. Ask students to solve the problem on a piece of scrap paper and then share their results. Ask one student to describe what they did to solve the

problem. As they explain, write the equations 2 + 3 = 5 and 5 - 2 = 3 on the transparency below the word solve. Have students copy this onto individual plans. Then model for students how to write a sentence using the answer. (The boy has 3 apples left.)

LOOK BACK:

a. Show students that the first three steps are done and now we are ready to **look back**. Look back means we will "think about" our answer. We need to make sure that we have answered the question and that the answer is makes sense.

Look back includes:

1) check to see if the answer makes sense and
2) try another way, if needed, to see if we get the same answer.

b. Ask students if they feel that the answer 3 makes sense for this problem and why. Show them that one way to check this answer might be to draw what happened. Draw 2 apples. Cross out 2 apples. Ask how many are left. Compare this answer with the one we got in step three. Ask if 3 apples is the correct answer. (Yes)

2. Use the transparency master/poster, The Problem Solving Plan, to review what was done for each step of the plan. In addition to posting this plan, if students are keeping a notebook, it might be helpful for them to have individual copies of this basic plan.

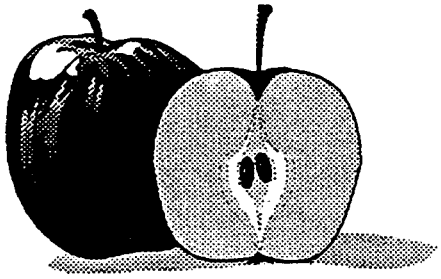
3. As a follow-up lesson, have students work together in pairs to complete a problem solving plan for The Soccer Game. When the students have finished the plan, share answers reinforcing the process and vocabulary for each of the four steps in the problem solving plan.

Note:

These written problem solving plans should be used throughout the year until the students become comfortable with the steps and show evidence of

internalizing the procedure when solving problems. Additional formats for this plan will be offered at later times. Exposure to several different versions of the plan will add variety to the lessons; however, plans should always be chosen with an understanding of students' individual English proficiency levels.

The Apple Problem



Carlos has 2 apples.
His mother gives him 3 more apples.
Then he eats two apples.
How many apples does he have left?

The Soccer Game

8 people are playing soccer.
2 people go home.
Then 4 more people come to play.
How many people are playing soccer now?

Problem Solving Plan

Understand

- check words you don't know
- retell the problem in your words
- tell what the problem is asking

Plan

- decide what to do with the data
- choose a special strategy, if needed

Solve

- write or show the math work to solve the problem
- write the answer

Look Back

- check to see if the answer makes sense
- try another way, if needed