FAST Math Curriculum

Volume 2

Department of Instructional Services

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FAST Math Curriculum

Volume 1

Volume 2

FAST Math Curriculum

Volume 1

General Information

EAME

Number Concepts and Theory

Operations: Whole Numbers
Addition and Subtraction

Operations: Whole Numbers
Multiplication and Division

Volume 2

Fractions

Measurement

Geometry

Data Analysis/Statistics/Probability

Problem Solving

Professional Resources

Fractions

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Fractions

Objective 1: Divide shapes into 2, 3, 4, 5, 6, 10 and 12 equal parts. Use the words halves, thirds, fourths, fifths, sixths, tenths, twelfths to describe the equal parts into which the shape is divided. Know that halves are equal only when the shapes or groups are equal in size.

Vocabulary

fraction
divide
half
halves
part
third
fourth
fifth
sixth
tenth
twelfth

Materials

Overhead
Geoboards
Overhead Geoboard
5"x 8" cards with
denominators
Fraction Bars -
1 set/per student pair
Overhead fraction bars

Language Foundation

1. Discuss the word fraction. A fraction is a number that tells us what part of something we are talking about. Explain that in the United States we use fractions to tell about parts of things instead of using decimals. Discuss where they may have seen fractions such as in measurements (2 1/2 in.), in recipes (2/3 cup of sugar), in a store (one-half off), on tools (a 7/12 socket wrench). Tell them that in this unit we will learn about fractions and how to add, subtract, multiply, and divide them.

2. Review the concept of dividing - breaking (separating , cutting) things, including numbers, into equal parts. Give an example such as dividing 6 into 3 equal groups of two.

Operations Objectives
(Draft)

Fractions		SOL Link
	Addition and Subtraction:	
Obj. 1	Explore equivalent fractions. Order and compare fractions to solve problems.	NS 4.2 NS 4.3
Obj. 2	Use concrete materials to add and subtract fractions and mixed numbers with like denominators (halves, thirds, fourths, fifths, sixths, eights, tenths,and twelfths).	CE 3.11 CE 4.10
Obj. 3	Add and subtract fractions and mixed numbers with like denominators. Record answers in simplest form.	CE 4.10 CE 5.7
Obj. 4	Use concrete materials to add and subtract fractions and mixed numbers with unlike denominators (halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths).	CE 4.10 CE 5.7
Obj. 5	Add and subtract fractions and mixed numbers with unlike denominators. Use rounding to the nearest half or whole number to estimate and check reasonableness of answers.	CE 5.7
Obj. 6	Use calculators to solve problems adding and subtracting fractions.	CE 4.10
	Decimals:	
Obj. 7	Estimate and use models to add and subtract decimals to hundredths.	CE 4.10
Obj. 8	Add and subtract decimal numbers through thousandths.	CE 4.10

Obj. 9	Use calculators to solve problems adding and subtracting decimals, including money amounts.	CE 4.5
Obj. 10	Choose an appropriate tool (calculator, mental math, estimation, paper/pencil) to solve problems using addition and subtraction.	CE 4.10

Operations Objectives
(Draft)

Fractions		SOL Link
	Multiplication and Division:	
Obj. 1	Use concrete materials to multiply fractions and whole numbers by fractions.	CE 6.6
Obj. 2	Estimate and multiply fractions and whole numbers by fractions.	CE 6.6
Obj. 3	Find fractional parts of a group of up to 100 objects.	
Obj. 4	Use models to multiply mixed numbers by whole numbers and fractions.	CE 6.6
Obj. 5	Use concrete materials to model and record division of whole numbers and fractions by fractions.	
Obj. 6	Estimate and divide mixed numbers by fractions and mixed numbers.	
Obj. 7	Use fractions to find percent of a number.	
Obj. 8	Use fractions to solve problems - (multiplication and division).	CE 6.6
	Decimals:	
Obj. 9	Estimate and multiply 1- and 2-digit decimal numbers by 1-digit whole and decimal numbers.	CE 5.4
Obj. 10	Multiply a 2-digit decimal by a 2-digit whole and decimal numbers.	CE 5.4

Obj. 11	Use decimals to find percent of a number.	
Obj. 12	Multiply whole numbers and decimal numbers by powers of ten.	CE 5.4
Obj. 13	Estimate and divide decimal numbers using 1- and 2-digit whole number divisors. Use multiplication to check division.	CE 5.6 CE 6.6
Obj. 14	Divide whole and decimal numbers by decimal numbers. Round answers to the nearest hundredth.	CE 6.6
Obj. 15	Use calculators to solve multi-step problems dividing decimals.	CE 6.7

Mathematics Component

Give each student a geoboard. Have the students form a square with 3 pegs on each side. Check the squares. Tell the students to stretch another rubber band to divide the area inside that path into 2 equal parts. Share the different ways students did this. Using the same square, have the class divide the area into 4 equal parts and share these. Explain that there is a special math symbol used to tell us to divide into equal parts. Draw — on the overhead. Explain that when the sign has a number beneath it, it means to divide into that number of equal parts. Put the overhead geoboard with a square divided into 4 equal parts on the overhead. Write 4 and tell them this means to divide into 4 equal parts "like this square". Explain that each part is called a "fourth". Write the word "fourth" on the overhead pointing out the word four with the th added. Tell them the "th" means to divide and the 4 tells us how many equal parts to divide into. Relate this to tenths and hundredths which they use when working with decimals. Remove one rubber band so that two equal parts are left. Ask how many equal parts are here. Elicit 2. Ask how you should write this; then write 2. Explain that each part is a half. 2 means divided into halves.

Have the students clear their geoboards and then make a rectangle that has 5 pegs on the top and bottom and 4 pegs on the 2 sides. Model this using the overhead geoboard. Show them the 3 card. Tell them this says to divide into thirds. Ask them to divide their rectangles into 3 equal parts, or "thirds", and show them to you.

Show them the 5 card. Ask how many equal parts they should divide this into. Elicit 5. Tell them that 5 equal parts are called fifths. Repeat with sixths, tenths and twelfths.

Collect the geoboards as you pass out sets of fraction bars to each pair of students. Have each pair find the 13 orange bars and arrange them in order from a 0 bar to the whole bar as you do it on the overhead. (See Fraction Bar Teachers Guide p.1) Ask them how many parts each orange bar is divided into (elicit 12) and how to write this (elicit 12). Remind them that 12 equal parts are called twelfths. Repeat for the white bars (tenths), red bars (sixths), purple bars (fifths), blue bars (fourths), yellow bars (thirds), and green bars (halves).

Have each student choose two bars (each a different color). Have them write a description of each of their bars. If there is time, have students exchange descriptions and see if they can find the correct bars based on the descriptions.

12

2

10

4

6

3

5

Objective 2: Identify a given portion of a shape or group using the correct oral and written fraction. Know that 2/2, 3/3, etc. describe the whole.

Vocabulary

numerator
denominator
out of

Materials

4 pieces of paper
-1 folded in halves
-1 folded in two parts that are <u>not</u> halves.
-1 folded in thirds
-1 folded in 3 parts that are <u>not</u> thirds
12 unit blocks
Overhead projector
Overhead pen
Fraction bars
1 set/student pair
Overhead fraction bars
Fraction Bingo mat - 1/student
1 set of fraction cards
Fraction Bars Worksheets, pp. 1 & 6
1/student
Bingo markers- 5/student
Representing Fractions Worksheets R-1 through R-4

Language Foundation

1. When you discuss unequal parts, discuss the prefix *un* which means "not". Help the students think of other words beginning with *un* such as *unfriendly, unhappy,* and *unsafe.*

2. When you begin naming fractions and use the terms <u>numerator</u> and <u>denominator</u>, help the students remember which is which. One way might be that the *bottom number* tells how many parts the thing is <u>divided</u> into. *Divide* starts with *D* and so does *denominator*. A wall chart with a large fraction, the words "numerator" and "denominator" and definitions will also help.

Mathematics Component

Show the students the paper folded in half. Ask how many equal parts there are and how to write this (2). Write this on the board or overhead. Show them the piece folded in 2 unequal parts. Ask them how many equal parts there are. Discuss that when the parts are *unequal*, we do not have fractions and can't use 2 to describe the parts. Show them both pieces of paper divided into 3 parts and ask which one shows you 3 or thirds. (Continue if needed until the students understand that fractional notation indicates division into equal parts.)

Put 12 unit blocks on the overhead projector. Write 4 beside them. Ask how many equal groups you should form. Elicit 4. Sort the blocks one by one into 4 groups. Ask how to check for equality. Regroup the blocks and repeat for sixths writing 6 on the overhead. Repeat for thirds. Then do fifths (5), dividing the blocks into 2 groups of 3 and 3 groups of 2. Explain that because the groups are not equal we can't divide 12 into fifths.

Pass out the fraction bars. Have the students find the 7 red bars. Ask how many equal pieces each bar is divided into. Write 6 on the overhead. Remind them the six says the bar is divided into sixths. Write the word *denominator* on the overhead (or use the wall chart suggested in Language Foundation #2) and explain this number under the line is called the denominator and tells us the number of parts something is divided into. Ask what makes each one of these bars different. Elicit the number of pieces that are shaded. Put the 4/6 bar on the overhead. Tell them that four of the six parts are shaded. Repeat 4 out of 6 are shaded. Write 4/6. Tell them that the four which tells the number of pieces we are talking about is called the numerator. Write the word on the overhead or use the chart and repeat the words. Ask them to find the bar that has two pieces out of 6 shaded. Write 2/6. Show them the zero bar (no parts shaded) and ask what they think you should write. Elicit $0/6 = 0$. Write 6/6 and ask the students to find the card that shows this fraction. Write $6/6 = 1$ explaining that when the numerator and the denominator are the same, we are talking about all the parts or the whole thing which is one.

Have students put away the red bars and get out the bars that show tenths (white). Have the students arrange them in order from 0/10 to 10/10 on their desks as you do it on the overhead. Have them read the bars with you (1/10, 2/10, 3/10,...) Have them locate one or two of the bars as you write the fractions on the overhead such as 7/10.

Have students put away fraction bars as you give each one a fraction Bingo mat. Have the students first work in pairs holding a sheet between them. As one student tells the fraction shaded on each circle from left to right, the other student checks the answers on the back. They then switch roles using the other mat. (You may need to model this activity with a student in front of the class.) While students are doing this, give each student five bingo markers.

Play 5 Bars Bingo. Draw a card from the deck and read the fraction to the class. Any student with that bar puts a marker beside it. As you play the game, circulate and let students take turns drawing and reading the fraction cards. (Use this to assess their ability to read fractions.) Continue until someone has a marker by all 5 bars.

Worksheets 1 and 6 may be assigned for classwork or homework. Be certain that the students know what to do on each sheet.

Collect all Bingo materials and pass out Representing Fractions Worksheets (R-1/R-2). Go over R-1 with the students, and have them shade in the indicated portion of each circle. Reinforce the concept of equal parts. As the students complete the worksheet, write each fraction on the board in numerical and written form. Some of the problems on R-2 should be easy for the students to draw and shade (1/2, 1/4, 3/8); however, the 1/3 may take some practice. You might point out that the sixths can be drawn easily once the thirds are drawn, and the tenths can be drawn by dividing each fifth in half.

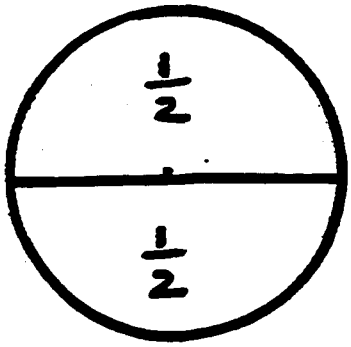
Worksheets R-3 and R-4 provide the students with further practice in drawing and shading parts of a whole, as well as writing the fractions in English. These may be assigned as classwork or homework activities.

Name _____

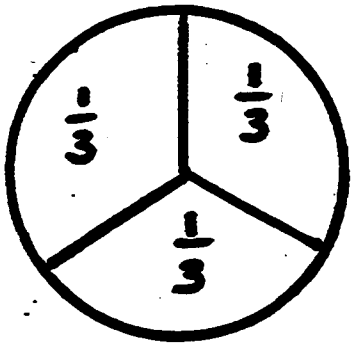
REPRESENTING FRACTIONS

Shade in the following fractions:

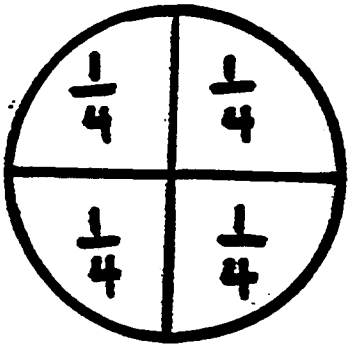
1) $\frac{1}{2}$



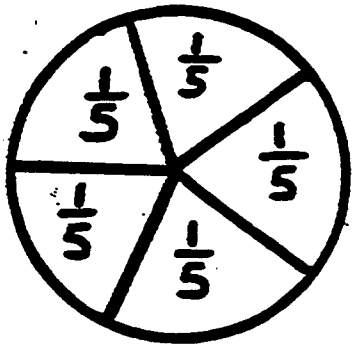
2) $\frac{1}{3}$



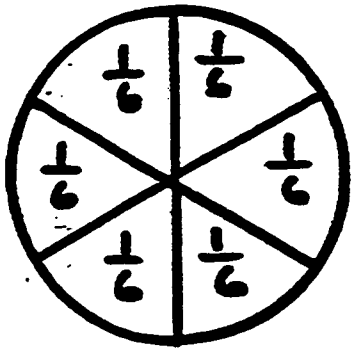
3) $\frac{3}{4}$



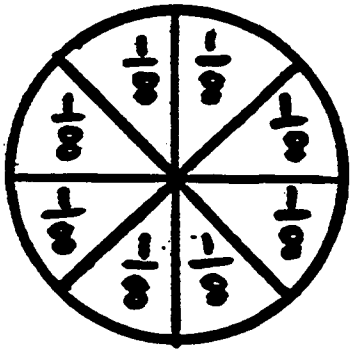
4) $\frac{3}{5}$



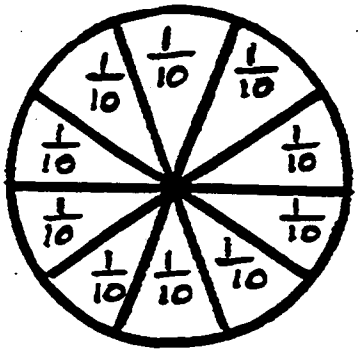
5) $\frac{1}{6}$



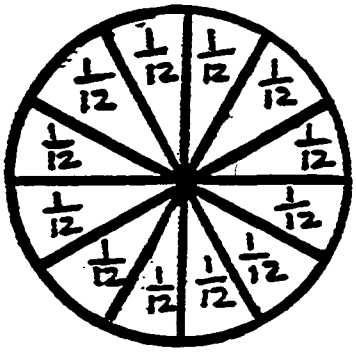
6) $\frac{5}{8}$



7) $\frac{5}{10}$

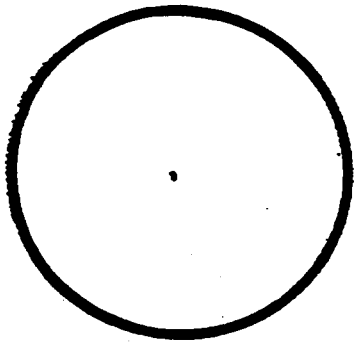


8) $\frac{3}{12}$

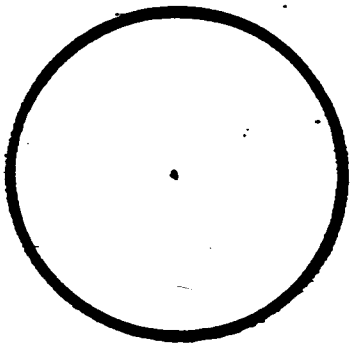


Draw and shade in the following fractions:

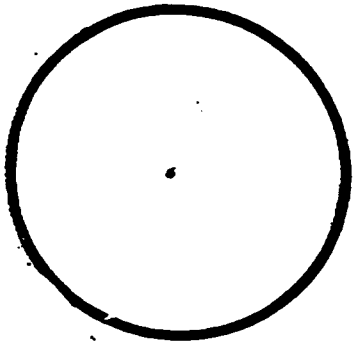
1) $\frac{2}{2}$



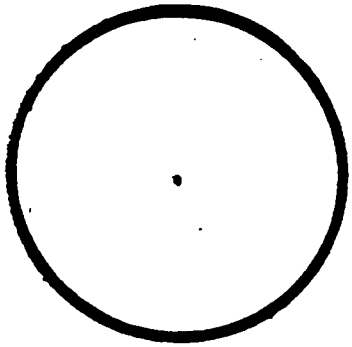
2) $\frac{3}{3}$



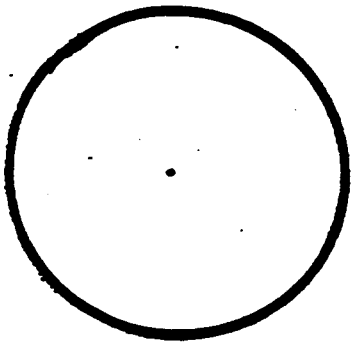
3) $\frac{1}{4}$



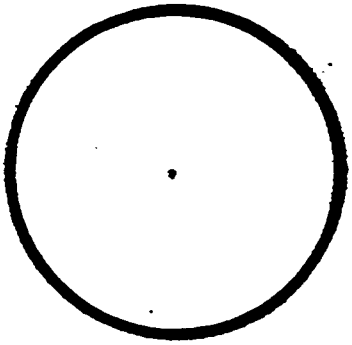
4) $\frac{2}{5}$



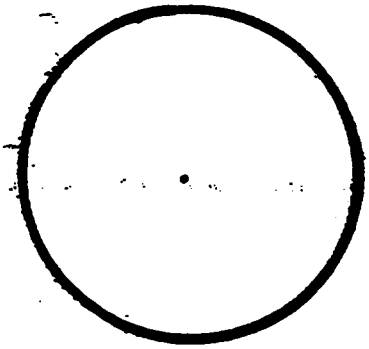
5) $\frac{3}{6}$



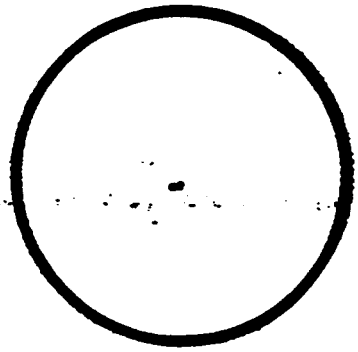
6) $\frac{4}{8}$



7) $\frac{3}{10}$



8) $\frac{5}{12}$

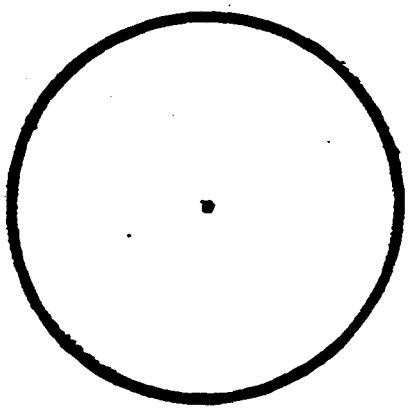


Name _____

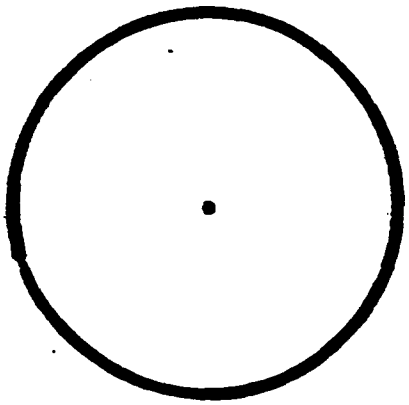
REPRESENTING FRACTIONS

Draw and shade in each fraction. Write the name of the fraction in English words.

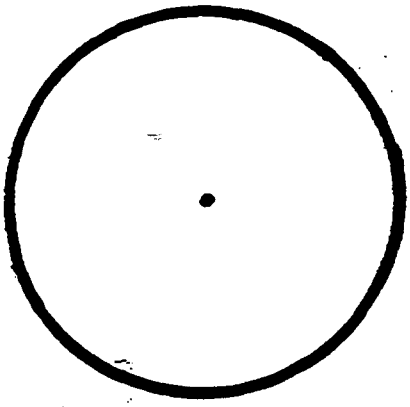
1) $\frac{1}{4}$



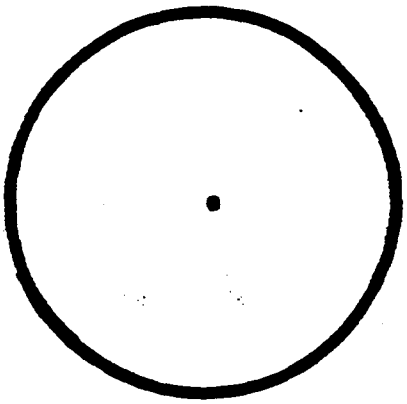
2) $\frac{1}{8}$



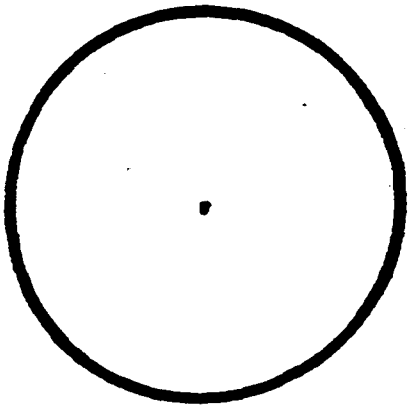
3) $\frac{1}{3}$



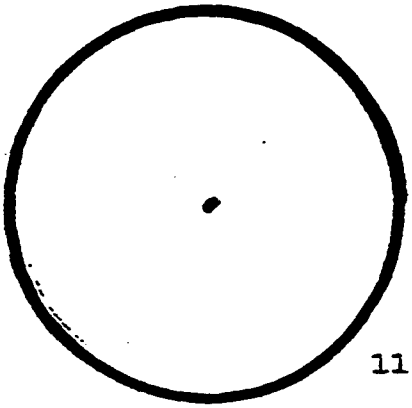
4) $\frac{5}{6}$



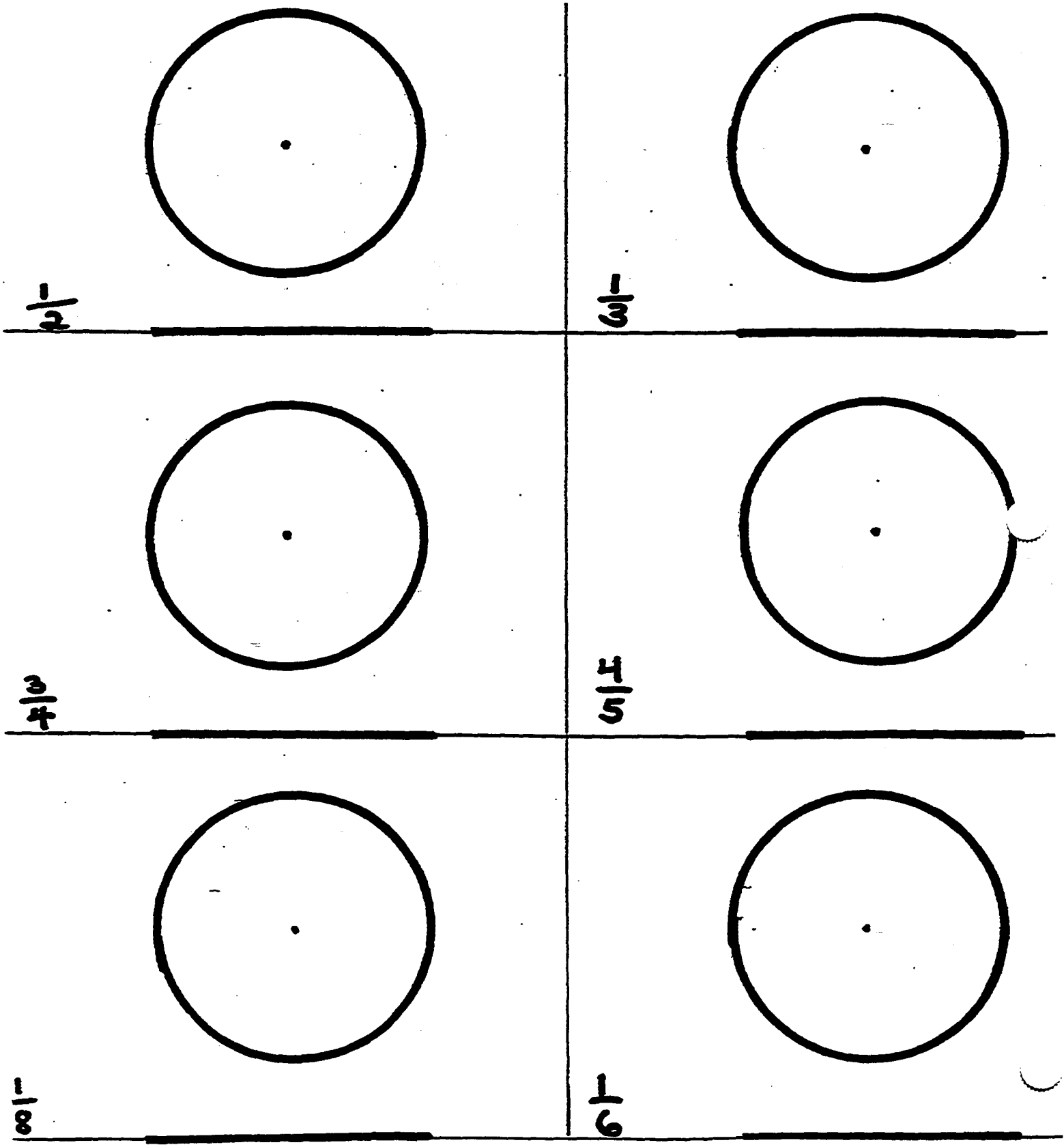
5) $\frac{2}{2}$



6) $\frac{1}{5}$



Draw and shade in each fraction. Write the name of the fraction in English words.



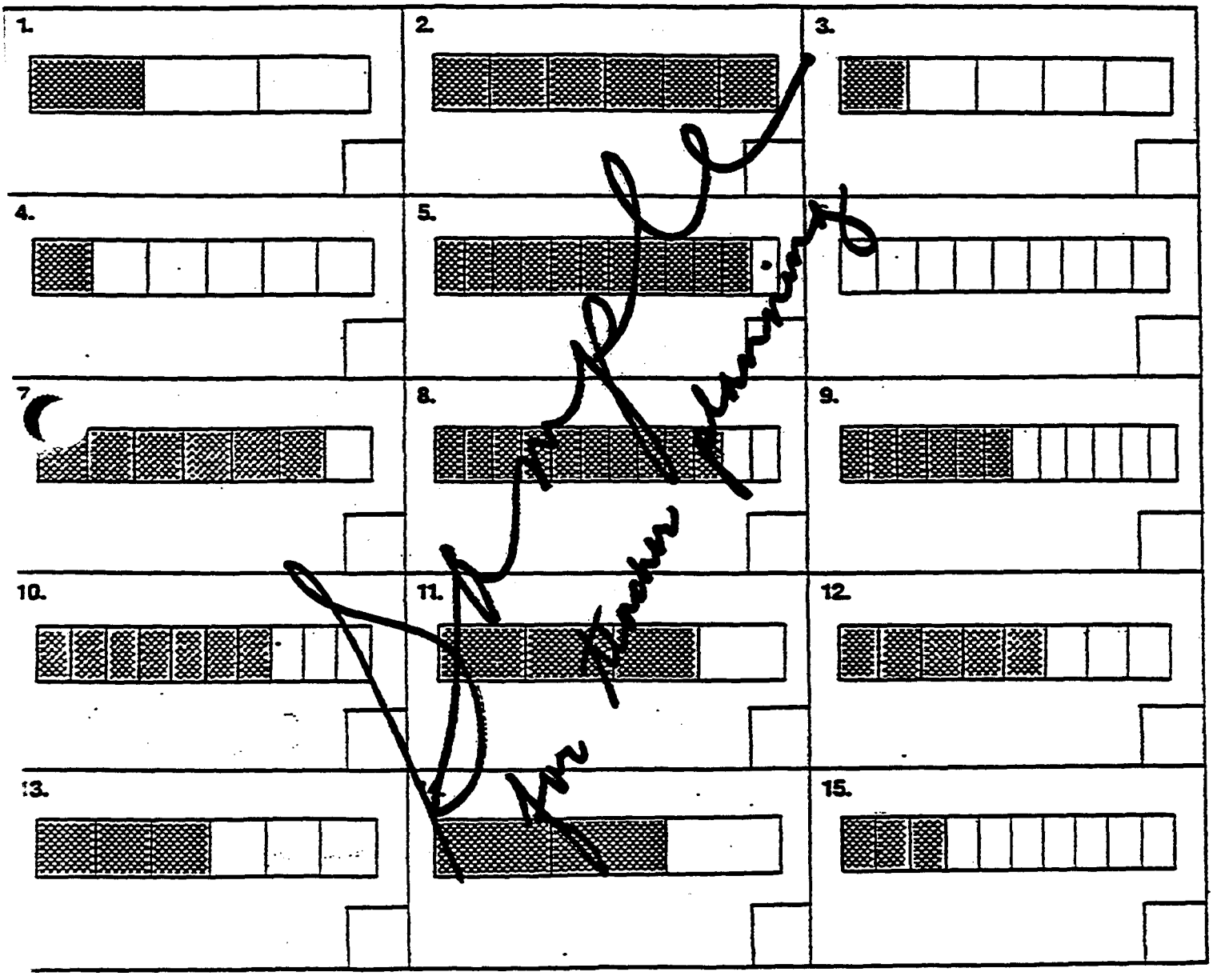
Name _____

On the following fraction bar, 5 out of 12 parts are shaded.

$$\frac{\text{number of shaded parts}}{\text{number of parts}} \quad \frac{5}{12}$$

$\frac{5}{12}$ (five-twelfths)

Write the fraction for the shaded amount of the bar.

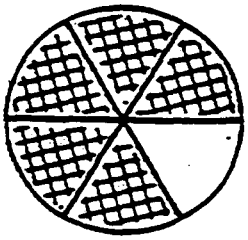


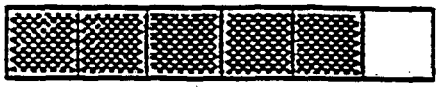
. How many of these bars are zero bars? ____

many of these bars are whole bars? ____

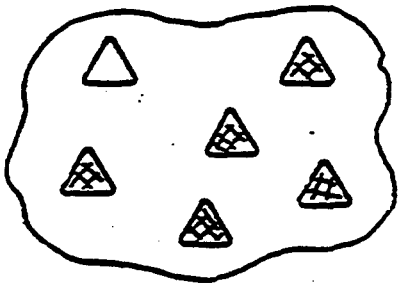
Name _____

The fraction for the shaded part of each figure is $\frac{5}{6}$ (five sixths).



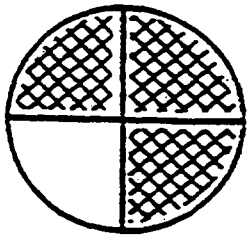


numerator 5
denominator 6



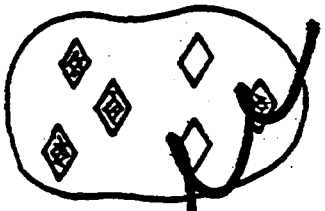
The "top number" of the fraction is called the numerator. (nú-mer-a-tor)
The "bottom number" of the fraction is called the denominator. (de-nom-i-na-tor)

Complete each sentence.



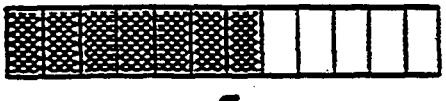
$\frac{3}{4}$

1. The numerator is _____.
2. The denominator is _____.



$\frac{4}{}$

3. The _____ is 4.
4. The _____ is 6.



$\frac{7}{12}$

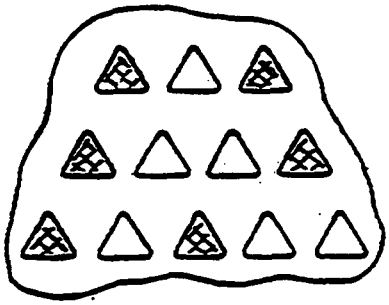
5. The _____ is 12.
6. The numerator is _____.

$\frac{2}{6}$

7. The numerator is _____.
8. The denominator is _____.

$\frac{0}{3}$

9. The denominator is _____.
10. The numerator is _____.



$\frac{6}{12}$

11. The denominator is _____.
12. The _____ is 6.

13. The _____ tells how many equal parts are in a whole.
14. The _____ tells how many of these parts are shaded.
15. The "bottom number" of a fraction is called the _____.
16. The "top number" of a fraction is called the _____.

Objective 3: Compare models of fractions using >, <, =, and ≠. Know that fractions can only be compared when the whole is the same size. Use >, <, =, and ≠ to compare fractions and mixed numerals.

Vocabulary

greater than >
less than <
mixed numeral

Materials

Overhead
Pattern Blocks
Overhead Pattern Blocks
Fraction Bars
Overhead Fraction Bars
Overhead Pen
Fraction Wheel page, 1/student
Scissors
Overhead Fraction Wheel
Fraction Bars Worksheets, pp. 32 & 33 1/student

Language Foundation

1. In addition to learning the > and < symbols, the students may have some difficulty with the idea of *greater than* and *less than*. Explain that *greater* means more, a bigger number. You may even use a comparison of *great* and *grande* (emphasizing the *gr*) to help the Hispanics. Explain that *less than* means smaller, a littler number. It may also help to show that greater than is longer than less than. A copy of a chart that can be made and put up is also included and found on p. 18.

Mathematics Component

Give each group of students a pile of pattern blocks. Have each student find a yellow hexagon, and explain that the hexagon equals 1 whole. Have them find a block that will cover one half of the hexagon. On the overhead write 1/2 beside the red trapezoid. Ask them to use different blocks (all the same color) to cover 1/2 of the hexagon. When they have discovered that 3 triangles cover it, ask how many triangles are needed to cover the entire hexagon. Write 6 on the overhead. Then ask how many sixths it took to cover 1/2. Write 3/6 on overhead. Ask if 3 triangles are equal to 1 trapezoid. Model this and write 1/2 = 3/6. Ask how many blue rhombuses cover the hexagon. Write 1/3 beside a rhombus on the overhead. Remind them that the triangle is 1/6. Ask how many sixths (triangles) equal 1/3 (rhombus). Elicit 2. Write 1/3 = 2/6.

Replace the pattern blocks with a set of fraction bars for each pair. On the overhead show the students the 6/12 bar and compare it with the 3/6 bar. Model the comparison for the students by saying "6/12 equals 3/6" and then write 6/12 = 3/6. Ask the students to find the 3/12 bar. Then ask them to find the blue bar with the same amount shaded. Ask a student to describe the comparison and to write it on the overhead (3/12 = 1/4). Repeat with the yellow 2/3 bar finding a red bar to match it (2/3 = 4/6). Ask the students to find the 1/2 bar and then find 4 other bars that are equal to it. When they are finished, have someone write 1/2 = 2/4 = 3/6 = 5/10 = 6/12.

Give each student a copy of the Fraction Wheel worksheet. Help them to follow the directions on their sheet to construct their wheel. Explain that the dotted portion will move to show them fractional parts of the wheel with numbers. (Help them see they are comparing parts of the white wheel.) Have the students compare 1/4 and 1/8. Ask which is greater. Write 1/4 > 1/8. Read the equation explaining the symbol using the chart or explaining that the point *points at the smallest number*. Work with the students to complete B through D. Have the students work in pairs to complete E through H. Check the answers.

Have the students get the fraction bars for 1/2, 1/3, 1/4, 1/5, 1/6, 1/10, and 1/12. Have them compare the shaded portions. Help them generalize a rule relating the denominator and the size of the shaded piece. Elicit the larger the denominator the smaller the piece. Ask if 1/15 would be larger or smaller than 1/12. Write 1/35 and 1/40 on the overhead and ask what sign should go between them. Have the students compare the 2/3, 2/5, and 2/10 bars. Does the "rule" still fit?

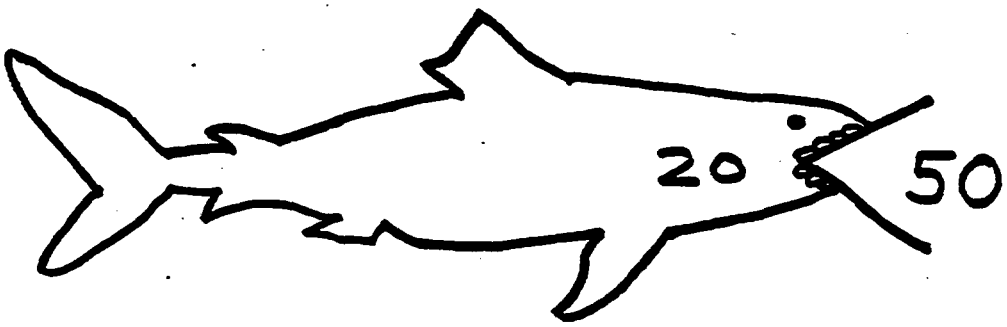
Have the students compare the 2/4 and 3/4 bars. Talk about what happens when the denominator is the same and the numerator changes. Remind them that this time we have the same size pieces, just more of them. Have them compare 11/12 and 5/12. Have them help you write the equation 11/12 > 5/12. Repeat with 2/10 and 4/10.

Write 1 1/2 and 2 1/2 on the overhead. Ask which is greater and why. Elicit 2 is greater than 1. Write 1 1/2 < 2 1/2. Explain that 2 1/2 and 1 1/2 are called mixed numerals because they have both whole numbers and fractions. Talk about what 1 1/2 pizzas would mean. Compare 2 2/5 and 2 4/5. Help the students look for the number that is different and then decide which is larger.

Pass out sheets 32 and 33. Help the students locate the > and < signs at the bottom of p. 32. Explain what they are to do and let them use the fraction bars if they still need help.

smaller number < greater number

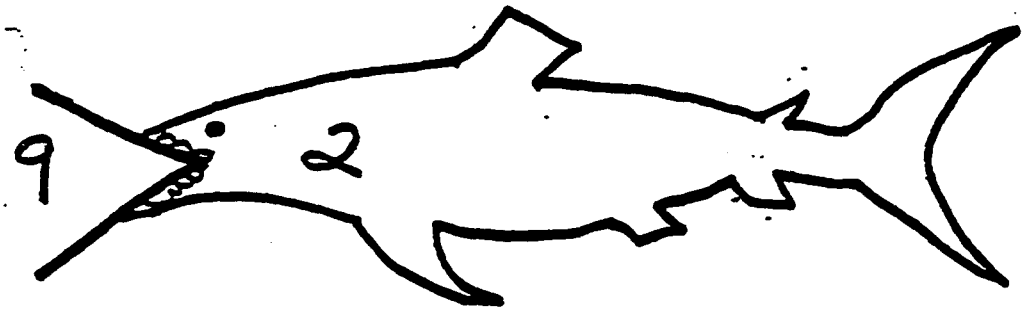
is less than



20 is less than 50.

greater number > smaller number

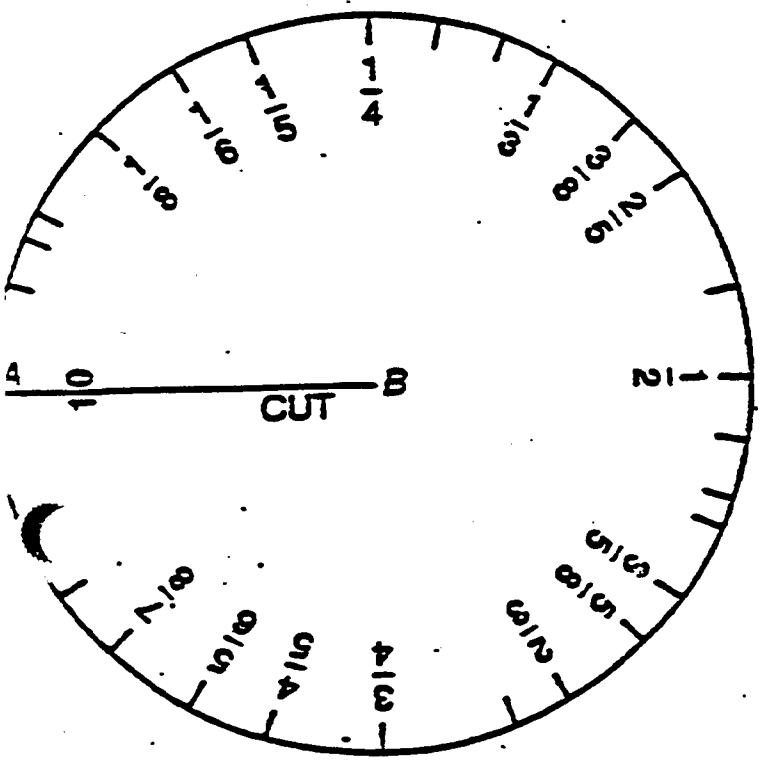
is greater than

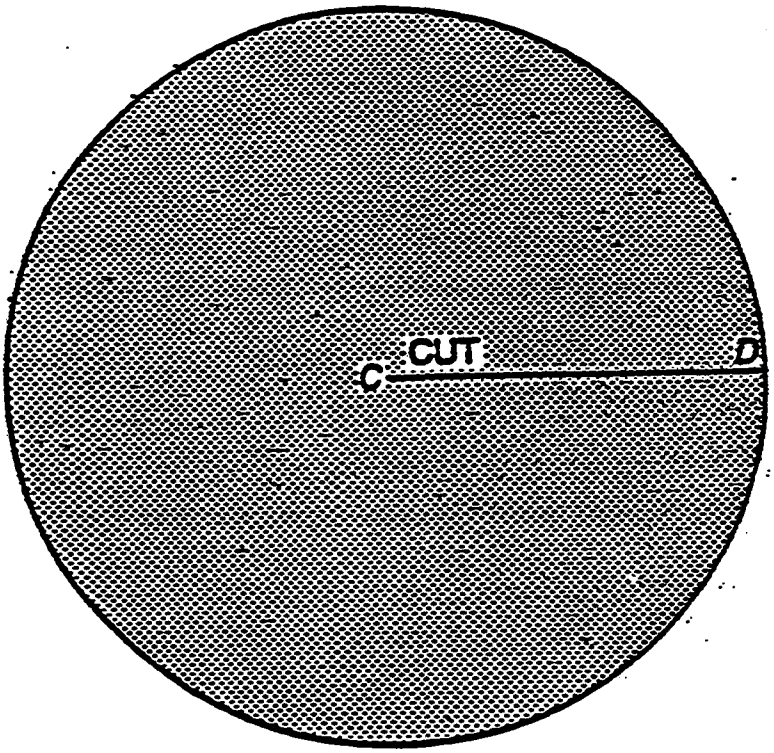


9 is greater than 2.

FRACTION WHEEL

1. Cut out the two circles.
2. Cut along AB and CD.
3. Fit the wheels together by inserting CD into AB.
4. Move the shaded wheel around, and it will show you the size of each fraction on the dial.





Name _____

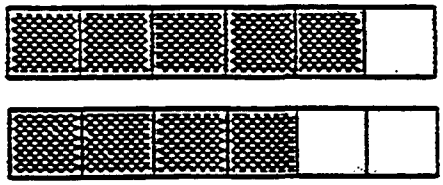
Complete the following expressions. Use symbols for greater than (>) or less than (<).

A. 1	1	B. 5	1	C. 2	5	D. 1	1
A. 8	4	B. 8	2	C. 3	6	D. 8	2

E. 2	3	F. 7	4	G. 3	5	H. 5	3
E. 5	4	F. 8	5	G. 5	8	H. 6	5

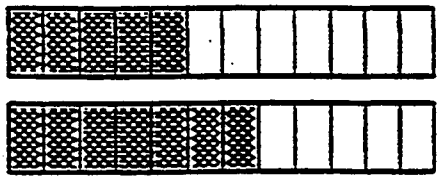
Name _____ Fraction Bars Worksheet, page 32

When two fractions have the same denominator, the greater fraction is the one with the greater numerator. The smaller fraction is the one with the smaller numerator.



$\frac{5}{6} > \frac{4}{6}$ since $5 > 4$

"is greater than" >



$\frac{5}{12} < \frac{7}{12}$ since $5 < 7$

"is less than" <

Write the correct inequality symbol (> or <) between each pair of fractions.

1. $\frac{2}{3}$ $\frac{1}{3}$	2. $\frac{5}{12}$ $\frac{7}{12}$	3. $\frac{1}{4}$ $\frac{2}{4}$
4. $\frac{0}{6}$ $\frac{1}{6}$	5. $\frac{7}{12}$ $\frac{11}{12}$	6. $\frac{6}{6}$ $\frac{5}{6}$
7. $\frac{3}{4}$ $\frac{1}{4}$	8. $\frac{1}{2}$ $\frac{2}{2}$	9. $\frac{3}{6}$ $\frac{2}{6}$

Sample for tutor planning

Here's a good way to remember the symbols. The lines point to the smaller number, or the mouth "bites" the bigger number.

greater number > smaller number

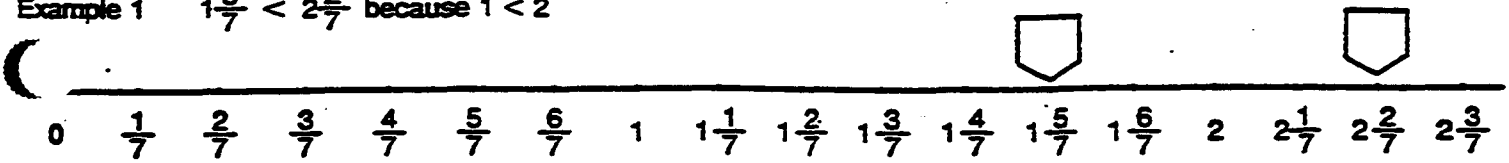
"is greater than"

smaller number < greater number

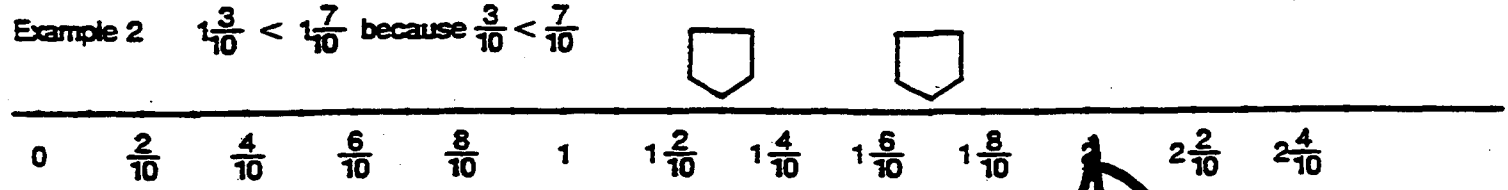
"is less than"

Name _____

Example 1 $1\frac{5}{7} < 2\frac{2}{7}$ because $1 < 2$



Example 2 $1\frac{3}{10} < 1\frac{7}{10}$ because $\frac{3}{10} < \frac{7}{10}$



Write the correct inequality symbol (< or >) between each pair of mixed numbers.

1. $2\frac{3}{4}$ $3\frac{3}{4}$	2. $4\frac{}{7}$ $1\frac{5}{7}$	3. $2\frac{9}{10}$ $2\frac{2}{10}$
$3\frac{1}{8}$ $3\frac{5}{8}$	5. $2\frac{1}{9}$ $1\frac{}{9}$	6. $1\frac{9}{12}$ $1\frac{4}{12}$
7. $2\frac{1}{6}$ $2\frac{5}{6}$	8. $3\frac{}{8}$ $4\frac{1}{8}$	9. $6\frac{1}{3}$ $4\frac{2}{3}$
10. $1\frac{3}{5}$ $1\frac{2}{5}$	11. $3\frac{1}{11}$ $2\frac{6}{11}$	12. $4\frac{1}{4}$ $2\frac{3}{4}$
13. $5\frac{1}{6}$ $5\frac{3}{6}$	14. $2\frac{3}{12}$ $1\frac{5}{12}$	15. $4\frac{1}{2}$ $5\frac{1}{2}$

Objective 4: Measure and compare lengths of object to nearest 1/2", 1/4", and 1/8".

Vocabulary

number line
measure
nearest to

Materials

Overhead
Fraction bars
Overhead fraction bars
Number lines mats
1/student
Markers - 2/student
Paper
Fraction Bars Work-
sheet, p. 11
1/student
Rulers - 1/student
Measurement
worksheet, 1/student
Transparency of
Measurement
worksheet
Things to measure

Language Foundation

1. This lesson allows some "catch-up/check-up" time. You may want to begin the lesson with a review of vocabulary. One way to do that would be to use the fraction Bingo game (see p.75 in Fraction Bar manual.) You could even say cover any circle with a numerator of 3 or with a denominator of 6 as well as the regular fractions.

2. In this lesson you will use the superlative *nearest to*. The students may not have been introduced to this form. Talk about words ending in *-est* which compare more than 2 things. Discuss who is *nearest to* the door, *nearest to* your desk, etc. You might even measure someone to the nearest foot.

3. Students will be dealing with mixed numbers in this lesson. You may want to use the number lines mats and have students place a marker on the thirds line 2/3 of the distance from one to two. Explain that the mixed number is read *"one and two-thirds."* Ask them to place a marker on the fourths line 1/4 of the distance from one to two. Ask them to read the mixed number (one and one-fourth.)

Mathematics Component

Pass out the fraction bars and sheets of paper. Have each student select a red sixth bar. As you model on the overhead, have each student draw a line segment the length of the bar placing a dot on the line to correspond to each division on the bar (See Fraction Bar manual p. 7). Have the students write 0, 1/6, 2/6, 3/6, 4/6, 5/6, 1, to match the appropriate points on the line. Explain that you have made a number line that can be used to measure things. Ask students to measure their pencils to the nearest sixth. If their pencil is too long, show them how to extend their line segment so they add 1 1/6, 1 2/6, 1 3/6, 1 4/6, 1 5/6, 2. Tell them their line segment is 2 units long.

Have the students take a white tenth bar and repeat the above activity using tenths. After they have finished measuring their pencils, ask whether tenths or sixths were more accurate.

Pass out copies of Worksheet 11. Work on it with the students until you feel they are ready to complete it with a partner.

Collect the fraction bars and give each student a ruler. Place a clear plastic ruler on the overhead. Ask the students to tell you how many units the ruler is divided into. Elicit 12 inches. Have the students locate the mark on the ruler that is half the distance between the first mark on the ruler and the 1 inch mark.Tell them this is the half inch mark. Show them the half inch marks between 2 and 3, 3 and 4, and 4 and 5. Help them locate the half inch marks on the entire ruler. Tell them you are going to draw a line 5 1/2 inches long. Model doing this. Write 7 1/2 inches on the overhead. Have the students draw a 7 1/2" line on their papers. Have them check each other's drawings.

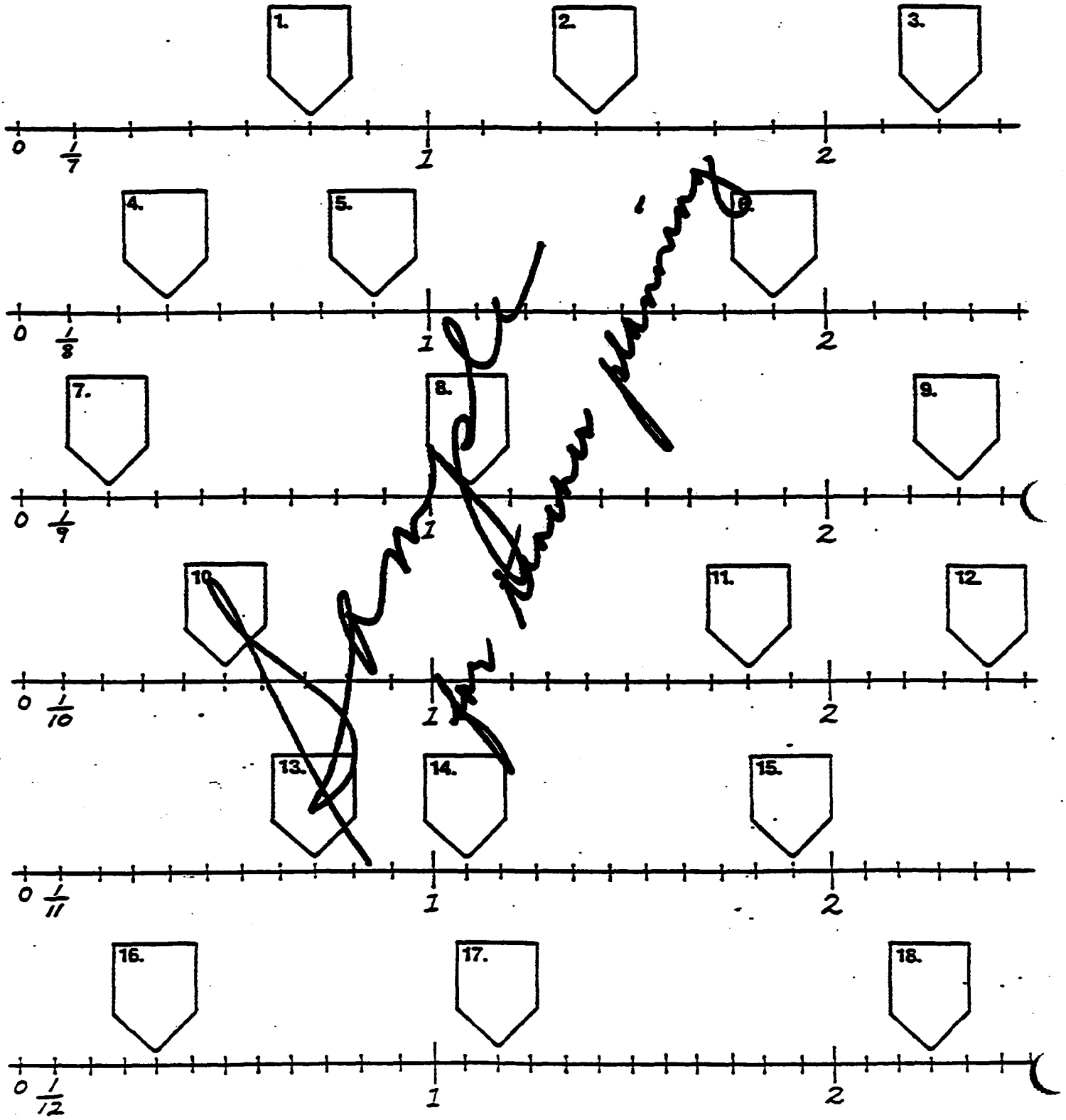
Ask the students to locate the marks that divide 1 inch into 4 equal pieces. Tell them each part is 1/4 of an inch. Under your ruler write 1/4, 2/4, 3/4 indicating the lines. Ask them what other name we call the 2/4 mark. Elicit 1/2. Repeat for the eighth inch marks. Explain that these rulers are divided into sixteenths of an inch.

Give each student a copy of the measurement worksheet. Put the transparency of the worksheet on the overhead. Model measuring line A to the nearest inch, nearest 1/2", nearest 1/4", and nearest 1/8". Have them work with you to measure line B. Have the students finish measuring lines C, D, and E.
(This part may need to be spread over a few days, depending on how well the students grasp the concept.)

If the students finish early, have a variety of small materials for students to measure such as paper clips, notecards, pens, etc.

Name _____

On each line, write a fraction or mixed number on the marker for each of the indicated points.

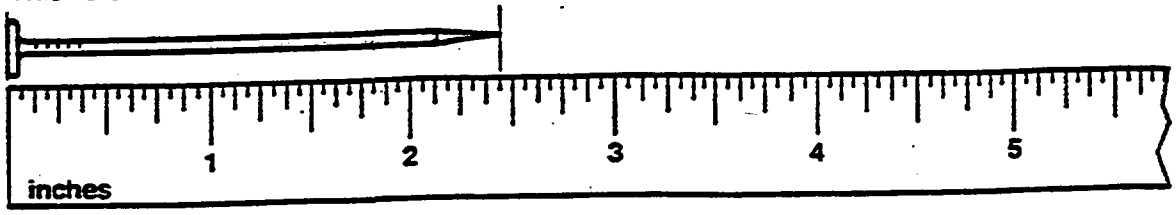


Fraction Bars
Basic Concepts, Step 3
Objective B-6

MEASUREMENT WORKSHEET

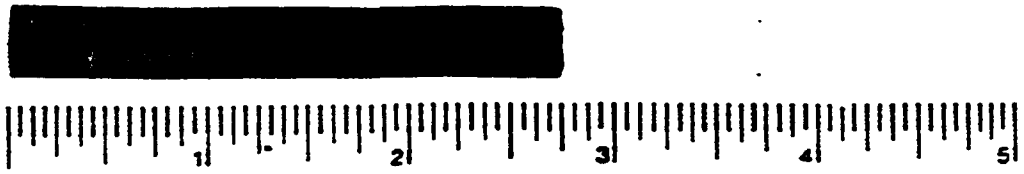
Here are some examples of using a ruler to measure in inches:

This ruler is marked in inches and parts of an inch.



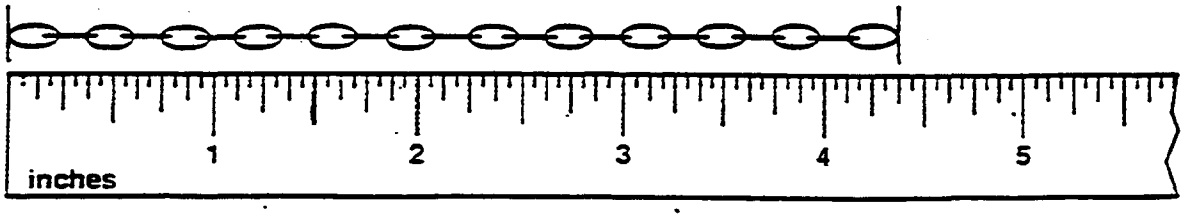
The nail is 2 inches long to the nearest inch.
It is $2\frac{1}{2}$ inches long to the nearest $\frac{1}{2}$ inch.
It is $2\frac{2}{4}$ inches long to the nearest $\frac{1}{4}$ inch.

This pencil is between 3 and 4 inches long.
It is almost halfway between 3 and 4 inches. It is about $3\frac{1}{2}$ inches long.



This piece of chalk is between 2 and 3 inches long. It is about $2\frac{3}{4}$ inches long.

How long is the chain to the nearest inch? $\frac{1}{2}$ inch?



How long is the chain to the nearest $\frac{1}{4}$ inch?

Measure each line segment using an inch ruler:

A 

nearest inch ____ nearest 1/4 inch ____

nearest 1/2 inch ____ nearest 1/8 inch ____

B 

nearest inch ____ nearest 1/4 inch ____

nearest 1/2 inch ____ nearest 1/8 inch ____

C 

nearest inch ____ nearest 1/4 inch ____

nearest 1/2 inch ____ nearest 1/8 inch ____

D 

nearest inch ____ nearest 1/4 inch ____

nearest 1/2 inch ____ nearest 1/8 inch ____

E 

nearest inch ____ nearest 1/4 inch ____

nearest 1/2 inch ____ nearest 1/8 inch ____

Objective 5: Identify and write fractions and mixed numbers as decimals.

Vocabulary

decimal number
decimal point
equivalent
repeating
tenth
hundredth
thousandth
imagine
one-place
two-place decimal

Materials

Overhead
Overhead pens
Transparency 1
Transparency 2
Fraction bars
Overhead fraction bars
Calculators
Overhead Calculator
Scissors
*Copies of Fraction - Decimal jigsaw puzzle (1 per pair)

*Run off each copy on different colored paper or mark back pieces of each puzzle with distinct symbols to make sorting easier.

Language Foundation

1. The word *point* (as in decimal point) may need to be explained. In many countries a comma is used instead of a decimal point. It may help to write $4.25 and indicate the decimal point here. Tell them that numbers to the right of the decimal point stand for a part of 1 whole thing. 25¢ is part of one whole dollar.

2. Point out the "ths" ending on the end of tenths, hundredths, (and thousandths). Explain that this ending is related to fractions and decimals. Tenths means divided into 10 equal parts, hundredths into 100 equal parts, etc.

3. The fraction - decimal number line in Activity 1 can be done using the overhead and transparencies, the chalkboard, or a bulletin board so it can be left up as a reference.

Mathematics Component

Put transparency 1 on the overhead (or use bulletin board or chalk board - see Other Language Foundation #3). Have students take turns labeling halves and fourths. Using decimal notation for fractional parts of a dollar, write the decimal equivalents for halves and fourths on the number line. Have students help you expand this between the whole numbers on the number line. Put transparency 2 on the overhead and have students take turns labeling the tenths. Discuss the number line using questions such as "Where would I mark the price of a can of soda?" "A school lunch?" "The length of a pencil 2 3/4 inches long?" "A distance of 2 6/10 miles?"

Pass out fraction bars. Ask students to find a bar with 10 equal parts with 3 parts shaded. Ask them to write the fraction for the bar (3/10). Ask how they write this as a decimal. Tell them we call this three tenths. Explain that it also is read as a point three. Have the students find all their white fraction bars and arrange them from smallest to largest as you do this on the overhead. Have them write the correct fraction under each bar as you do this on the overhead. Model writing the first 3 or 4 equivalent decimals under the appropriate fractions. Have the students finish the list. Have them read the decimal numbers aloud. Write some mixed numerals on the overhead (1.7, 3.4, 25.5) and ask the students to read them explaining that we read the decimal point as "and".

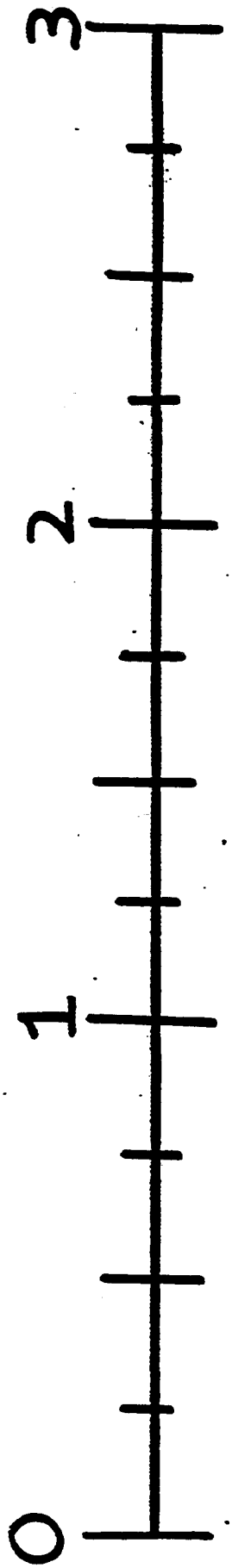
Have the students look at the 0/10 bar and imagine it divided into 100 parts. Have them imagine 35 of the 100 parts shaded. Ask them to write the fraction for the bar and compare to see if they agree with their partners. Write 35/100 on the overhead and read it to them. Write the decimal .35 and tell them that this is a 2 place decimal because its denominator is 100. Write these decimals on the overhead and ask students to write the fraction for each bar .86, .9, .14, .63, .99, .3. Then put .6 and .60 on the overhead. Ask them the difference in the two. (Remind them about 1-place and 2-place decimals.) After you write .6 = 6/10 and .60 = 60/100, write .06 and ask what fraction that equals. (Elicit the 2-place decimal 6/100.)

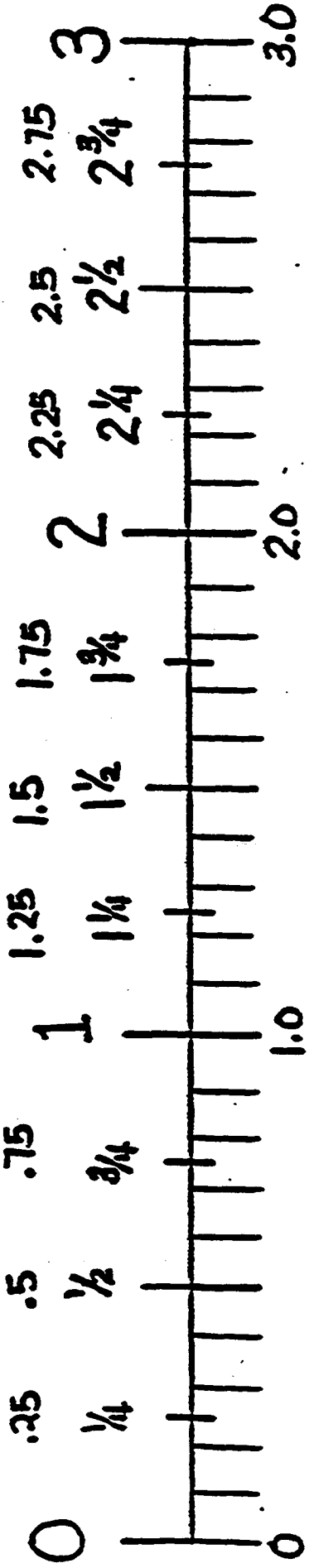
Have the students find the 2/5 bar. Ask them to find the white bar it matches. Write 2/5 = 4/10. Ask them to write the decimal for 2/5. Have them find the 4/5 bar, match it to a tenths bar and write the equivalent decimal. Repeat with the red 3/6 bar. After the students have written 3/6 = .5, ask them how to write 4 3/6 as a decimal. Elicit 4.5.

Ask the students to find the orange 9/12 bar. Ask them to match it to a tenth bar. When they have discovered that it won't match, tell them that we will now learn to use the calculator to find a decimal equivalent for a fraction. Pass out the calculators. Model dividing 9 by 12. Write 9/12 = 0.75 on the overhead. Remind them that they decided 3/6 = .5. Have them check it by dividing 3 by 6. Repeat for 9/10. Now write 8 3/4 on the overhead. Tell them they do not have to change the 8, only the 3/4. Write 8.75. Point out that we are only changing the fraction to the decimal. Do 2 1/2 with the students. Have them do 15 3/5. Repeat with 3 1/8. Point out that the decimal is a 3-place decimal. Repeat with 9 3/8.

Have the students find the decimal for 1/3 as you model on the overhead calculator. Point out that this is a repeating decimal. It says 3 over and over again. We can write this as 1/3 = .3. Repeat with 2/9 (.2). Repeat with 1/22 = .045. Point out that the line is only over the *repeating* part. Repeat with 2/3 (.6).

Pass out copies of the Fraction-Decimal Jigsaw Puzzle. Have the students cut it out and then put it back together so that the fractions and decimals match. They should be encouraged to use their calculators to solve the puzzles.


3
2
1
0


3
2.75
2¾
2.5
2½
2.25
2¼
2
1.75
1¾
1.5
1½
1.25
1¼
1
.75
¾
.5
½
.25
¼
0
3.0
2.0
1.0
0

Fraction - Decimal Jigsaw Puzzle

$\frac{3}{4}$ 0.75 $\frac{1}{4}$ 0.25 0.4 $\frac{2}{5}$ 0.25 0.5 $\frac{9}{10}$ $2\frac{3}{5}$			
$\frac{1}{4}$ $\frac{1}{2}$ 0.9 2.6 $2\frac{2}{5}$ 2.4 0.1 $\frac{1}{10}$ $\frac{20}{10}$ 2 $5\frac{4}{5}$ $\frac{3}{10}$ $\frac{4}{8}$ $\frac{5}{5}$			
5.9 0.3 .0.5 1 0.4 $\frac{2}{5}$ $\frac{3}{6}$ 0.5 $\frac{4}{5}$ 0.8 $\frac{3}{3}$ $3\frac{1}{5}$ 0.6 1.1			
1 3.2 $\frac{3}{5}$ $\frac{11}{10}$ $\frac{3}{12}$ 0.25 0.3 $\frac{3}{10}$ $\frac{6}{8}$ 0.75			

Objective 6: Determine if 2 fractions are equivalent; find a fraction equivalent to a given fraction.

Vocabulary

equivalent fractions

Materials

Overhead
Rectangular pieces of paper (1/student)
Markers (water-based)
Fraction bars
Overhead fraction bars
Blank transparency
Overhead pens
Fraction Bars Worksheets, pp. 19,22,23 1/student
Fraction cards

Optional
Worksheets 14-18 & 21)
Multiplication tables (or calculators)

Language Foundation

1. Tell the students they are going to work with equivalent fractions. Remind them that equal means the same size. Write the word equivalent on the overhead and ask them to find the word equal in the word equivalent. Underline the letters that spell *equal* (equivalent). Tell them equivalent is another way to say equal.

2. Those students who do not yet know the multiplication facts will need to use a multiplication table (or calculator) to complete the worksheet.

3. Included at the end of this lesson are two other sheets on equivalent fractions. Neither of these sheets is used in the lesson. The first, Number-Line Equivalents, can be passed out to the students to use or can be enlarged as a wall chart. The second, Equivalent Fraction Strips, can be passed out and explained to students as needed. Some students may need this aid in finding equivalent fractions. Use these 2 sheets as you see fit.

Mathematics Component

Pass out the rectangular pieces of paper, markers, and fraction bars. Ask the student to fold the paper into halves, 2 equal pieces and to color one half. Model doing this. Write 1/2 on the overhead as you tell them "1/2 is colored." Have them now fold the paper into fourths, 4 equal pieces. Ask them to unfold the paper and tell you how many parts the paper is divided into (4) and how many are colored (2). Write on the overhead 1/2 = 2/4. Explain that the same amount of paper is colored. Have the students find the green 1/2 bar and compare it with the blue 2/4 bar. Have the students refold their paper and fold it once more into eighths. Ask them to unfold the paper and tell you what part is now colored. Write 1/2 = 2/4 = 4/8. Explain that 1/2 and 2/4 and 4/8 are equivalent fractions because they name the same amount. Point out the similarity in equal and equivalent.

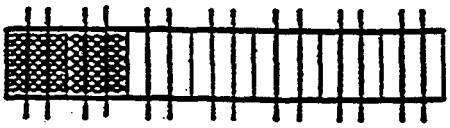
Ask the students to find the 1/3 bar. Place the 1/3 bar on the overhead and place a clear transparency over it. Using the overhead pen, split each part of the bar into 2 equal pieces by drawing dotted lines on the overhead. Ask how many parts the bar now has (6) and how many are shaded (2). What fraction does this bar now represent? Elicit 2/6. Have them find the red 2/6 bar and check to see if it is equivalent to 1/3. Repeat the process using the 3/5 bar. Lead the students to see that splitting each part in half actually multiplies the total number of parts and the number of shaded parts. Have the students find the red 4/6 bar. Have them think about dividing each piece of the bar in half. Ask if they can predict (guess) what the new equivalent fraction will be.
Write $\frac{4}{6} \cdot \frac{x\,2}{x\,2} = \_$ and fill in the $\frac{8}{12}$.

Ask the students to find the green 1/2 bar. Model splitting each part into thirds as they do it using the water-based markers (which will wash off). Ask how many total parts (6) and how many shaded parts (3). Write 1/2 = 3/6. Guide them to see that splitting each part of a bar into thirds is like multiplying the numerator and denominator by 3. Repeat by having the students divide a 2/3 bar into 4 equal parts to get 8/12. Pass out copies of sheet 19 from the fraction bar kit and, after modeling how to do it, have students work in pairs to complete the sheet.

Tell the students that for any fraction an equal fraction can be obtained by multiplying both the numerator and denominator by the same number. Write these sets of equivalent fractions on the overhead and ask the students to tell you what number both the numerator and denominator were multiplied by - 3/4 = 15/20, 1/3 = 7/21, 2/5 = 20/50. Write 4/6 = ?/12 on the overhead. Ask what number we multiplied 6 by to get 12. Elicit 2. Therefore, how can we find the missing numerator? Elicit, multiply by 2. Write 4/6 = 8/12. Show the red 4/6 bar is equivalent to the orange 8/12. Repeat with 2/5 = /10. Repeat with a missing denominator 3/4 = 9/?. Have students practice with 3/5 = 6/?, 2/3 = 6/?, and 1/4 = 4/?. Pass out worksheets 22 and 23. Model how to complete them and have students work together to do so.

Have students work in groups of 4. Have them shuffle the fraction bar cards and turn 3 face-up. (You may need to explain *face-up*.) Place the rest of the cards in a pile face down. Each player in turn turns 1 card face-up and compares it with the face-up cards. If the player's card is equivalent to one of the face-up cards, the player wins both cards and may continue by turning over another card. If the card does not match, it is placed face-up with the others and the next player takes his/her turn. When there are no more cards in the face-down stack, the player with the most cards wins. Students can also use the fraction cards to play concentration by laying all the cards face down in an array and turning over 1 pair at a time. A player keeps equal pairs. If the pairs are not equivalent, the cards are put back face down and the next player tries to find a matching pair.

Splitting all the parts of a bar gives you more parts on the bar and more shaded parts. It's like multiplying the numera and denominator of a fraction by the same number.



$$\frac{2}{7} = \frac{6}{21}$$

Multiply numerator and denominator by 2. Complete the equations.

1.	2.	3.	4.
$\frac{1}{10} =$	$\frac{3}{11} =$	$\frac{3}{5} =$	$\frac{7}{10} =$

Multiply numerator and denominator by 3. Complete the equations.

5.	6.	7.	8.
$\frac{1}{10} =$	$\frac{5}{9} =$	$=$	$\frac{1}{6} =$

Multiply numerator and denominator by 5. Complete the equations.

9.	10.	11.	12.
$\frac{1}{10} =$	$\frac{9}{10} =$	$\frac{3}{8} =$	$\frac{5}{7} =$

Multiply numerator and denominator by 10. Complete the equations.

13.	14.	15.	16.
$\frac{1}{10} =$	$\frac{9}{10} =$	$\frac{3}{5} =$	$\frac{4}{9} =$

Name _____

To find a missing numerator in an equation, compare the denominators.

$\frac{3}{4} = \frac{\quad}{8}$

Think!

$? \times 4 = 8$

Since 8 is equal to 2 x 4, the missing numerator is 2 x 3.

$$\frac{2 \times 3}{2 \times 4} = \frac{6}{8}$$

Write the missing numerators.

1. $\frac{1}{2} = \frac{\quad}{12}$	2. $\frac{2}{3} = \frac{\quad}{6}$	$\frac{1}{2} = \frac{\quad}{4}$
4. $\frac{2}{3} = \frac{\quad}{9}$	5. $\frac{\quad}{8} = \frac{\quad}{24}$	6. $\frac{5}{6} = \frac{\quad}{12}$
7. $\frac{0}{3} = \frac{\quad}{12}$	8. $\frac{2}{2} = \frac{\quad}{6}$	9. $\frac{1}{4} = \frac{\quad}{8}$

10. I'm equal to $\frac{1}{3}$ and my denominator is 6. What fraction am I?

$\frac{\quad}{\quad}$

11. My denominator is 16 and I am equal to $\frac{3}{4}$. What fraction am I?

$\frac{\quad}{\quad}$

Find the missing number in the following equation by comparing the denominators.

$$\frac{7}{5} = \frac{}{30}$$

Think!

? x 5 = 30

Since 6 x 5 = 30, the 7 must also be multiplied by 6.

$$\frac{7}{5} = \frac{6 \times 7}{6 \times 5} = \frac{42}{30}$$

Write the missing numerator or denominator in each equation.

1. $\frac{3}{7} = \frac{}{28}$	2. $\frac{}{40} = \frac{}{40}$	3. $\frac{3}{4} = \frac{}{20}$
4. $\frac{2}{11} = \frac{8}{}$	5. $\frac{5}{}{} = \frac{15}{}$	6. $\frac{2}{5} = \frac{4}{}$
7. $\frac{2}{3} = \frac{}{18}$	8. $\frac{3}{5} = \frac{}{50}$	9. $\frac{7}{10} = \frac{}{100}$
10. $\frac{1}{6} = \frac{4}{}$	11. $\frac{4}{5} = \frac{12}{}$	12. $\frac{1}{2} = \frac{8}{}$

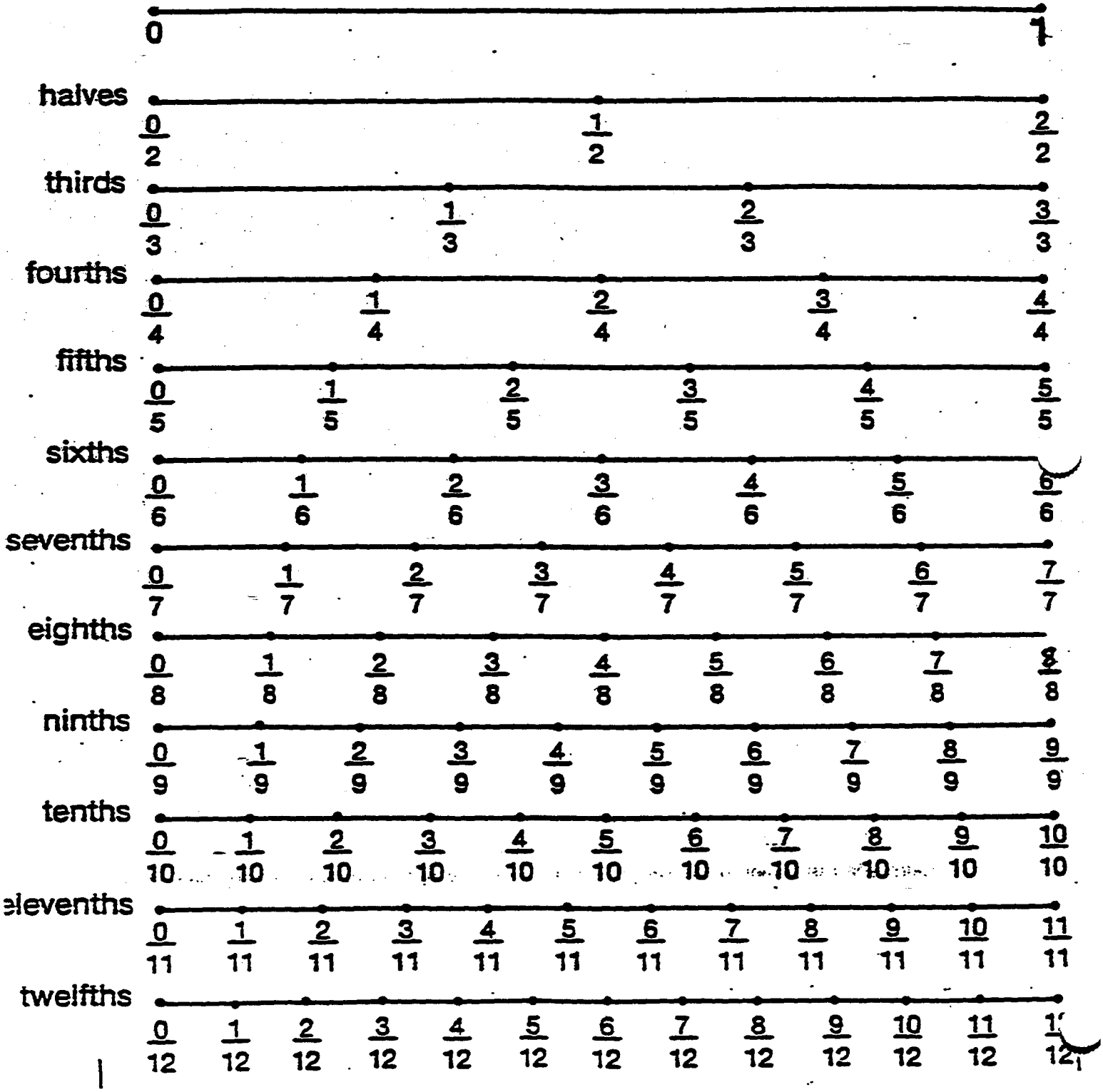
Sample for tutoring planning

Fraction Bars
Equality, Step 4
Objective E-5

MULTIPLICATION TABLE

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

NUMBER-LINE EQUIVALENTS



EQUIVALENT FRACTION STRIPS

Lay one ruler just above the numerator and one just below the denominator to find a fraction equivalent to a given fraction.

Laying 1 ruler above the 3 and another below the 4 would give:

3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80

The set of fractions equivalent to $\frac{3}{4}$ is shown to be: $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}$ ..... $\frac{60}{80}$ ....

EQUIVALENT FRACTION STRIPS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500

Adapted from an idea of Bonnie Litwiller and David Duncan, University of Northern Iowa.

Objective 7: Find the factors of a number. Find the greatest common factor of 2 numbers. Use the Greatest Common Factor to write a fraction in lowest terms.

Vocabulary

factor
common factor
greatest common factor
lowest terms

Materials

Overhead
Overhead pens
Fraction bars
Overhead fraction bars
Multiplication table (p. 39)
Blank transparency
Overhead of multiplication table
Factor worksheet
Fraction Bars Worksheet, p. 25

Language Foundation

1. Review the *-est* ending meaning *the most* and that we use it to compare more than 2 things. Talk about *the smallest*, comparing at least 3 concrete objects (i.e. paper clip, stapler, yardstick). Talk about *the greatest* and *the lowest* in the same way.

2. Discuss the meaning of common (or "in common") meaning that 2 or more things have something alike. You and I both have black hair so we have black hair in common. You are all students in Individualized Math. You all have this class in common. The numbers 10, 11, 12, and 13 all start with 1. The numbers all have the common first digit of 1.

3. Tie the word factor to factory. A factory is a place where things are made. A number is made of factors.

Mathematics Component

Have students find the orange 8/12 bar. Have them find the red bar and the yellow bar that are equivalent to the 8/12 bar (4/6 and 2/3). Ask which fraction has the smallest denominator (2/3). Put those bars away. Ask them to find the red 3/6 bar. Have them find the other four bars equivalent to the 3/6 bar (1/2, 2/4, 5/10, 6/12). Ask them to tell you the names of the fractions equivalent to 3/6. Write them as they tell you. Ask them which fraction has the smallest denominator (1/2). Tell them that we are going to work with fractions to find the smallest denominator. Put away the fraction bars.

Review finding equivalent fractions using 1/2 = ?/4, 1/2 = 3/?, 1/2 = ?/8. Remind them we found equivalent fractions by multiplying the numerator and denominator by the same number. Tell them we can also find equivalent fractions by dividing the numerator and denominator by the same number. Illustrate using these fractions. (Be sure to write in the ÷ sign between both numerators and denominators.)

$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$ $\frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$ $\frac{2}{4} \div \frac{2}{2} = \frac{1}{2}$

Tell them that numbers by which we can divide other numbers are called factors. We can divide 4 by 2 and so 2 is a factor of 4. Give 2 or 3 other examples. Tell them that we are going to learn to find factors. Write $\square \times \triangle = 16$ on the overhead. Ask them to tell you a pair (two) numbers that can be written in the $\square$ and the $\triangle$ to make the equation true. Continue until you have 1 and 16, 2 and 8, 4 and 4. Tell them you can divide 16 by 16, 8, 4, 2, and 1. These numbers are factors of 16. On a transparency, write 16: 1, 2, 4, 8, 16. Repeat with $\square \times \triangle = 12$. When you've gotten the factors, write 12: 1, 2, 3, 4, 6, 12 on the transparency.

Put the transparency of the multiplication table on the overhead and give students a copy. (If they have not used a multiplication table before, you may need to spend some time showing the students how to use one.) Tell the students we are going to find the factors of 20. Tell the students we always write 1 and the number we want to find the factors for because we can divide any number by 1 and by itself. Look at the 2's row. Are there any 20's in it? Yes, there is a 20 in the 2's row so 2 is a factor of 20 and 10 (above the 20) is a factor of 20. Add 2, 10 to the list of factors of 20 on the transparency sheet. Now look at the 3's row. No 20. Look at the 4's row. There is a 20. Ask what row (4's) it is in and what number is above it (5). Add 4, 5, to the factor list. Go to the 5's row. 20 is there but we already have 5 and 4 in our factor list. Continue row by row to the 10's row. Again 2 and 10 are in the list so we don't need to add them. Continue on to 12. There are no more 20's. Pass out the factor worksheet. Have the students work with you to find the factors of 18 (18: 1, 2, 3, 6, 9, 18). Have them do 15 with a partner. After you have checked 15 (15: 1, 3, 5, 15), let them finish the sheet. Tell them we will use this list to find common factors.

Check the factor worksheet and make sure the factors are all listed correctly. Have students use the worksheet to list the factors of 8 (8: 1, 2, 4, 8). Have them list the factors of 12 (12: 1, 2, 3, 4, 6, 12). Have them list the common factors of 8 and 12 (the factors that are in both the lists). (1, 2, 4.) Ask which of these numbers is the greatest? Have them circle 4. Tell them that 4 is the greatest common factor of 8 and 12. Explain that this means that 4 is the biggest number that will divide into both 8 and 12. Write $\frac{8}{12} \div \frac{4}{4} =$ on the overhead. Ask how many 4's are in 8. Write 2 as the numerator . Ask how many 4's are in 12. Write 3 as the denominator. Have them look at the factors of 2 and of 3. The common factor is one. Because the common factor of 2 and 3 is one, meaning that the only thing we can divide both 2 and 3 by is one, we say that 2/3 is in lowest terms. Repeat this process with 18 and 27. Repeat for 6 and 24. Pass out worksheet 25. Do the first 2 problems together. Have students complete the sheet. (Let them omit the word problem if you think the language is too difficult.)

As students finish, work on individual areas of difficulty with them or let them play the lowest terms game on p. 79 in the Fraction Bars Teacher's Guide.

Factor Worksheet

16: 1, 2, 4, 8, 16

12: 1, 2, 3, 4, 6, 12

20: 1, 2, 4, 5, 10, 20

4: 1, 2, 4

9: 1, 3, 9

22: 1, 2, 11, 22

18:

15:

2:

3:

5:

6:

8:

10:

14:

21:

24:

30:

Name _____ Fraction Bars Worksheet, p. 25

(Not in lowest terms) $\frac{9}{24} = \frac{9 \div 3}{24 \div 3} = \frac{3}{8}$ (In lowest terms)

(Not in lowest terms) $\frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$ (In lowest terms)

Complete each of the following equations by writing the fraction in lowest terms.

1. $\frac{5}{30} =$	2. $\frac{4}{14} =$	3. $\frac{12}{18} =$
4. $\frac{21}{30} =$	5. $\frac{10}{16} =$	6. $\frac{6}{22} =$
7. $\frac{12}{15} =$	8. $\frac{3}{9} =$	9. $\frac{8}{20} =$
10. $\frac{50}{100} =$	11. $\frac{20}{100} =$	12. $\frac{70}{100} =$

Simple for fraction planning

The fraction I'm thinking of is less than 1.

If I put my fraction in lowest terms, the sum of the new numerator and denominator is 7.

The denominator is 2 more than the numerator.

What is my fraction? _____

Fraction Bars
Equality, Step 5
(Parts 1 and 2)
Objective E-6

Objective 8: Compare and order fractions and mixed numbers.

Vocabulary

order
cross-multiply

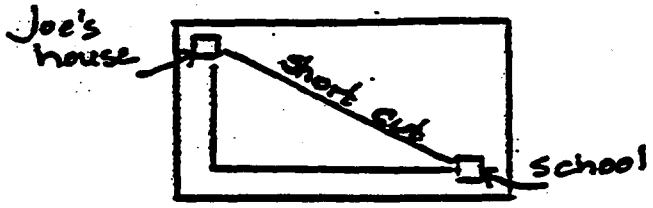
Materials

Overhead
Tower of Bars Mats
1/student pair
Paper markers for mats
Overhead Tower of Bars mat
Rulers
Fraction Bars Worksheet, p. 37
1/student
Transparency of worksheet, p. 37
Overhead fraction bars
Multiplication Table (p. 39)
Comparing Fractions worksheet, 1/student

Optional:
Fraction Bars Worksheets, p. 38 & 39
1/student

Language Foundation

1. In step 5 the word short-cut is used. This may need to be explained to the students. Put a small map on the board showing



a short cut from Joe's house to school. Explain that a short-cut is a faster way to get something done.

2. You may need to review the term mixed numeral explaining that it is a whole number and a fraction.

Mathematics Component

Spend a few minutes reviewing the <, >, and = symbols. Use fractions with a numerator of 1 to also review the fact that the bigger the denominator the smaller the fraction.

Give each pair of students a Tower of Bars Mat, some paper markers, and a ruler. Put your mat on the overhead. Have the students put a marker on 1/2, 1/3, 1/4, ...1/12. Spend some time discussing the amount of space represented by the fractions with 1 as a numerator. Have them remove all the markers except those on 1/8 and 1/12. Ask which is greater. Ask whether 1/15 is greater or less than 1/12. Write 1/8 > 1/15 on the overhead. Write 1/15, 1/10, 1/6, 1/32, 1/20 on the overhead. Ask the students to work together to arrange them in order from largest to smallest by filling in __> __> __> __> __. Check these.

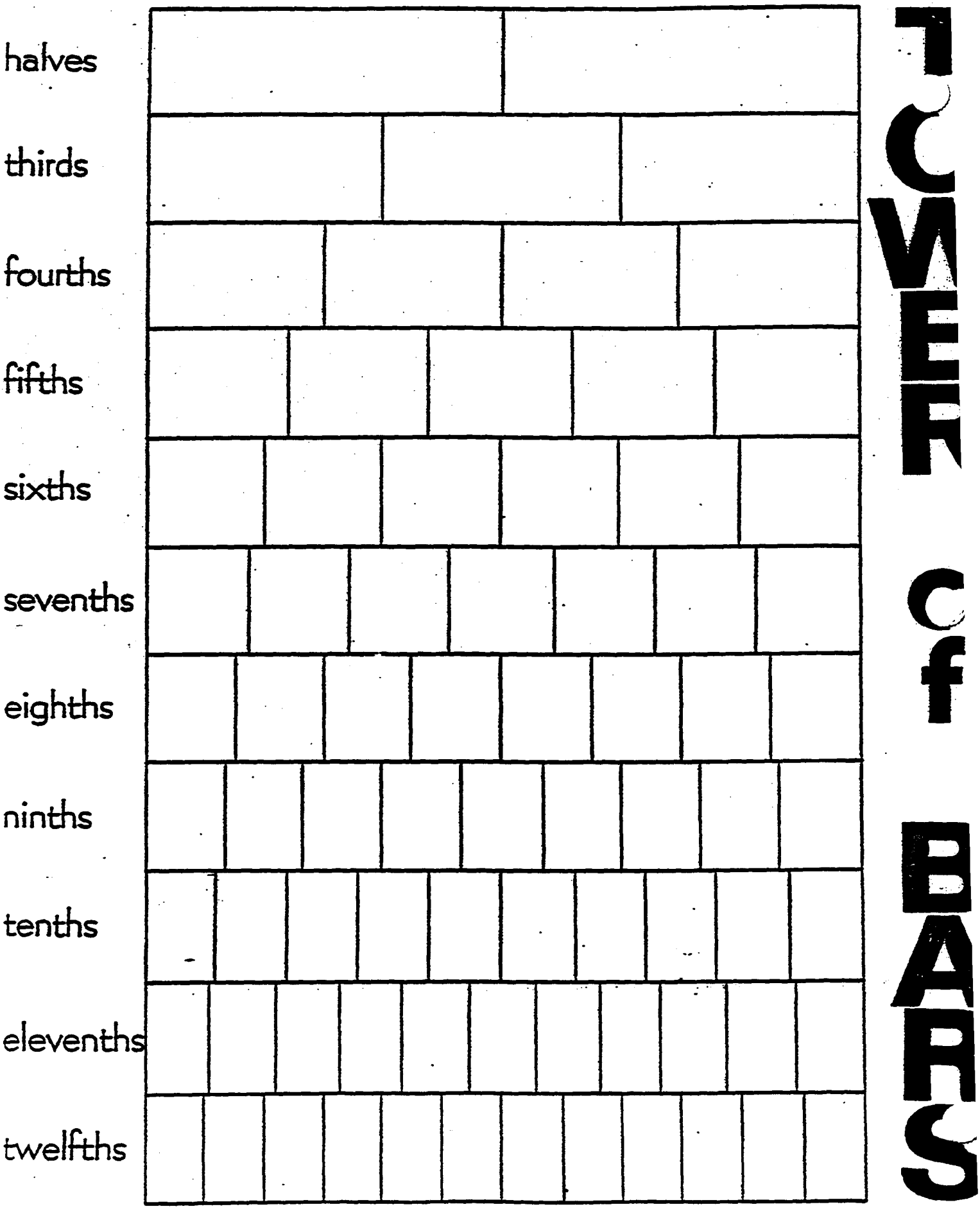
Have students place markers on their Tower of Bars Mat on 2/5 and 4/7 as you do it on the overhead. Show the students how to place a ruler vertically on the mat to compare the vertical lines for 2/5 and 4/7. Have the students write a comparison (2/5<4/7). Repeat for 5/6 and 4/5. Repeat until all students are able to compare the fractions. Put transparency of p. 37 on the overhead. Write the fraction beside each bar in the sample and have the students help you fill in the inequality below the example. Model problem 1. Have them work problem 2 with you and then complete the sheet. Check it. Collect Tower of Bars Mats and Markers as they complete this sheet.

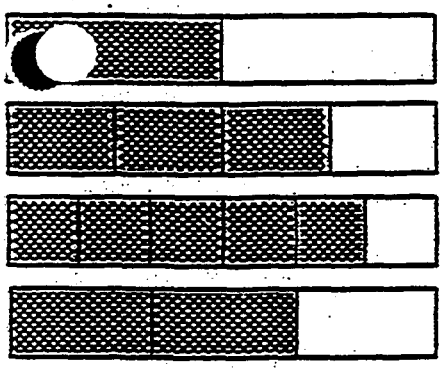
Put the yellow 2/3 bar and the blue 3/4 bar on the overhead. Tell the students it is easy to compare the bars when they are there. Have someone give you the comparison. Write it on the overhead. Tell them we are going to learn how to compare 2 fractions without using bars. Write 2/3 ☐ 3/4 on the overhead. Ask what 3x4 is, showing the students that we are using the denominators of the 2 fractions. Tell them we are going to change the fractions to 12ths. Write 2/3 x 4/4 = 8/12 and 3/4 x 3/3 = 9/12. Write 8/12 by 2/3 and 9/12 by 3/4. Ask which is greater 8/12 or 9/12. Put in the < sign. Compare it with the 2 fraction bars as a check.

Tell the students you are now going to show them a short-cut, a faster way, to compare fractions. Write 2/3 ☐ 5/6 on the overhead. Then draw arrows showing the direction to multiply (2/3 ☐ 5/6). Multiply 2x6 and write 12 beside the 2. Multiply 3x5 and write 15 beside the 5. Ask them to now compare 12 and 15 and help you complete the comparison. Write < in the box. (You can pull out the yellow 2/3 bar and red 5/6 bar to show the students that the comparison is correct.) Repeat the process with 8/10 and 3/4. Be sure to use the arrows to show the direction. Write 2 1/3 ☐ 2 2/5. Tell the students because the 2's are the same we compare the fractions by cross-multiplying to see which is larger. Complete the comparison. Write 6 1/2 ☐ 3 3/4. Tell the students they should compare the 6 and 3. Because 6>3 they don't need to compare the fraction. (You may want to ask them if $6.50 is greater than or less than $3.75 to help them see this.)

Pass out the Comparing Fractions Worksheet and model for the students what they are to do. Have them complete it. (Students who find the worksheet easy could do worksheet 38 in the Fraction Bar Kit.)

TOWER of BARS





$\frac{1}{2}$

$\frac{3}{4}$

$\frac{5}{6}$

$\frac{2}{3}$

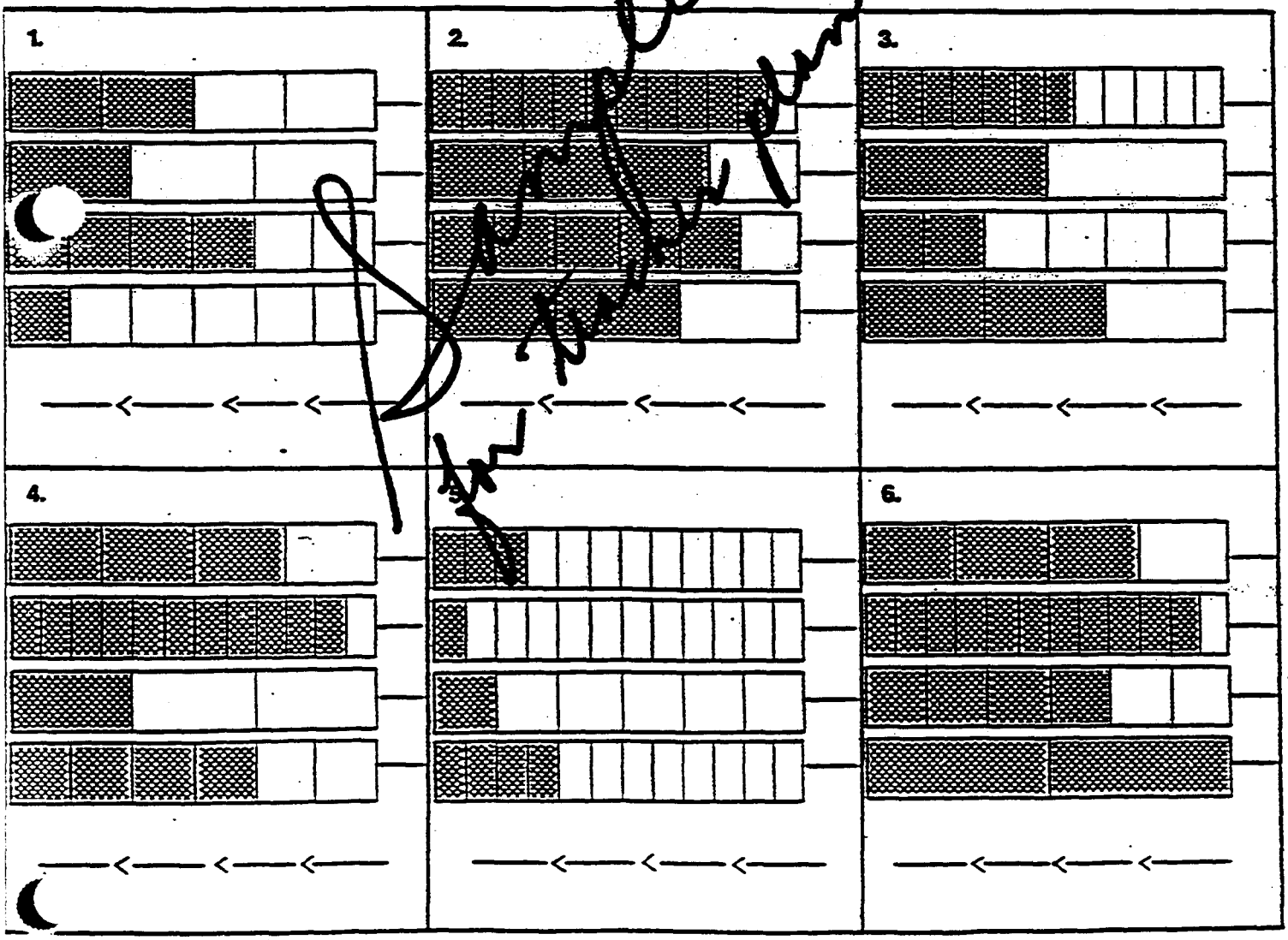
The fractions for these bars can be put in order from smallest to greatest.

$$\frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{5}{6}$$

Think

$$\frac{6}{12} < \frac{8}{12} < \frac{9}{12} < \frac{10}{12}$$

Beside each bar, write the fraction for the bar. Then write these fractions in order from smallest to greatest.

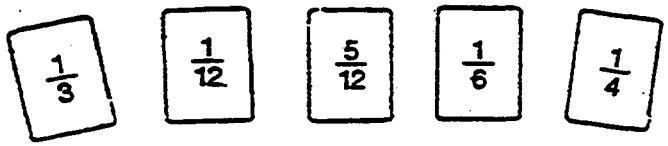


Fraction Bars
Inequality, Step 3
Objective I-3

Name _____

The game of SUPER FRIO involves putting fractions in order. Write each set of fractions in order from smallest to greatest.

1.



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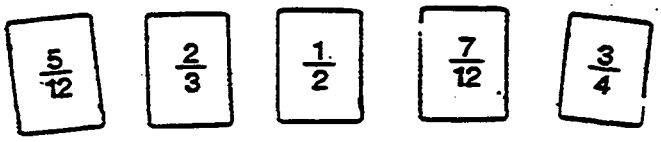

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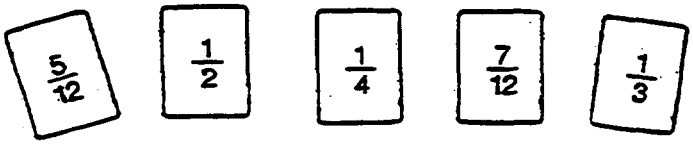

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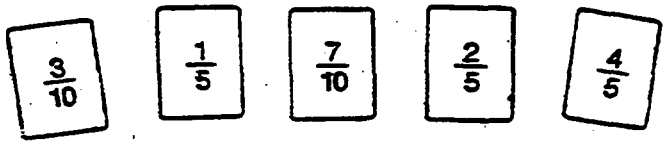

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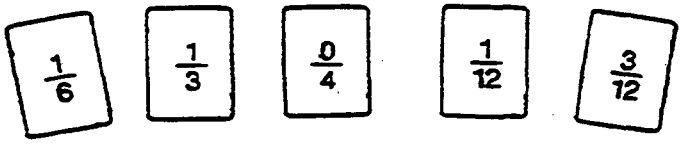
Write each set of fractions in order from greatest to smallest.

5.



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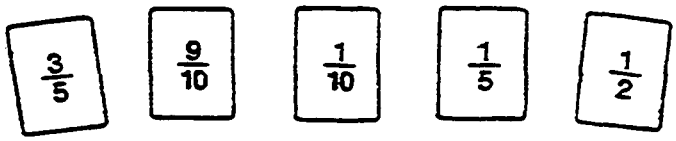

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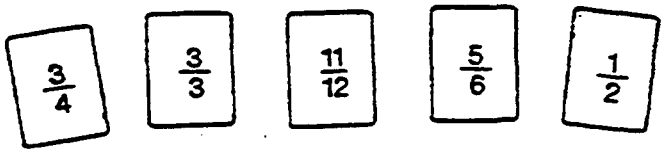
7.



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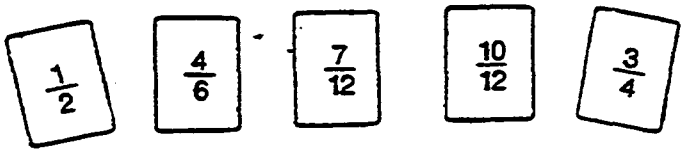

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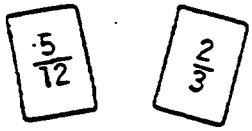
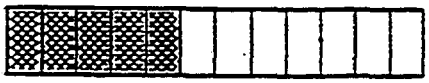
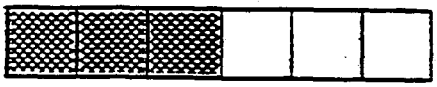
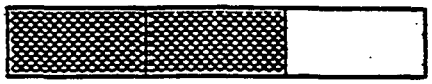
Fraction Bars
Inequality, Step 3
Objective I-3

Name _____

game, TAKE A CHANCE, fractions are compared to $\frac{1}{2}$.



Numerator less than half the denominator	Numerator half the denominator	Numerator greater than half the denominator
$\frac{5}{12} < \frac{1}{2}$	$\frac{3}{6} = \frac{1}{2}$	$\frac{2}{3} > \frac{1}{2}$
		
less than half-shaded	half-shaded	greater than half-shaded

ite the correct symbol ($<$, $=$, $>$) between each pair of fractions.

1. $\frac{4}{6} \quad \frac{1}{2}$	2. $\frac{5}{6} \quad \frac{1}{2}$	3. $\frac{3}{8} \quad \frac{1}{2}$	4. $\frac{5}{10} \quad \frac{1}{2}$
5. $\frac{1}{3} \quad \frac{1}{2}$	6. $\frac{5}{9} \quad \frac{1}{2}$	7. $\frac{7}{12} \quad \frac{1}{2}$	8. $\frac{3}{3} \quad \frac{1}{2}$
9. $\frac{4}{7} \quad \frac{1}{2}$	10. $\frac{6}{12} \quad \frac{1}{2}$	11. $\frac{5}{6} \quad \frac{1}{2}$	12. $\frac{2}{5} \quad \frac{1}{2}$

Fraction Bars
Inequality, Step 4
Objective I-4

Comparing Fractions

Name _____

Write > or < in each box.

1. $\frac{5}{6} \square \frac{2}{5}$

2. $\frac{1}{5} \square \frac{1}{4}$

3. $\frac{7}{10} \square \frac{3}{4}$

4. $2\frac{5}{8} \square 2\frac{1}{2}$

5. $\frac{7}{8} \square \frac{2}{3}$

6. $9\frac{1}{8} \square 9\frac{1}{6}$

7. $\frac{5}{8} \square \frac{3}{4}$

8. $\frac{3}{10} \square \frac{1}{4}$

9. $7\frac{3}{10} \square 8\frac{3}{10}$

10. $\frac{4}{5} \square \frac{3}{4}$

11. $4\frac{1}{6} \square 4\frac{2}{3}$

12. $12\frac{1}{2} \square 12\frac{3}{8}$

13. $\frac{4}{6} \square \frac{3}{4}$

14. $5\frac{1}{8} \square 4\frac{2}{7}$

15. $3\frac{4}{5} \square 4\frac{3}{5}$

16. $5\frac{3}{4} \square 5\frac{7}{12}$

Objective 9: Write mixed numbers as improper fractions; write improper fractions as mixed numbers.

Vocabulary

improper fraction

Materials

Overhead
Pattern blocks
Overhead pattern blocks
Lowest terms worksheet 1/student
Number Line transparency
Worksheet #1 1/student
Sheets of paper
Markers
Worksheet #2 1/student

Language Foundation

1. When working with pattern blocks, use both the *color* and the correct *geometric name* to refer to them. After hearing the term "yellow hexagon" many times, the student will use the word yellow to help visualize the hexagon when they hear the term out of context or without the word "yellow".

2. Explain the word improper as not correct, not right, not the way we usually do things. You may want to use examples of improper conduct or not following the rules.

Mathematics Component

Review lowest terms with the students. Write 12/15 on the overhead. Help them find the factors of 12 and 15, then the common factors, and the greatest common factor. Ask them how many 3's are in 12 and 15 and then write 12/15 = 4/5. Give them each a copy of the Lowest Terms Worksheet. Have them work in small groups to work the problems. Tell them each member of the group must decide on the correct answers.

Give each group pattern blocks. (Students may work in pairs or groups of 4.) Remind the groups that the yellow hexagon equals 1 whole. Have the students get 3 red trapezoids. Remind them that the red trapezoid = 1/2. Ask how many halves we have. Write 3/2. As you model, have them put 2 of the halves together to make 1 whole. Ask what mixed numeral we have now (1 1/2). Write 3/2 = 1 1/2. Tell them that 3/2 is an improper fraction because it doesn't follow the rule that numerators must be smaller than denominators. Have the students get 5 blue rhombuses and form 1 hexagon with 2 left over rhombuses as you do it on the overhead. Ask them what mixed numeral these represent. (You may need to remind them that 1 blue rhombus equals 1/3). Write 1 2/3. Have them split the 5 pieces apart. Ask what fraction you have now. Write 1 2/3 = 5/3. Ask which number is the improper fraction and why. Repeat with 6 green triangles writing 6/6 = 1. Collect the pattern blocks.

Put the number line on the overhead. Write 13/4. Count on the fourths line until you reach 13. Ask how many whole numbers you passed (3). Ask how many fourths are left beyond 3 (1). Write 13/4 = 3 1/4. Tell the students that we are going to do this another way. Divide 13 by 4. Tell them that 4 goes into 13 three times as you write 3. Tell them that since 4x3=12 there is one left over and you write it as the numerator. Write 1/4. Repeat this division process using 24/8. Pass out worksheet 1. Go over the example and do 1 or 2 problems with them. Let students work together to complete the sheet.

Give each student 2 sheets of paper. Have them fold each sheet in fourths. Have them shade in all of 1 sheet and 1/4 of the second sheet. Ask them to tell you how much paper is shaded (1 1/4). Ask how many fourths that is (5/4). Write 1 1/4 = 5/4. Tell them we are now going to go from a mixed numeral to an improper fraction. Put the number line back on the overhead. Write 2 3/4 = ?/4. Use the number line to locate 2 3/4. Count the number of fourths. Replace the ? with 11. Now show them a short cut. Tell them that in each whole number there are 4 fourths so in 2 there are 8 fourths as you write 2x4=8. Tell them we have 3 more fourths so we have 11 fourths as you write 8+3=11. Write 2 3/4 = 11/4. Repeat with 1 2/3. Repeat with 3 7/8. Pass out the worksheet. Go over the sample problems and do 1 or 2 as examples with them. Have the students complete the worksheets.

Lowest Terms Name _____

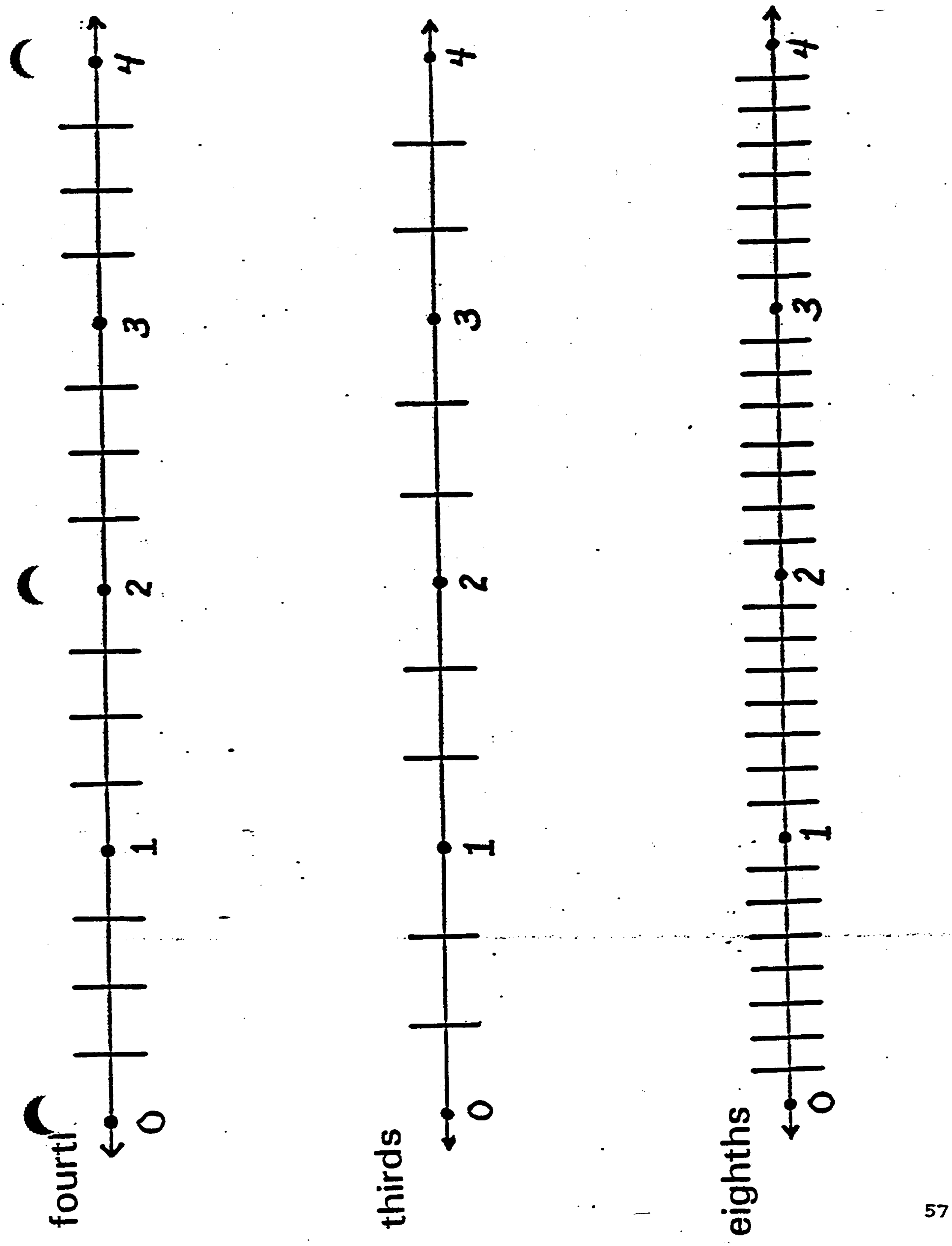
1. $\frac{9}{12} =$

2. $\frac{4}{6} =$

3. $\frac{5}{20} =$

4. $\frac{8}{12} =$

5. $\frac{20}{50} =$


fourths
0
1
2
3
4
thirds
0
1
2
3
4
eighths
0
1
2
3
4

Worksheet 1 Name _____

Examples

$\frac{9}{4} = 2\frac{1}{4}$ (4)9, 2, −8, 1

$\frac{26}{7} = 3\frac{5}{7}$ 7)26, 3, −21, 5

1. $\frac{7}{4} =$

2. $\frac{8}{3} =$

3. $\frac{27}{5} =$

4. $\frac{9}{2} =$

5. $\frac{16}{8} =$

6. $\frac{19}{6} =$

Worksheet 2 Name _____

Examples: $5\frac{1}{2} = \frac{11}{2}$ ($\times 2$, $+1$) $3\frac{2}{5} = \frac{17}{5}$ ($\times 5$, $+2$)

1. $1\frac{3}{4} =$

2. $2\frac{1}{10} =$

3. $4\frac{1}{2} =$

4. $4\frac{3}{8} =$

5. $6\frac{2}{3} =$

6. $8\frac{4}{5} =$

Objective 10: Use physical materials to add common fractions. Add fractions, whole numbers, and mixed numerals with like and unlike denominators.

Vocabulary

common denominator

Materials

Overhead
Number line transparency (p. 55)
Overhead pens
Fraction bars
Adding Mixed Numerals Transparency
Fraction Bars Worksheet, p. 45 1/student
Addition Worksheet 1/student

Optional:
Fraction Bars Worksheets 49 - 51 for additional practice

Language Foundation

1. The rules that are written in bold type and underlined should be written on a chart and posted on the wall.

2. You may need to review the meaning of the word *common* as something that 2 things share. In this lesson it will refer to a common denominator as a denominator that 2 fractions can share.

Mathematics Component

Put the number line transparency on the overhead. Under the fourths line write 2/4 + 1/4. Put an x on the 2/4 line. Then count over 1 more fourth. Ask what the answer is (3/4). Repeat using the eighths line with 3/8 + 4/8. Write the answer.

Pass out the fraction bars. Have the students find the blue 1/4 and 2/4 bars. Model placing the shaded parts end to end and then help them write the equation 1/4 + 2/4 = 3/4. Now have the students find the orange 7/12 bar and 8/12 bar. Have them put the shaded parts end to end and write the equation 7/12 + 8/12 = 15/12. Remind them that 15/12 is an improper fraction. We need to change it to a mixed numeral. Help them divide 12 into 15 and then add = 1 3/12 to the equation. (Don't worry about lowest terms at this point.) Repeat with the purple 1/5 and 4/5 bars. Have each pair of students solve their own equation by having one of the pair select a bar of any color and the other student selecting another bar of the same color. Have them write their equation and solve it. Monitor this and choose 1 or 2 to do on the overhead.

Ask the students to help you write a rule for adding fractions with equal denominators such as 2/6 + 3/6. Help them come up with <u>Add the 2 numerators and keep the denominator.</u>

Put the Adding Mixed Numerals Transparency on the overhead covering the answer portion. Ask for student input about how to find the answer. They should have no difficulty arriving at 4 17/10. Ask them what to do about the improper fraction 17/10. Write 17/10 = ?. Help them to divide to get 1 7/10. Write 4 17/10 = 4 + 1 7/10. Ask them to add 4 +1 and then add on the 7/10. (Elicit 5 7/10.) Uncover the answer so they can compare. Remove the transparency and work with them to solve 2 5/6 + 4 2/6 and 2 2/5 + 3 3/5. Ask them to help you write a rule for adding mixed numerals with equal denominators. Help them come up with <u>Add the whole numbers and fractions separately and write the answer as a mixed number.</u> Pass out worksheet 45 and model how to do it. (Some students may be ready to convert answers to lowest terms. Help them with this step. However, it is more important for them to understand the process than to reduce the answer.)

Review addition of fractions by writing 3/6 + 2/6 = 5/12 on the overhead. Ask the students if this is correct. Pass out the fraction bars and ask them to use the bars to prove whether the answer is right or not. Have someone explain what you did wrong. Ask what the right answer is. A similar activity can be done in writing as an assessment and/or homework.

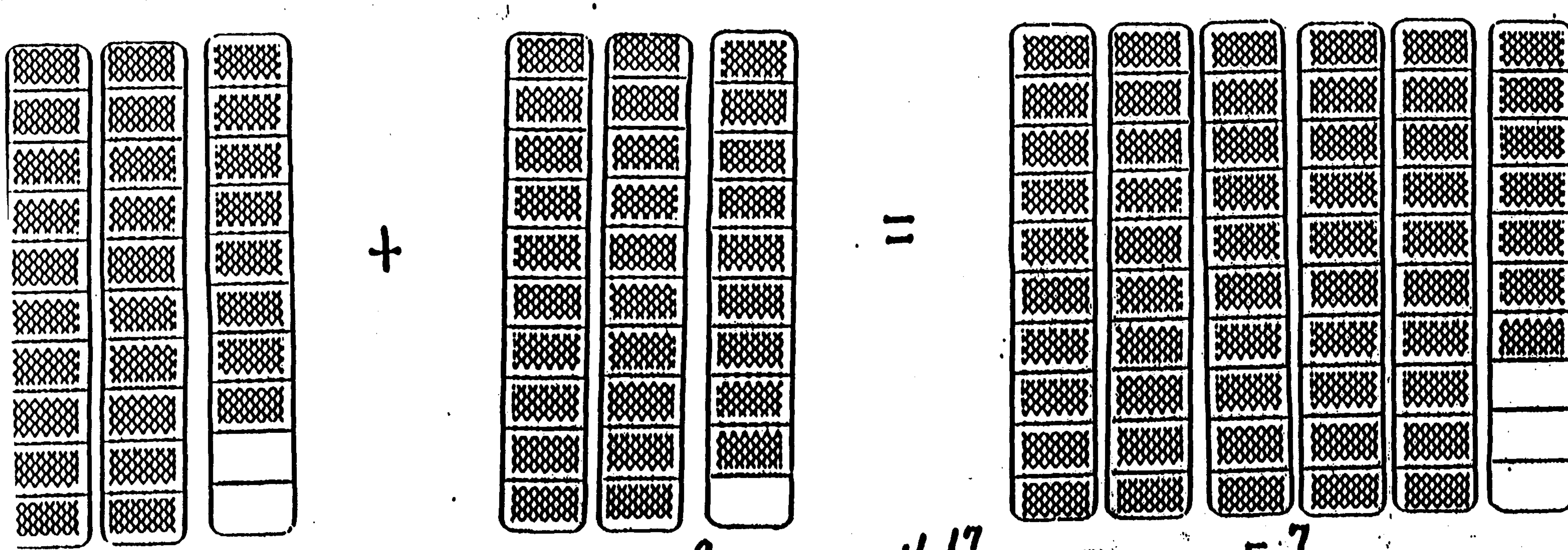
Tell the students they are now going to learn how to add fractions with unequal (different) denominators. Have them get the green 1/2 bar and the yellow 1/3 bar. As you model on the overhead, have them place the shaded portions end to end. Ask if the total shaded amount is less than or greater than 1. Ask them to find the red bar equal to the green 1/2 bar. Ask which bar it is (3/6). Write 1/2 = 3/6. Ask them to find the red bar equal to the yellow 1/3 bar and to tell you which it is (2/6). Under the 1/2 = 3/6 write + 1/3 = 2/6 to form an addition problem. Tell them they can now add the 3/6 and the 2/6. Ask for the answer. Write 5/6. Have them compare the red 5/6 bar with the 1/2 and 1/3 bar to check the answer. Repeat with the yellow 2/3 bar and blue 1/4 bar changing them to orange twelfths bars to get 11/12. (If the students need another concrete example use the green 1/2 bar and the purple 2/5 bar changing them to tenths to get 9/10.)

Tell the students they are now going to add fractions with unlike denominators without using the fraction bars. Write 1/5 = / and + 2/3 = / vertically to form an addition problem. Tell the students we will find the common denominator by multiplying 3x5. Write 15 as the new denominator in both addends. Help the students find the numerator for each fraction and then add to find the sum. When you have finished, talk through the whole process one more time. Repeat with 2/3 and 5/6.

Redo the last problem by making it a mixed numeral problem (1 2/3 + 2 5/6). (Be certain the students understand they only need to change the fractions before adding. The whole numbers do not change.) When you get to 3 27/18, help them remember what they did before to change the improper fraction so they get 3 + 1 9/18 = 4 9/18. Repeat with 1 1/6 + 1 1/4.

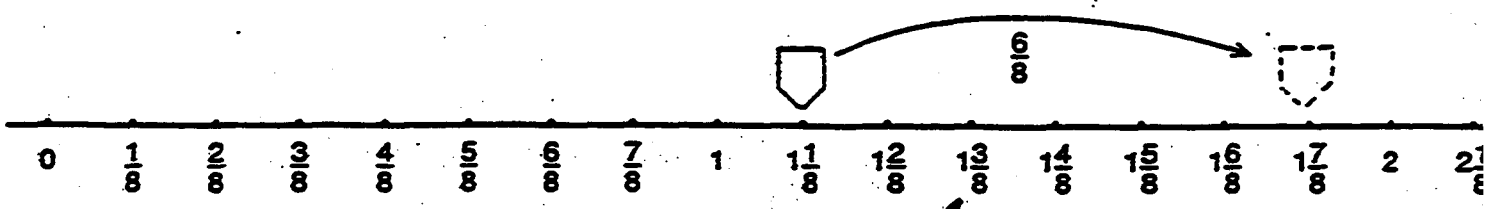
Pass out Addition Worksheet. Help the students work through the examples and the first problem. Have them finish the worksheet. If you need to spend additional time on this concept, use worksheet 51 in the Fraction Bar Kit.

Adding Mixed Numerals



$$2\tfrac{8}{10} + 2\tfrac{9}{10} = 4\tfrac{17}{10} = 5\tfrac{7}{10}$$

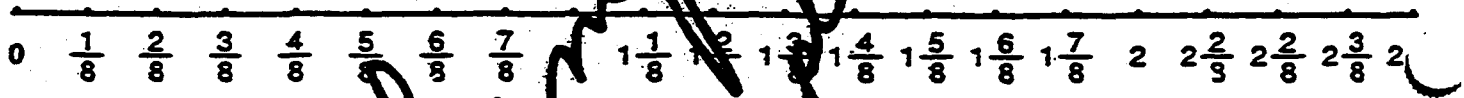
Suppose you had a marker at the point $1\frac{1}{8}$ on the NUMBER LINES mat. (Side 2). The sum, $1\frac{1}{8} + \frac{6}{8}$ can be computed by moving the marker 6 steps to the right to the point $1\frac{7}{8}$.



$$1\frac{1}{8} + \frac{6}{8} = 1\frac{7}{8}$$

$$\begin{array}{r} 1\frac{1}{8} \\ +\ \frac{6}{8} \\ \hline 1\frac{7}{8} \end{array}$$

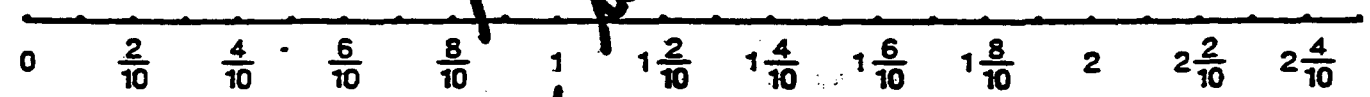
Compute the sums. Use the number lines to help you.



1. $\frac{7}{8} + 1\frac{3}{8} =$

2. $1\frac{5}{8} + \frac{5}{8} =$

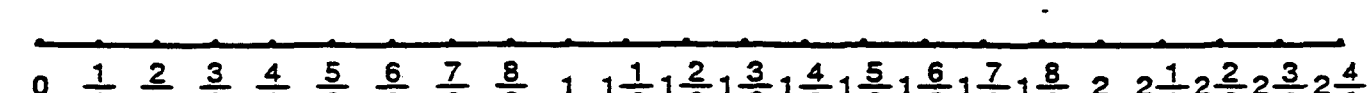
3. $1\frac{1}{8} + 1\frac{2}{8} =$



4. $1\frac{2}{10} + 1\frac{3}{10} =$

5. $\frac{6}{10} + 1\frac{6}{10} =$

6. $1\frac{5}{10} + \frac{7}{10} =$



7. $\frac{7}{9} + 1\frac{3}{9} =$

8. $1\frac{4}{9} + \frac{8}{9} =$

9. $1\frac{1}{9} + 1\frac{2}{9} =$

Addition Worksheet Examples

Name _____

$\frac{3}{8} = \_\_$

$+ \frac{1}{3} = \_\_$

$2\frac{4}{5} =$

$+ 1\frac{2}{3} =$

1. $\frac{1}{2} =$

   $+ \frac{3}{4} =$

2. $\frac{4}{5} =$

   $+ \frac{1}{6} =$

3. $\frac{4}{5} =$

   $+ \frac{3}{4} =$

4. $2\frac{1}{3} =$

   $+ 1\frac{3}{6} =$

5. $2\frac{1}{2} =$

   $+ 1\frac{2}{5} =$

6. $4\frac{2}{3} =$

   $+ 2\frac{5}{6} =$

Objective 11: Use physical materials to subtract common fractions. Subtract fractions, whole numbers, and mixed numerals with like and unlike denominators.

Vocabulary

trade
trading

Materials

Overhead
Number Line Transparency (p. 55)
Overhead Pens
Fraction Bars
Overhead Fraction Bars
Subtraction Sheet 1
1/student
Fraction Bars Worksheet, p. 64
1/student
Number Line Mats
Paper Markers
Subtraction Sheet 2
1/student
Overhead pattern blocks

Language Foundation

1. When the students need to subtract a larger number from a smaller number, the word trade (or trading) is used instead of borrow or regroup. If your students are unfamiliar with the term, introduce it as being the same thing as whatever term you are using. Put a yellow hexagon on the overhead, then "trade" it for 2 red trapezoids. Repeat by "trading" 2 green triangles for a blue rhombus. Explain that when you trade, the 2 things or groups have to be equal.