Answer Key
Obj. 36

Solving Equations Using Inverse Operation

Solution	Check	Solution	Check
1) $3 + y + 2 = 26 + 2$	$3 + y + 2 = 26 + 2$	2) $x - 16 = 37$	$x - 16 = 37$
$y + 5 = 28$	$3 + 23 + 2 = 26 + 2$	$x - 16 + 16 = 37 + 16$	$53 - 16 = 37$
$y + 5 - 5 = 28 - 5$	$28 = 28$	$x = 53$	$37 = 37$
$y = 23$			
3) $-5 + 3 + y = 12 - 6$	$-5 + 3 + y = 12 - 6$	4) $p - 11 = 36 + 2$	$p - 11 = 36 + 2$
$-2 + y = 6$	$-5 + 3 + 8 = 6$	$p - 11 = 38$	$49 - 11 = 36 + 2$
$-2 + 2 + y = 6 + 2$	$6 = 6$	$p - 11 + 11 = 38 + 11$	$38 = 38$
$y = 8$		$p = 49$	
5) $6w = 42$	$6w = 42$	6) $\frac{x}{9} = 4$	$\frac{(x)}{9} = 4$
$\frac{6w}{6} = \frac{42}{6}$	$6(7) = 42$	$\frac{9(x)}{9} = 4(9)$	$\frac{36}{9} = 4$
$w = 7$	$42 = 42$	$x = 36$	$4 = 4$
7) $15 + y - 24 = 32$	$15 + y - 24 = 32$	8) $3y + 2y = 15$	$3y + 2y = 15$
$y - 9 = 32$	$15 + 41 - 24 = 32$	$5y = 15$	$3(3) + 2(3) = 15$
$y - 9 + 9 = 32 + 9$	$32 = 32$	$\frac{5y}{5} = \frac{15}{5}$	$9 + 6 = 15$
$y = 41$		$y = 3$	$15 = 15$
9) $34 + 2 = 2x - 3 + x$	$34 + 2 = 2x - 3 + x$	10) $\frac{y}{8} = 4$	$\frac{y}{8} = 4$
$36 = 3x - 3$	$34 + 2 = 2(13) - 3 + 13$	$(8)\frac{y}{8} = 4(8)$	$\frac{32}{8} = 4$
$36 + 3 = 3x - 3 + 3$	$36 = 26 - 3 + 13$	$y = 32$	$4 = 4$
$39 = 3x$	$36 = 36$		
$\frac{39}{3} = \frac{3x}{3}$			
$13 = x$			

Answer Key
Obj. 36

Using Inverse Operation (Students should show all steps as modeled on the previous page.)

1)	m = 50	11)	y = 1
2)	z = 16	12)	a = 2
3)	h = 4 1/2	13)	m = 2
4)	r = 4	14)	r = 5
5)	z = 7.5	15)	b = 3
6)	x = 1	16)	t = 2
7)	m = 32	17)	m = 22
8)	q = 10	18)	p = 29
9)	a = 0	19)	v = 0
10)	s = 60	20)	k = 9

Objective 37: Solve Problems Using Equations

Vocabulary

inverse
variable
represent
situation
sequence

Materials

Equations and Problem Solving
Problem Solving Assessment
student copies

Language Foundation

1. Put a "+" sign on the board. Ask students what this represents. (plus or addition) Tell them that we can use symbols of signs to represent words or operations. For instance, we use variables to **represent** numbers.

2. The word **sequence** means the order in which something happens, or the order in which we perform certain tasks. Making bread or going to a friend's house are examples of when you would need to follow a special sequence or a special order. We also use a special sequence when we put words into alphabetical order.

3. **Situation** in this lesson refers to a problem to be solved. In representing the situation, the students will be showing what the problem is in an equation.

Mathematics Component

1. Provide students with a copy of the activity sheet Equations and Problem Solving.
   - Read over the directions together.
   - State that the students will "define the variable" and discuss the meaning of this phrase. (Each student will select a variable to represent the unknown.)
   - Tell students that they are going to use the skills developed in past lessons to write equations which **represent a situation** and then solve the equation using **inverse operation.** Students often have difficulty with this task because they want to jump right in and write an equation which shows how to solve the problem, instead of how to **represent** the problem. Therefore, it is suggested that the first few equations be written together. For example, for the first problem:

     Chan paid a total of $15 for a shirt on sale. It had been reduced by $9.
     What was the regular price of the shirt?

     Students will often want to write: $15 + $9 = r (regular price)

   - Explain that this shows how to **solve** the problem, but does not represent what took place. It also **does not** require the use of an inverse operation to find a solution! (To find the solution for 15 + 9 = r, students would merely simplify 15 + 9 = 24.)
   - Go back over the sequence of what happened in the problem. The original price was reduced by $9 and then Chan paid a total of $15. Choose the letter "r" as the variable to represent the regular price. The problem can be represented by:

     r - 9 = 15
     r - 9 + 9 = 15 + 9
     r = 24 (regular price was $24)

   - Work the second problem on the sheet together in the same way, explaining that the way to **represent the information** is 7 + n = 16. The cost of one item and the cost of the second item combine to total sixteen dollars.
   - Allow students to work with a partner to complete the remaining problems. However, it is best to have students complete individual activity sheets.
   - When completed use a transparency copy to read over each problem and discuss as a group. Problem number 7 is one that students seem to have the most difficulty with. Remind students that we are trying to represent the information in a way that will help us determine the number of **wins**. The best way to represent the information in a way that requires the use of one inverse operation would be w - 9 = 14.

Additional Activity

- Assessment- Ask students to make up a problem based on the information given on the Problem Solving Assessment sheet which is provided. A space is provided for all students to illustrate the problem. This can be used in place of the writing activity for those students who have limited writing skills.

Name _____
Date _____

Equations and Problem Solving

Directions:

a) **Define** a variable for each problem.
b) **Write** an equation to represent the information.
c) **Be sure** the equation requires the use of one inverse operation to find the solution!
d) **Show** a check for each solution.

1) Chan paid $15 for a shirt on sale. It was reduced by $9. What was the regular price of the shirt?

2) Together two items cost $16. One item costs $7. What does the other item cost?

3) A four-person seventh grade relay team ran a race in 48 seconds. What was the average time for each person?

4) A student earned $175 last week. She worked 35 hours. How much did she earn per hour?

5) In a recent year, 4635 runners started a race. Only 2338 finished the race. How many runners did not finish the race?

6) Two baseball players each hit a total of 117 home runs. One of the players hit 61. How many did a second player hit?

7) A school basketball team lost 14 games. They won 9 games more than they lost. How many games did they win?

8) Maria paid a total of $26 for two new jackets. Maria's first jacket cost $11. What was the price of the second jacket?

Name _____
Date _____

Problem Solving Assessment

Make up a problem to solve using the following information.

x = the number of students in one class
Use the equation x + 5 = 33 to solve the problem.

Problem:

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Solution

Answer Key
Obj. 37

Equations and Problem Solving (Students should show all work and a check as modeled in the previous lesson.)

1) $r - 9 = 15$
$r = 24$

2) $p + 7 = 16$
$p = 9$

3) $4t = 48$
$t = 12$

4) $35h = 175$
$h = 5$

5) $4635 - q = 2338$ or $2338 + r = 4635$
$q = 2297$

6) $p + 61 = 117$
$p = 56$

7) $w - 9 = 14$
$w = 23$

(Remind students that they need an equation that uses one inverse operation in order to be solved! The number of wins minus 9 represents the number of games lost.)

8) $11 + j = 26$
$j = 15$

Objective 38: Simplify and solve two-step equations.

Vocabulary

equation
inverse operation
integers
simplify
combine
distributive property

Language Foundation

1. Review vocabulary presented in previous lessons.

2. Relate **distributive** to distribution (noun) and distribute (verb). Explain that 'distribute' means to give out something to each member of a group. When we distribute things, we spread them out . With the distributive (adj.) property we can spread out numbers so we can work with them more easily.

Materials

Solving Two-Step Equations Practice Sheet
student copies

Mathematics Component

1. Review with students the procedure for solving one-step **equations**; that is, **simplify** by combining like terms, and then solve using **inverse operation**.
   - For example:

     Gina worked 2 hours on Tuesday, 8 hours on Wednesday, 6 hours on Thursday, and 1 hour on Friday. If she worked a total of 20 hours last week, how many hours did Gina work on Monday?

     $x + 2 + 8 + 6 + 1 = 20$

     $x + 17 = 20$ (simplify by combining like terms)

     $x + 17 - 17 = 20 - 17$ (subtraction is the inverse of addition)

     $x = 3$ Gina worked 3 hours on Monday.

2. To solve some equations, we must use more than one inverse operation.
   - For example:

     Tony is paid $5 per hour plus $3 each day for his bus fare. In order to determine how many hours Tony worked to earn $38 one day, we can set up the following equation:

     $5x + 3 = 38$ ($5 per hour * x hours plus $3 bus fare = $38 total)

     $5x + 3 - 3 = 38 - 3$ Since there are no like terms to combine, we undo the addition by subtracting 3 from both sides of the equation.

     $\frac{5x}{5} = \frac{35}{5}$ Next, we undo the multiplication by dividing both sides of the equation by 5.

   - In general, to solve two-step equations, first undo the addition or subtraction, using the inverse operation; then undo the multiplication or division, using the inverse operation.

     How many hours did Tony work if he was paid $40.50?

     $5x + 3 = 40.5$

     $5x + 3 - 3 = 40.5 - 3$

     $\frac{5x}{5} = \frac{37.5}{5}$

     $x = 7.5$ Tony worked 7.5 hours to earn $40.50

- Ask students which inverse operations should be used to solve the following equations:

$3x - 6 = 24$ Add 6 to both sides, then divide both sides by 3

$\frac{x}{2} + 3 = 8$ Subtract 3 from both sides, then multiple both sides by 2

- Have students work through the steps in solving the following equations:

$2x + 2 + x = 17 - 3$ Combine like terms on the left side and on the right side of the = sign

$3x + 2 = 14$

$3x + 2 - 2 = 14 - 2$ Subtract 2 from both sides

$\frac{3x}{3} = \frac{12}{3}$ Divide both sides by 3

$x = 4$

$\frac{x}{3} + 7 - 3 = 8 + 3$ Combine like terms on the left side and on the right side of the = sign.

$\frac{x}{3} + 4 = 11$

$\frac{x}{3} + 4 - 4 = 11 - 4$ Subtract 4 from both sides

$\frac{x}{3} = 7$

$3 * \frac{x}{3} = 7 * 3$ Multiply both sides by 3

$x = 21$

3. Next, have students work through an equation that involves **integer** rules:

$2x + 5 - 18 = -3 + -4$ Use integer rules to simplify both sides by combining like terms

$2x - 13 = -7$

$2x - 13 + 13 = -7 + 13$ Add 13 to both sides

$\frac{2x}{2} = \frac{6}{2}$ Divide both sides by 2

$x = 3$

4. Demonstrate how to solve a 2 step equation with the **distributive property**.

$2(x + 5) = 20$ Before this can be solved the distributive property must be used to simplify the left side of the equation.

$2(x) + 2(5) = 20$

$2x + 10 = 20$

$2x + 10 - 10 = 20 - 10$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

6. Distribute Solving Two-Step Equations Practice Sheet

Name_____

Solving Two-Step Equations Practice Sheet

Solve:

1.	$6m + 3 = 15$	2.	$6m + 3 = -15$	3.	$-6m + 3 = 15$
4.	$6m - 3 = 15$	5.	$6m - 3 = -15$	6.	$-6m - 3 = 15$
7.	$\frac{x}{5} + 4 = 12$	8.	$\frac{t}{2} + 3 = 18$	9.	$\frac{p}{3} - 7 = 13$
10.	$6 + 7x = 27$	11.	$-5 + 2t = 19$	12.	$1 - 3x = 28$
13.	$2(x+ 4) = 32$	14.	$3(n + 2) = 18$	15.	$4(t - 5) = 24$
16.	$-3(t + 7) = 3$	17.	$-2(x - 4) = 30$	18.	$-5(p + 6) = -30$

Solve using an equation:

19. Lynn's second test score was 60 points less than twice her first score. If her second score was 86, what was Lynn's first test score?

20. The sum of three times a number and seven is 55. Find the number.

21. Concert tickets cost $18 each and parking costs $6.50. How many people went to the concert if the total cost was $78.50?

22. At his new job, Rodrigo earns $100 more than twice the salary he earned as a college intern. He now earns $2100. How much did he earn as an intern?

23. Frank's second bowling score was 72 points less than twice his first score. If Frank's second score was 156, what was his first score?

24. On her thirteenth birthday, Diana's allowance was increased to $33 per month. This was $5 more than twice her original allowance. What was Diana's allowance before the increase?

25. The number of drummers in a band is two more than three times the number of tuba players. If there are eight drummers in the band, how many tuba players are there?

Answer Key
Obj.38

Solving Two-Step Equations

1.	2	2.	-3	3.	-2	4.	3
5.	-2	6.	-3	7.	40	8.	30
9.	60	10.	3	11.	12	12.	-9
13.	12	14.	4	15.	11	16.	-8
17.	-13	18.	0	19.	73	20.	16
21.	4	22.	$1000	23.	114	24.	$14
25.	2						

Obj. 39: Solve Inequalities

Vocabulary

equal
greater than
less than
equation
inequality
rational numbers
number line
solution
reverse
direction
constant
variable
sum
difference
product
quotient

Materials

Solving Inequalities Practice Sheet
Solving Special Inequalities Practice Sheet
student copies

Language Foundation

1. Explain that the prefix 'in' often means NOT. Give examples using words such as injustice, incomplete, indefinate. Ask students what they think **inequality** means. (Not equal to) An inequality in math means the left hand side of a number sentence is NOT equal to the right hand side.

2. Tell students **reverse** means to do the opposite or to go in the opposite direction. Ask students what will happen if a car is put in reverse. (It will go backwards.) Have a student walk forward. Tell them to reverse direction. (Student should walk backward or turn in opposite direction 180°.)

Mathematics Component

1. Begin by reviewing the concepts of equal, greater than, and less than.
   - For instance, $2 + 5 = 7$ states that the sum of 2 and 5 is equal to the quantity, 7. The statement is true because the sum of 2 and 5 is 7, and 7 is equal to 7.
   - We solve **equations** such as $x + 5 = 7$ by finding a value for x which will make the statement true. What number added to 5 will produce a quantity equal to 7?

$x + 5 = 7$	What number added to 5 equals 7?
$x + 5 - 5 = 7 - 5$	Add negative 5 to both sides; or subtract 5 from both sides
$x = 2$	The solution is 2.

   - We can compare numbers or quantities by using symbols for greater than and less than.

$5 > 2$	Five **is greater than** two. The symbol > means greater than.
$2 < 10$	Two **is less than** 10. The symbol < means less than.
$3 > -3$	Three **is greater than** negative 3.
$-1 < 0$	Negative one **is less than** zero.

2. The statement $x + 5 > 7$ is called an **inequality**, and it can be solved by using a method similar to the one used to solve the **equation**, $x + 5 = 7$.
   - In the equation, we find a value for x to add to 5 which will produce a value of 7. In the inequality, we are looking for values for x to add to 5 which will produce a value **greater than** seven. We will see that there are many numbers which apply, and which will make the statement true.

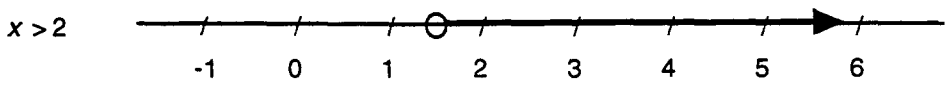
$x + 5 > 7$	What number, added to 5, will produce a quantity greater than 7?
$x + 5 - 5 > 7 - 5$	Add negative five, or subtract five, from both sides
$x > 2$	Any number **greater than two** will produce a quantity **greater than seven.**

   - Ask students for some examples of numbers greater than two. Substitute their answers for x in the inequality, and see if the statement is then true.

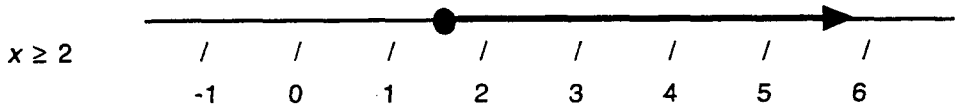
$3 + 5 > 7$	True
$10 + 5 > 7$	True
$3.1 + 5 > 7$	True

   - Discuss rational numbers such as 3.1, 3.01, 3 1/2, 3 1/5, etc. Since we cannot determine the **first** rational number which will make the statement true, we can use the number line to show the set of numbers which will make the statement true.

- To graph $x > 2$, we place an open dot at 2, and draw a line to the right, indicating that all numbers greater than two will make the inequality true.



- To graph $x \geq 2$, find values for x which are **greater than or equal to two.** Since two **is equal to** two, we must include two in the set of numbers which will make the statement true. To graph the set of numbers which are **equal to or greater than two,** we place a closed dot at two, and draw a line to the right, indicating that two and all numbers greater than two will make the statement true.



3. Practice solving simple inequalities with students, such as the following:

$x + 7 > 15$	What number added to 7 will produce a quantity greater than 15?
$x + 7 - 7 > 15 - 7$	Add negative seven, or subtract seven from both sides.
$x > 8$	Any number greater than 8 will make the statement true.

- Ask students how they would show the solutions on a number line. (Open dot at 8, with a line drawn to the right.)
- Repeat the procedure with the inequality, $x + 7 \geq 15$:

  Add negative seven, or subtract seven, from both sides.

  $x \geq 8$ Eight, or any number greater than 8, will make the statement true
- The graph of the solution would be a closed dot at eight, with a line drawn to the right.
- Graph the same equation using the symbol $\leq$ ($x + 7 \leq 15$) and compare the results.
- Practice solving and graphing these examples with students:

$x - 15 < 3$ $\quad 3x + 15 \leq 60$ $\quad 5x - 7 > 23$ $\quad \frac{x}{2} + 2.5 \geq 7.5$

4. Review with students the procedure involved in translating sentences into inequalities.
   - For example:

The sum of a number x and 3 is less than or equal to 10	$x + 3 \leq 10$
The product of 5 and a number t is greater than 35	$5t > 35$
The difference of a number p and seven is less than two	$p - 7 < 2$
The quotient of a number n divided by 3 is greater than or equal to 5 equal to 5	$\frac{n}{3} \geq 5$

- Distribute Solving Inequalities Practice Sheet.

5. After students have practiced solving inequalities and graphing them on the number line, explore the special case of reversing the direction of the inequality symbol.
   - It is necessary to reverse the direction of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.
   - Have students consider the following statements:

$6 > 2$	Six is greater than 2. True
$3(6) > 3(2)$	Three times six (18) is greater than three times 2 (6). True
$\frac{6}{2} > \frac{2}{2}$	Six divided by 2 (3) is greater than 2 divided by 2 (1). True

   - However, consider the following:

$6 > 2$	Six is greater than 2. True
$-1(6) \; ? \; -1(2)$	Negative one times six is negative 6. Negative one times two is negative two.
$-6 \; ? \; -2$	
$-6 < -2$	In order to make the statement true, we must use the symbol <, for **less than**, because negative six is a lesser quantity than negative two.

   - Consider this example:

$10 < 20$	Ten is less than 20. True
$\frac{10}{-5} \; ? \; \frac{20}{-5}$	Ten divided by negative 5 is negative 2. Twenty divided by negative 5 is negative 4.
$-2 \; ? \; -4$	
$-2 > -4$	In order to make the statement true, we must use the symbol > , for **greater than**, because negative two is greater than negative 4.

   - Rule: When multiplying or dividing an inequality by a negative number, **reverse the direction** of the inequality symbol to make the statement true.

- Here is another example:

  $-x + 3 > 7$ can be solved in two ways.

First way:	Second way:
$-x + 3 - 3 > 7 - 3$	$-x + x + 3 > 7 + x$
$-x > 4$	$3 > 7 + x$
$-1(-x) < -1(4)$	$3 - 7 > 7 - 7 + x$
$x < -4$	$-4 > x$ or $x < -4$

  The first way involves multiplying both sides of the equation by negative one. In order to make the statement true, the direction of the inequality symbol must be reversed.

  The second way involves moving the variable to the other side of the inequality. The direction of the symbol remains the same, but the answer is read, *x is less than negative four.*

- Practice solving the following examples with students:

  $-2x + 3 > -13$ $\qquad$ $5 - 4x \leq 17$ $\qquad$ $\frac{x}{2} + 3 < 7$

- Distribute Solving Special Inequalities Practice Sheet.

Name_____

Solving Inequalities Practice Sheet

Write an inequality representing each statement. Then solve each inequality.

1. The sum of a number n and 2 is greater than or equal to 4.

2. The product of 3 and a number t is less than 15.

3. The quotient of a number p divided by 6 is greater than 12.

4. The difference of a number x and 12 is less than or equal to 27.

Solve each inequality.

5. $y - 3 < 1$

6. $6t \geq 12$

7. $2p \leq -1$

8. $\frac{5x}{2} \geq 10$

9. $5r > 45$

10. $m + 1.5 \leq 10.5$

11. $3x < -33$

12. $7 + y \geq 10$

13. $2t + 12 > 4$

14. $5.7 + 3x < 2.1$

15. $\frac{(x + 3)}{4} \leq 5$

16. $\frac{x}{2} - 7 \geq 12$

17. $3 + \frac{t}{5} \leq 3.4$

Solve each inequality and graph solutions on a number line.

18. $3p + 27 < 36$

19. $5(x - 2) > 20$

20. $7 - 3x + 12x \geq 25$

21. $2 + 3t - 18 \leq 5$

Name _____

Solving Special Inequalities Practice Sheet

Write an inequality representing each statement. Then solve each inequality.

1. The product of -3 and a number t is greater than 27.

2. The quotient of a number x divided by 4 is less than 36.

3. The sum of a number $-5y$ and 7 is greater than or equal to -28.

4. The difference of a number $-3p$ and 12 is less than or equal to 48.

Solve each inequality.

5. $-4t + 3 < 19$

6. $2 - 6p \geq -22$

7. $-18b \leq 90$

8. $37 - x > 36$

9. $\frac{2}{3} - x < \frac{17}{3}$

10. $-4(y + 4) \geq 24$

Solve each inequality and graph solutions on a number line.

11. $-4x > 12$

12. $-2(t - 7) \leq 10$

13. $\frac{x}{-2} \geq 3$

14. $8 - p < -1$

Answer Key
Obj. 39

Solving Inequalities Practice Sheet

1.	$n + 2 \geq 4$ $n \geq = 2$	2.	$3t < 15$ $t < 5$	3.	$p/6 > 12$ $p < 72$
4.	$x - 12 \leq 27$ $x \leq = 39$	5.	$y < 4$	6.	$t \geq 2$
7.	$p \leq -1/2$	8.	$x \geq 4$	9.	$r > 9$
10.	$m \leq 9$	11.	$x < -11$	12.	$y \geq 3$
13.	$t \geq -4$	14.	$x < -1.2$	15.	$x \leq 17$
16.	$x \geq 38$	17.	$t \leq 2$	18.	$p < 3$
19.	$x > 6$	20.	$x \geq 2$	21.	$t \leq 7$

Solving Special Inequalities Practice Sheet

1.	$-3t > 27$ $t < -9$	2.	$x/4 < 36$ $x < 144$	3.	$-5y + 7 \geq -28$ $y \leq 7$
4.	$-3p - 12 \leq 48$ $p \geq 16$	5.	$t > -4$	6.	$p \leq 4$
7.	$b \geq -5$	8.	$x < 1$	9.	$x > -5$
10.	$y \leq -10$	11.	$x < -3$	12.	$t \geq 2$
13.	$x \leq -6$	14.	$p > 9$		

Obj. 40: Graph inequalities

Vocabulary

function table
equation
point
line segment
line
region
above
below
boundary line
shade

Materials

rulers
graph paper

Graphing Inequalities Practice Sheet
student copies

Language Foundation

1. Explain the difference between **point** used as a decimal, a period, and a point on a graph. In graphing, a point marks the location of an ordered pair on a coordinate plane.

2. Tell students a segment is a small piece of a larger object. For example, if you peel an orange, you will see all the segments. Cut a piece of string into smaller pieces and show students the segments. A **line segment** is a small piece or section of a line.

3. Explain that **region** means a certain area. What region of the world do you come from? East coast, west coast, southern states,etc. In this lesson, students will be working with certain regions or areas of a graph.

4. Ask students for synonyms for **above.** (on top of , over) The sky is **above** us.

5. Ask students for synonyms for **below.** (on the bottom of, under) The floor is below the desk.

6. A **boundary** is a dividing line. The line between states is a boundary. Ask students where the boundary lines are in football or soccer. (sidelines)

7. **Shade** means to color in softly. Shade some colors on a sheet of paper, showing the difference between shading and deep color. You might also point out the difference between shade the verb, and a shade, used as a noun.

Mathematics Component

1. Begin by reviewing the procedure for graphing the equation, $y = x + 1$.
   - Make a function table and plot the following points on a transparency.

x	y
6	7
3	4
0	1
-3	-2
-6	-5

   - Now draw a line through the points which extends to the edges of the graph.
   - Explain to students that the graph of the equation $y = x + 1$ separates the coordinate plane into three sets of points:

     (1) those above the line, such as (-4, 5)
     (2) those below the line, such as (3, 1), and
     (3) those on the line, such as (2, 3).

   - The region *above* the line is the graph of the set of solutions of the inequality $y > x + 1$.
   - The region *below* the line is the graph of the set of solutions of the inequality $y < x + 1$.
   - The line $y = x + 1$ forms the ***boundary line*** of the graphs of the inequalities $y > x + 1$ and $y < x + 1$. For any inequality we can get the boundary line by replacing the inequality symbol with the "equals" symbol.
   - Since the graph of an inequality consists of all the points above or below a boundary line, we use shading to indicate the region. If the boundary line is part of the graph, it is drawn with a solid line. If the boundary line is not part of the graph, we use a dashed line.

2. Show students the following graphs on a transparency:

   $y > x + 1$ $\quad y \geq x + 1$ $\quad y < x + 1$ $\quad y \leq x + 1$

   - To check whether the shading is correct for an inequality, choose a point in the shaded region.
   - Then substitute its coordinates for x and y in the inequality. If the coordinates satisfy the inequality, then the shading is correct.

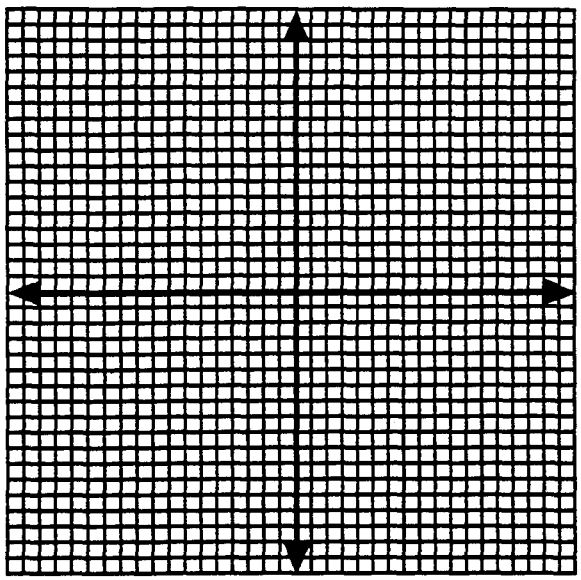
3. Practice graphing the following on a transparency:

   $y - x < 7$ (Rearrange to $y < x + 7$) $\quad x > 3$ $\quad y \leq 1$ $\quad y > x$

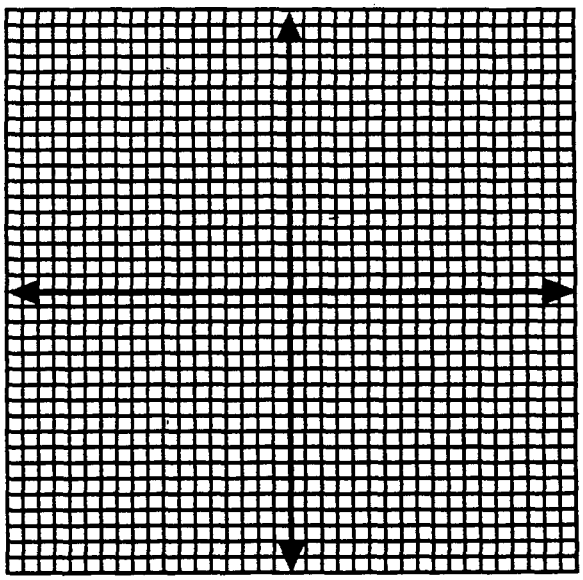
   - Distribute Graphing Inequalities Practice Sheet.

Name _____
Date _____

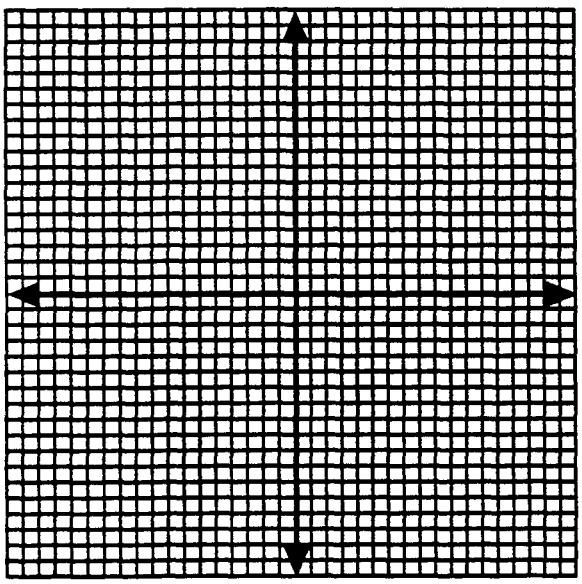
Graphing Inequalities Practice Sheet



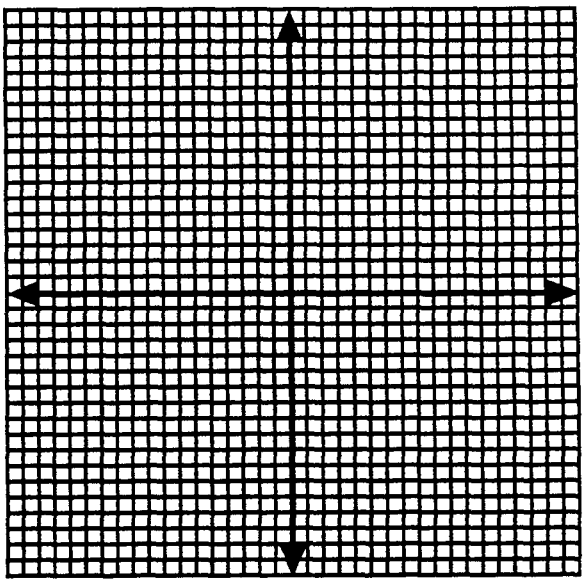
$y < x - 3$



$y > x + 2$

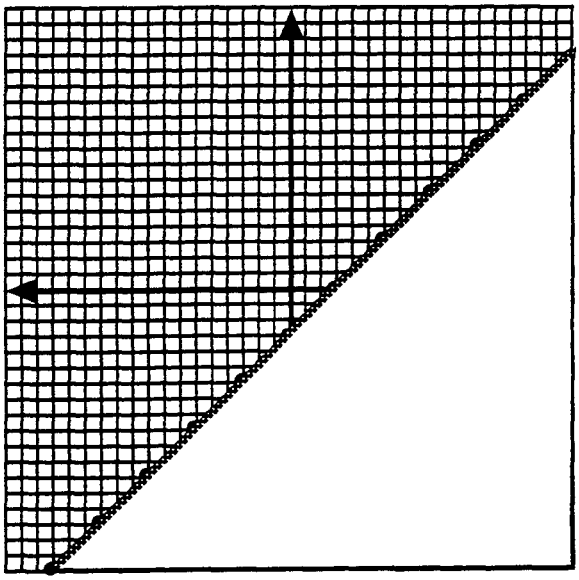


$y \leq \frac{x}{3}$

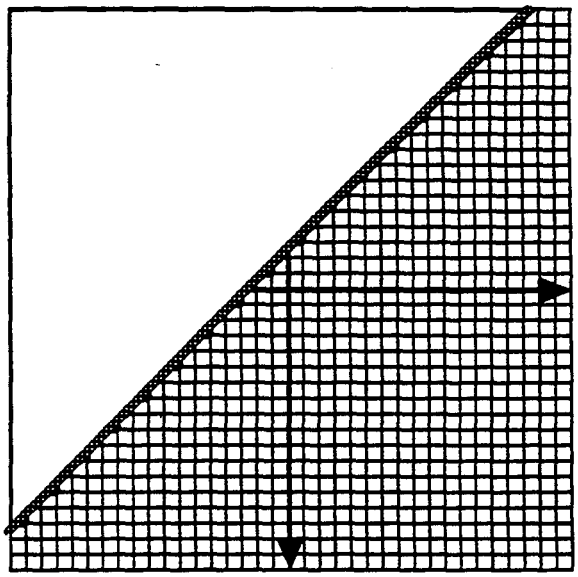


$y - x \geq 8$

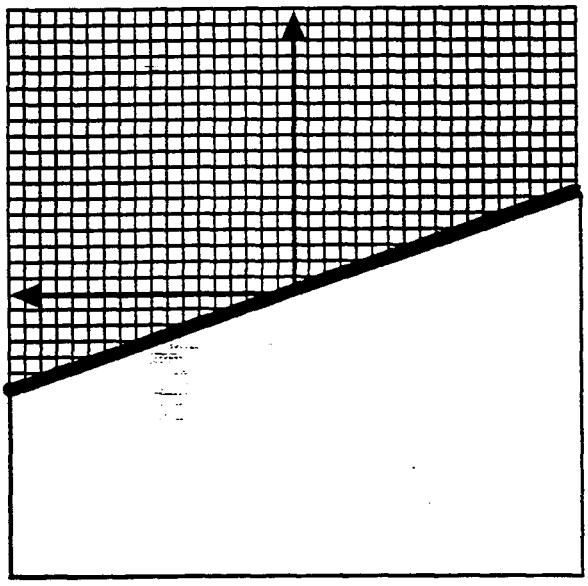
Answer Key
Obj. 40



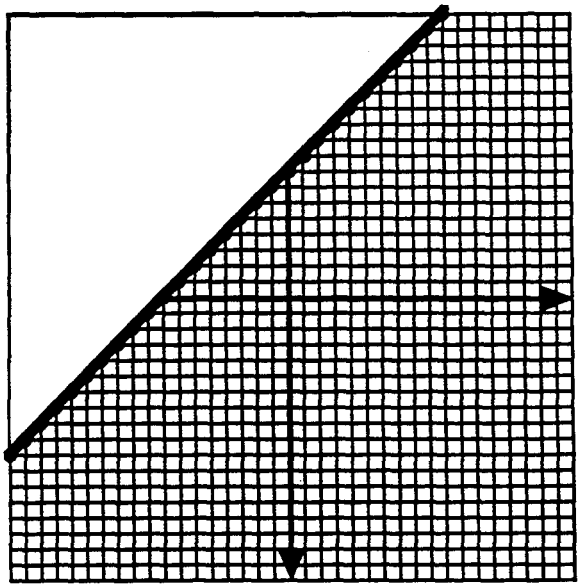
$y < x - 3$



$y > x + 2$



$y \leq \frac{x}{3}$



$y - x \geq 8$

Geometry

Objective 42: Identify, draw, and define the characteristics of polygons through ten sides. Explore quadrilaterals.

Vocabulary

polygon
regular polygon
irregular polygon
triangle
quadrilateral
pentagon
hexagon
heptagon
octagon
nonagon
decagon
interior angles

Materials

calculators
overhead geoboard
markers/colored pencils
protractors

Transparencies:
- Name that Polygon!
- Regular Polygons
- Quadrilaterals
- Family
- Quadrilateral Venn Diagram

Student Copies:
- Drawing Regular and Irregular Polygons
- Practice With Polygons
- Learning About Quadrilaterals
- Quadrilateral Review and Diagram
- Polygon Vocabulary Practice
- Vocabulary Challenge

Language Foundation

1. Introduce and/or review prefixes with students. Point out that a prefix is a word part that comes before a word and can change the meaning of a word. Knowing prefixes could help students determine the meaning of unfamiliar words.

   Give them the prefix **poly** which means many. Explain that in this lesson, students will be learning about **polygons**, figures with many sides.

   Tell students that there are many numerical prefixes. Give students the following number prefixes:

   - **tri** = three
   - **quad** = four
   - **penta, pent** = five
   - **sexa, sex** = six (in Greek =**hexa**)
   - **septi, sept** = seven (in Greek =**hepta**)
   - **octa** = eight
   - **nona** = nine
   - **dec, deca, deka** = ten

   Explain that if students add these prefixes to other word parts, they can make a word that means a figure with a certain number of sides. Have them tell you how many sides they think there are in a **decagon**, **nonagon**, **octagon**, **septagon**, **hexagon**, **pentagon**, **quadrilateral**, and a **triangle**.

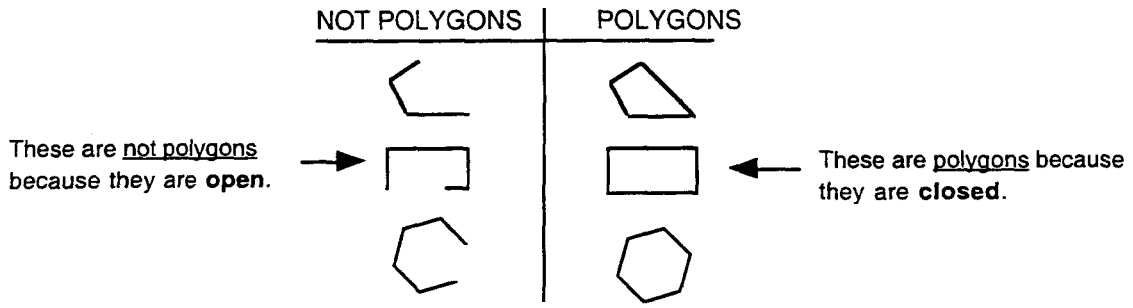
   Give students a few other examples of words that use these prefixes such as triplets, tricycle, tripod, quadruplets, quadrangle, quarter, octopus,and decade. Have students brainstorm other examples.

   For Spanish speakers, point out that many numbers in Spanish resemble these prefixes. Have students tell the class numbers 3, 4, and 6 - 10 in Spanish: tres, cuatro, seis, siete, ocho, nueve, and diez.

2. Teach students the prefix **ir** (not). Explain that words that begin with the prefix **ir** often change the meaning of the root word to the opposite. Brainstorm examples with the class such as irregular, irresistible, and irresponsible.

Mathematics Component

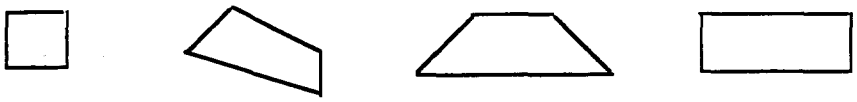
1. Identify and name polygons (closed figures made of line segments) through ten sides and distinguish between regular and irregular figures.
   - Write the word **polygon** on the board and underline the prefix "poly." Remind students that "poly" means MANY. Review that in math a polygon is a closed figure made of line segments with MANY sides and angles."
   - Illustrate that polygons are closed figures by drawing the following chart.



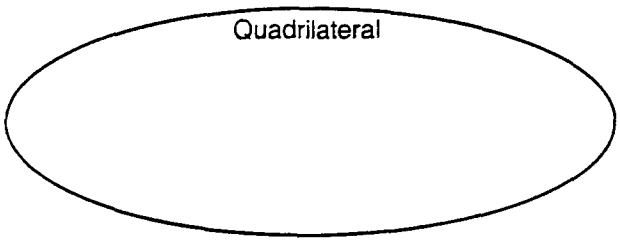
   - Draw a circle and ask students to explain why a circle is not a polygon even though it is a closed figure. (It is not made of line segments. Line segments are straight.)
   - Ask students to work with a partner to draw a polygon which has the least possible number of sides and angles. Have a student model the solution using an overhead geoboard. (Students should draw a triangle.) Discuss why a triangle is the polygon with the least number of sides. (The line segments of a polygon can not be closed with less than three sides.)
   - Place the transparency Name that Polygon! on the overhead, covering the last line. Review number prefixes discussed in the Language Foundation. Tell students that these prefixes are used to name polygons. Have students choose the appropriate name for each of the polygons from the word bank at the bottom of the page. Discuss why each name is appropriate as you write and pronounce the word.
   - Point to the last row of polygons. Tell students that there is something that makes these three polygons different from the rest of the polygons. Have students talk with a partner about how they are different. Share students' ideas. Lead them to understand that the bottom three polygons are **regular polygons** because for each one, the line segments are the same length and the angles are the same measure. Label the bottom three as "regular polygons."
   - Tell students that the polygons which do not have equal line segments and equal angles are called **irregular polygons**. Label the first two rows as "irregular polygons."
   - Show the demonstration transparency Regular Polygons. Have students name each polygon and tell the number of equal sides and equal angles in each. The wall poster /transparency with the answers to Regular Polygons can be posted as a wall chart for reinforcement.

- Distribute student copies of Drawing Regular and Irregular Polygons. Review directions together, reminding students that they must include at least two **regular polygons**.
- The sheet Practice With Polygons is included for further practice on this topic..

2. Explore quadrilaterals.
   - Review that a **quadrilateral** is a polygon made of four line segments. Remind students that quadrilaterals can look very different from each other. Draw several different examples of quadrilaterals on the board.

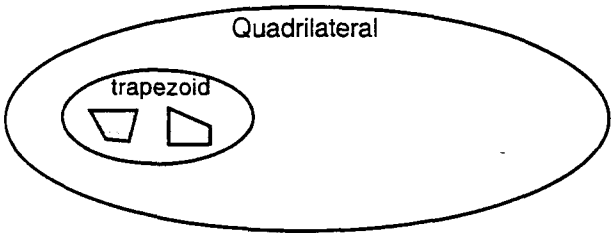


   - Tell students that some quadrilaterals have special characteristics so they are given special names.
   - Using the transparency/wall poster Quadrilaterals, discuss the five special quadrilaterals shown. For each one: say the name, give a description, and point out its special traits on the illustration provided.
   - Have students complete the activity sheet Learning About Quadrilaterals.
   - Place the Family transparency on the overhead. Tell students that in a family one person may have more than one family name. Explain that in this picture the man is called "husband" by his wife. He is also called "father" by his son and "grandfather" by the small boy.
   - Tell students that many quadrilaterals can also be described using more than one name.
   - Point to the large **oval** on the transparency Quadrilateral Venn Diagram and tell students that an oval is shaped like an egg.



   - Tell students that the word "quadrilateral" gives us some information about all of the figures inside this oval. Have students tell you specific characteristics which will be true about all of the quadrilaterals inside the oval. (Quadrilaterals are: four sided, closed figures, and made of line segments.)
   - Have students refer back to the wall poster Quadrilaterals . (It would be beneficial for students to also have a copy for their notebook.) Ask them to talk with a partner about each of the five special quadrilaterals shown: trapezoid, parallelogram, rectangle, rhombus, and square.
   - Explain that the five quadrilaterals can be divided into two groups - figures with only one set of parallel lines and figures with two sets of parallel lines.

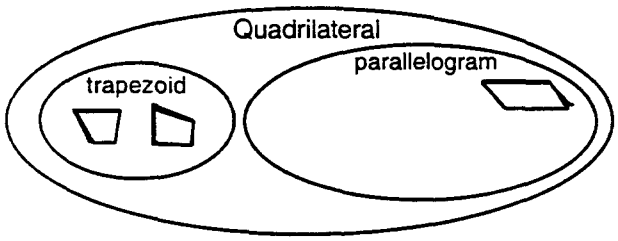
- Ask students to name the quadrilateral which has only one set of parallel lines. (trapezoid)
- Draw an oval inside the larger oval and label it "trapezoid" as shown below. Have a couple of students come up and draw examples of a trapezoid inside the appropriate oval.



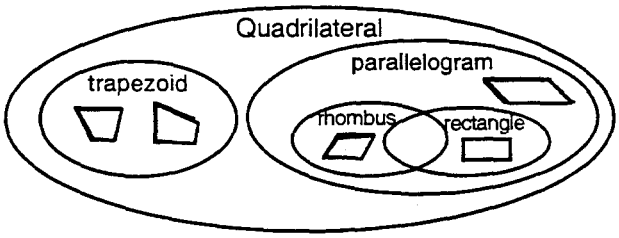
- Have students brainstorm a list of the special quadrilaterals which have two sets of parallel lines. Record the list on the board.

  parallelogram
  rectangle
  rhombus
  square

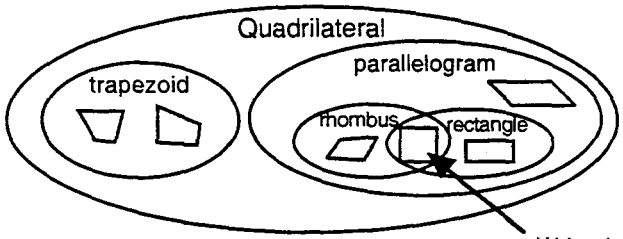
- Look back with students at the transparency Quadrilaterals. Review the "definition" of a parallelogram - "opposite sides the same length and parallel."
- Have students look at the "examples" of each of the remaining three quadrilaterals: rectangle, rhombus, and square. For each one, have students decide if it fits the description of a parallelogram. For example, point to the illustration of the rectangle and say, "Does a rectangle have opposite sides the same length and are opposite sides parallel?" (Yes)
- Repeat the same question for the rhombus and the square. Then explain that since they all fit the description, they are all parallelograms.
- Draw an oval and label it "parallelogram" as shown below. Draw the figure shown below in the parallelogram oval.



- Draw a rectangle and a rhombus on the board. Remind students that these are parallelograms because they fit the description of a parallelogram. Have them look at the rectangle and the rhombus and tell how these parallelograms are different from the one drawn inside the oval. (The rectangle has four right angles and the rhombus has four congruent sides.)
- Draw two ovals as shown below and label them as "rectangle" and "rhombus" Have a students come up and draw each shape as they explain how they are different from the first parallelogram.

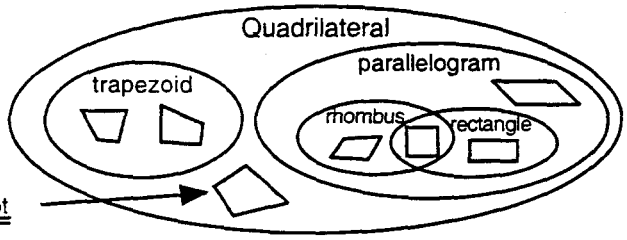


- Reinforce the concept of a Venn diagram by pointing to the figures and reviewing that:
    - all figures are **quadrilaterals** because they are inside the quadrilateral oval
    - **trapezoids** are quadrilaterals with one set of parallel lines
    - **parallelograms** are quadrilaterals with two sets of parallel lines
    - a **rhombus** is a parallelogram with four congruent sides
    - a **rectangle** is a parallelogram with four right angles
- Have students help decide where a square should be placed in the Venn diagram. You may want to use the following questions to guide their thinking.
    - Is a square a quadrilateral? (Yes, so it belongs inside the largest oval.)
    - Does a square have one or two sets of parallel lines? (Two, so it belongs inside the parallelogram oval.)
    - Does a square have four congruent sides like a rhombus or four right angles like a rectangle? (It has both, so it belongs in both the rhombus and the rectangle ovals.)
- Have students suggest a way to include the square inside both the rhombus and the rectangle ovals. Draw the final quadrilateral into the appropriate space as shown below.



Write the word "square" inside this figure.

- Ask students if there is a quadrilateral that would not belong inside any of the other ovals. Allow time for students to discuss this and try to draw one on paper. Add the quadrilateral to the Venn diagram.



This figure is JUST a quadrilateral and does not fit any other description.

- Additional practice classifying quadrilaterals is provided on the activity sheets Quadrilateral Review. The Quadrilateral Diagram may be used for oral discussion about quadrilaterals.

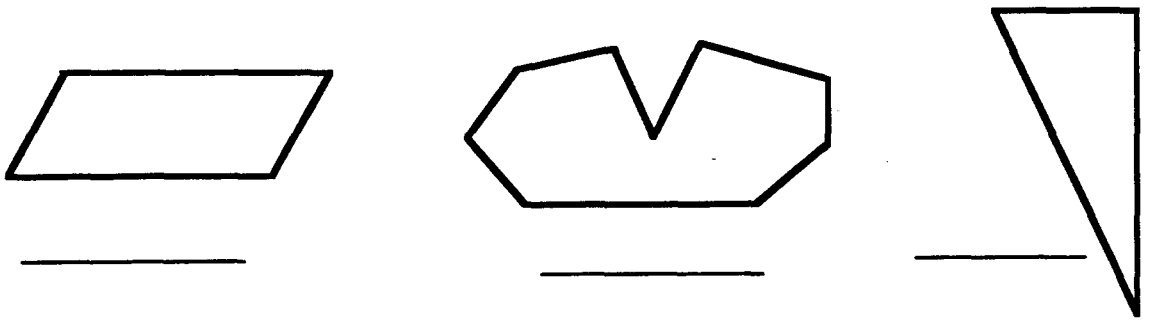
Extension: Geometer's Sketchpad may be used to draw and explore polygons.
The best way to familiarize yourself with this software is to work through the Guided Tours before the lesson. The Sketchpad User Guide contains a full set of tours, and there's a shorter set, suitable for duplication, in Exploring Geometry with *The Geometer's Sketchpad*. As you learn the program, be sure to use Sketchpad's extensive on-line help system. For instance, to get help on using any menu command, press the ZF1 key while highlighting that command.

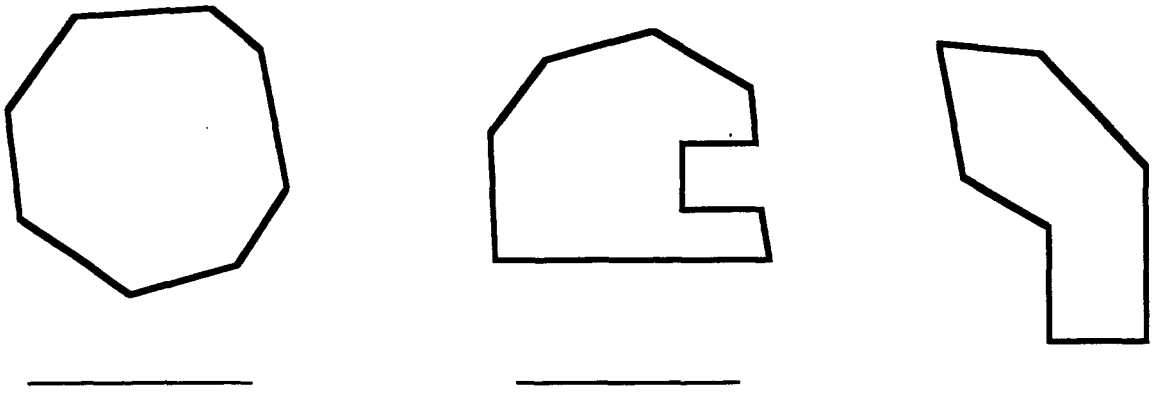
Language Development Activities

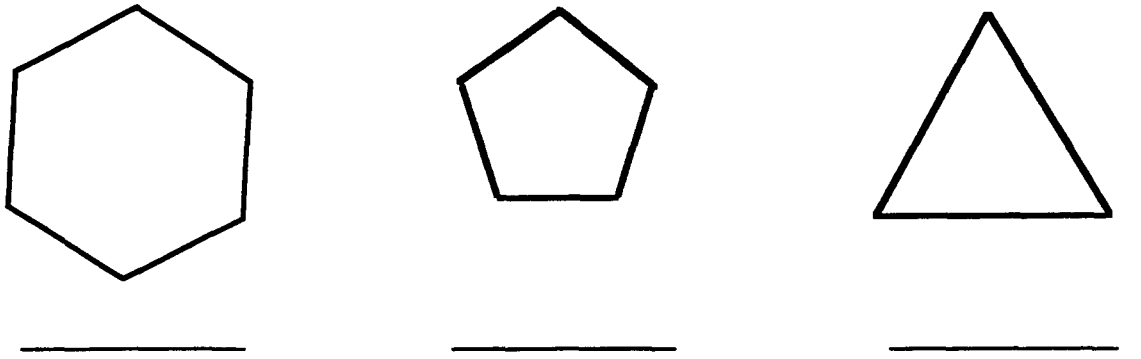
- The activity sheets Polygon Vocabulary Practice and Vocabulary Challenge will give students further reinforcement and oral practice using the vocabulary in this lesson.

Transparency

Name That Polygon !







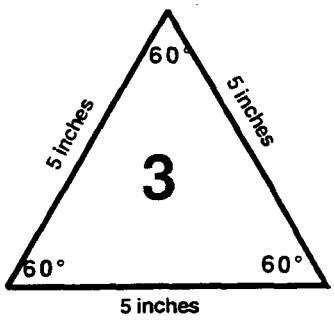
WORD BANK	Quadrilateral	Triangle	Septagon	Octagon
	Pentagon	Hexagon	Nonagon	Decagon

Think!! These polygons are regular and irregular. Why??

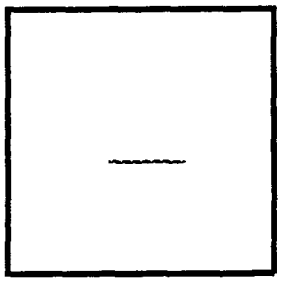
Regular Polygons

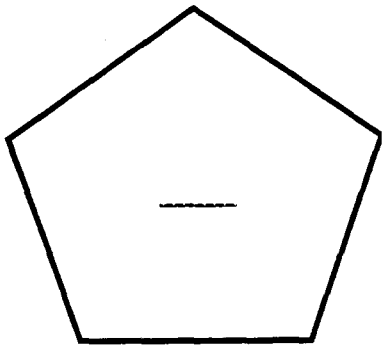
Demonstration transparency

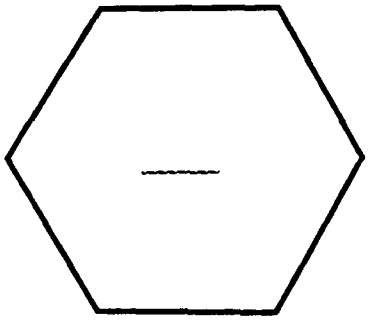
A **regular polygon** has all sides the same length and all angles the same measure.

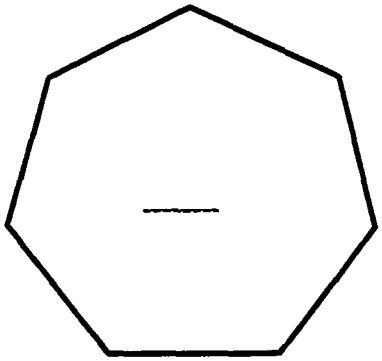


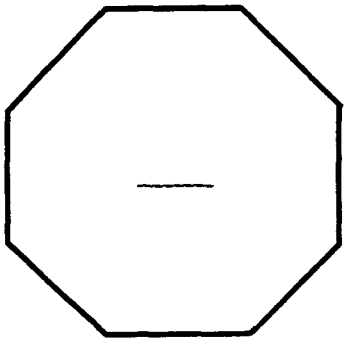
Triangle

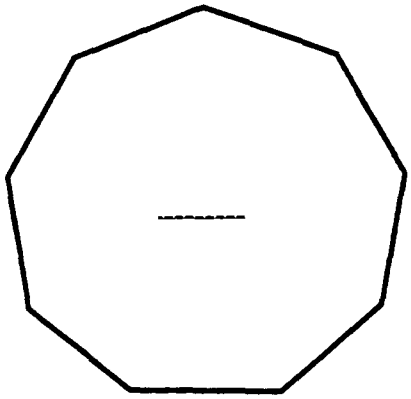


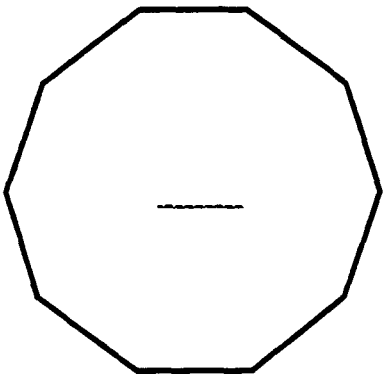


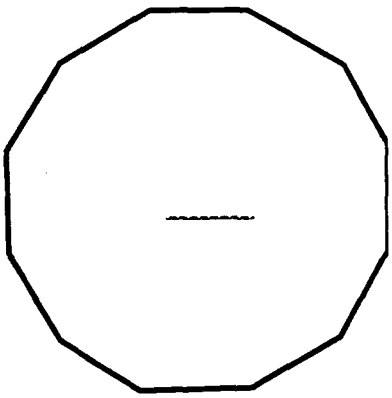








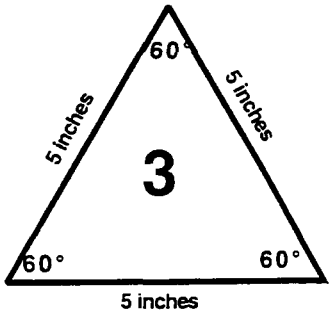




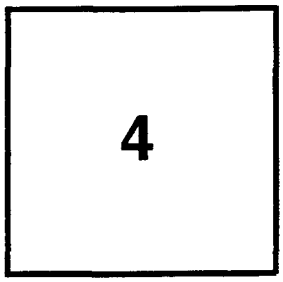
Regular Polygons

Wall Poster/Transparency

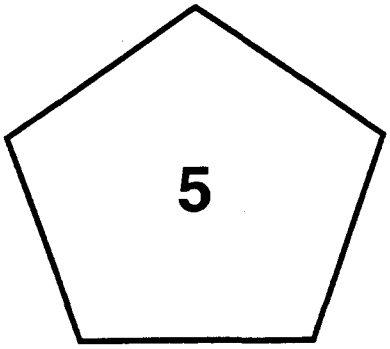
A **regular polygon** has all sides the same length and all angles the same measure.



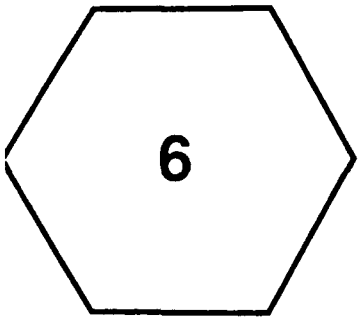
Triangle



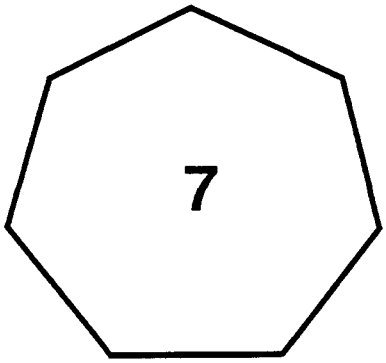
Quadrilateral



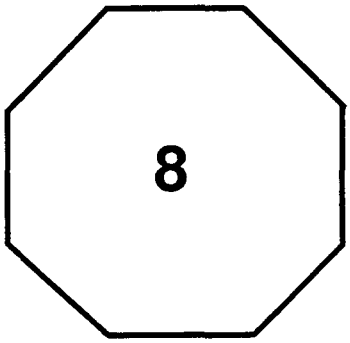
Pentagon



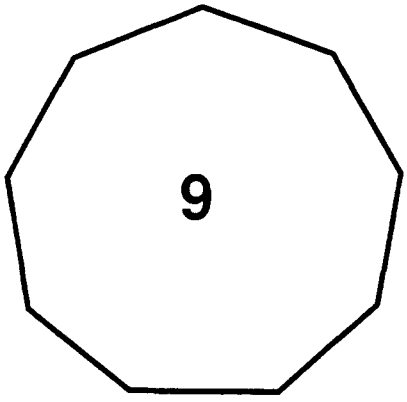
Hexagon



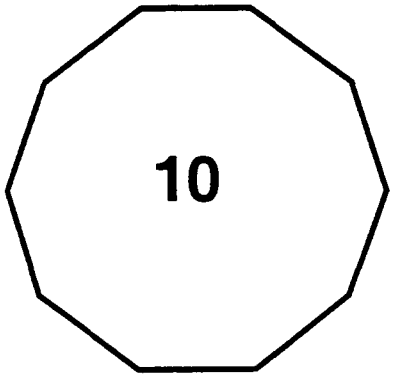
Septagon



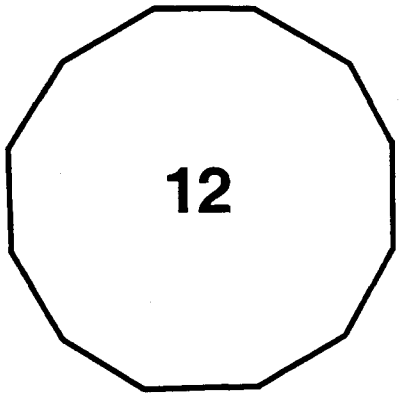
Octagon



Nonagon



Decagon

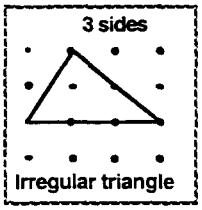


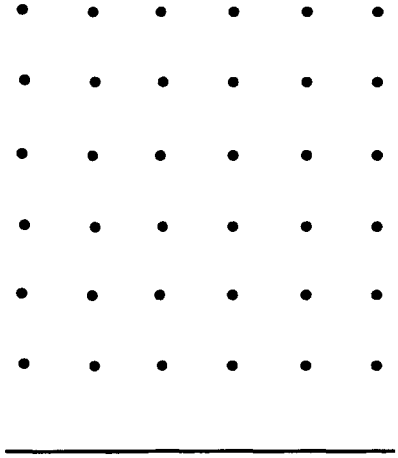
Dodecagon

Name: _____ **Drawing Regular and Irregular Polygons**

- Draw a regular or irregular polygon with the number of sides given.
- Write the name of the polygon.
- Make sure you have at least 2 regular polygons!

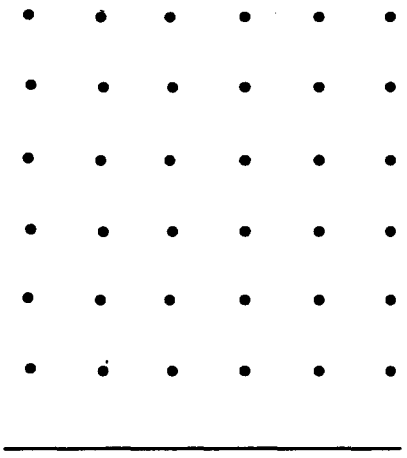
EXAMPLE





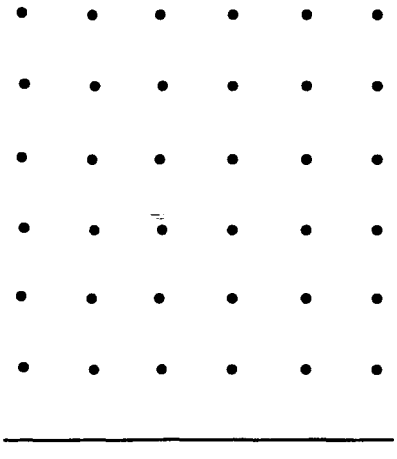
3 sides

_____



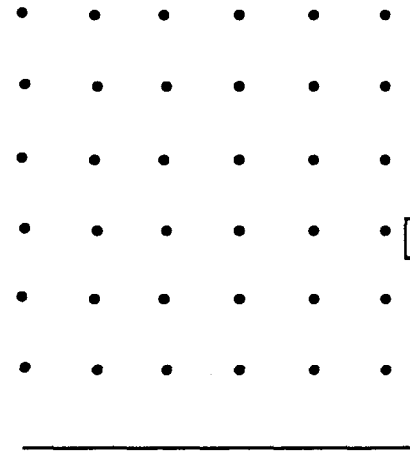
4 sides

_____



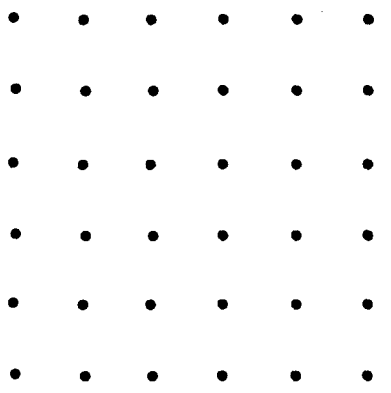
5 sides

_____



6 sides

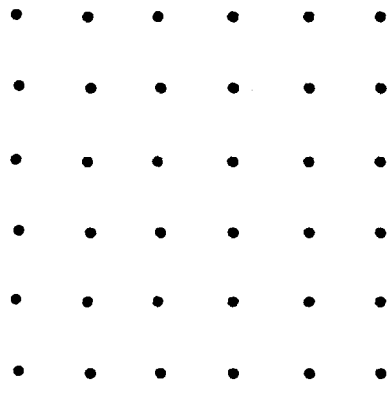
_____



7 sides

_____

_____



8 sides

_____

_____

Name: _____

Practice With Polygons

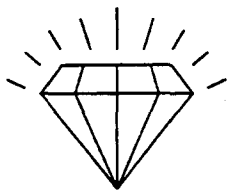
equal		regular	
	closed		open
polygons			
	sides	angles	

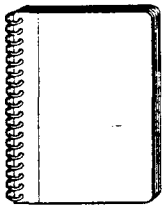
A geometric figure can be _____ or _____. Closed figures are called _____. Polygons are named by the number of _____ and _____ they have. If the sides and angles are _____, it is called a _____ polygon.

Fill in the missing spaces in the chart:

Polygons			
Name	Sides	Angles	Picture
Triangle		3	
	4		
			
Octagon			
		9	
	10		

Name a polygon that looks like each object:

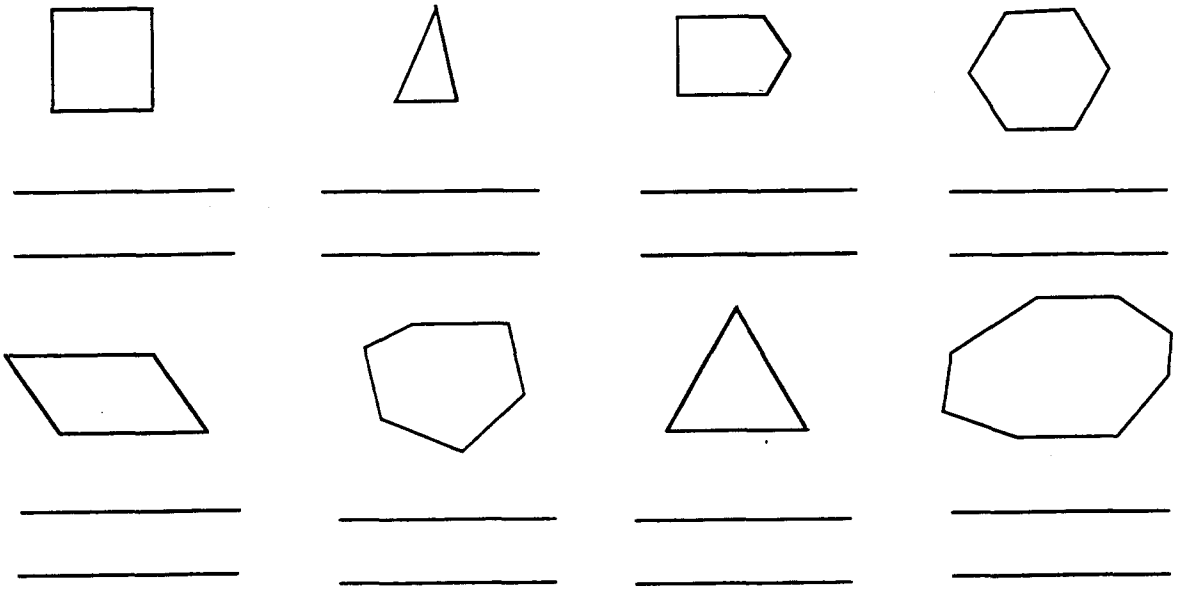




_____ _____ _____ _____

Practice With Polygons

Name these polygons. Tell if they are regular or irregular polygons.



Use your geoboard or dot paper. Make a polygon. Answer the questions about your polygon. Tell the class about your polygon.

1) My figure is open or closed? _____

2) My figure has _____ sides and _____ angles.

3) My figure has equal or unequal angles? _____

4) My figure has equal or unequal sides? _____

5) My figure is a regular or irregular polygon? _____

6) My polygon is called a _____.

TRUE or FALSE?

1) A triangle is an open figure.

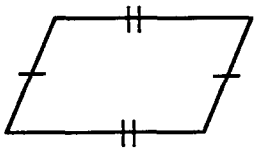
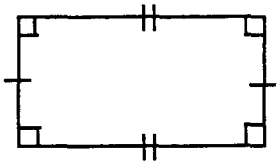
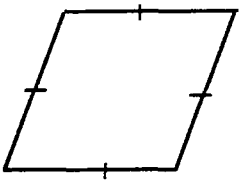
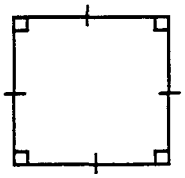
2) A hexagon has six sides and six angles.

3) A regular pentagon has two sides of unequal length.

4) A dodecagon has nine angles.

5) A figure with seven sides is a septagon.

Quadrilaterals

Name	Description	Figure
Trapezoid	One set of parallel sides	
Parallelogram	Two sets of parallel sides and opposite sides the same length	
Rectangle	Two sets of parallel sides, opposite sides the same length, and four right angles	
Rhombus	Two sets of parallel sides and four sides the same length	
Square	Two sets of parallel sides, four sides the same length, and four right angles	

Name:_____

Learning About Quadrilaterals


Parallelogram


Rectangle

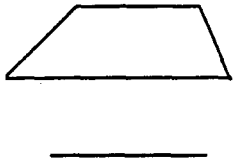

Square

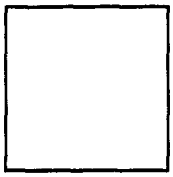

Rhombus

Trapezoid

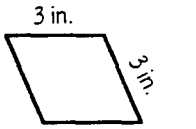
Name the quadrilateral


_____


_____


_____

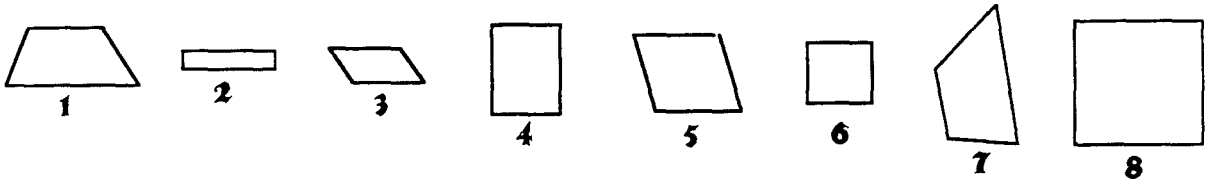

_____



_____

Which of the polygons are.....?

SQUARES	TRAPEZOIDS	RECTANGLES	PARALLELOGRAMS
----------	----------	----------	----------



Complete the chart.

Figure	Name	Sides	Angles
	Parallelogram		
	Rectangle		4 right angles
□			
		4 equal sides	2 acute angles 2 obtuse angles
		1 set of parallel sides	

- Use your geoboard or dot paper and make a quadrilateral.
- Answer the questions about your quadrilateral.
- Describe your quadrilateral to the class.

1) My quadrilateral has _____ sides and _____ angles.

2) My figure has _____ right angles.

3) My figure has _____ acute angles.

4) My figure has _____ obtuse angles.

5) My figure has _____ sets of equal sides.

6) My figure has _____ sets of parallel sides.

7) The quadrilateral I have is called a _____.

Draw and name the quadrilateral.

1) Four sides the same length and parallel
No right angles

2) One set of parallel sides

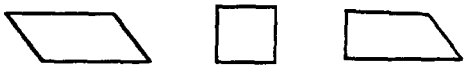
3) Four sides equal
Four right angles

4) Two sets of parallel sides
Four right angles

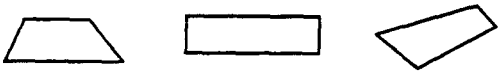
Circle the correct quadrilateral.

1) Four congruent angles

2) One pair of parallel sides



3) Two pairs of parallel sides

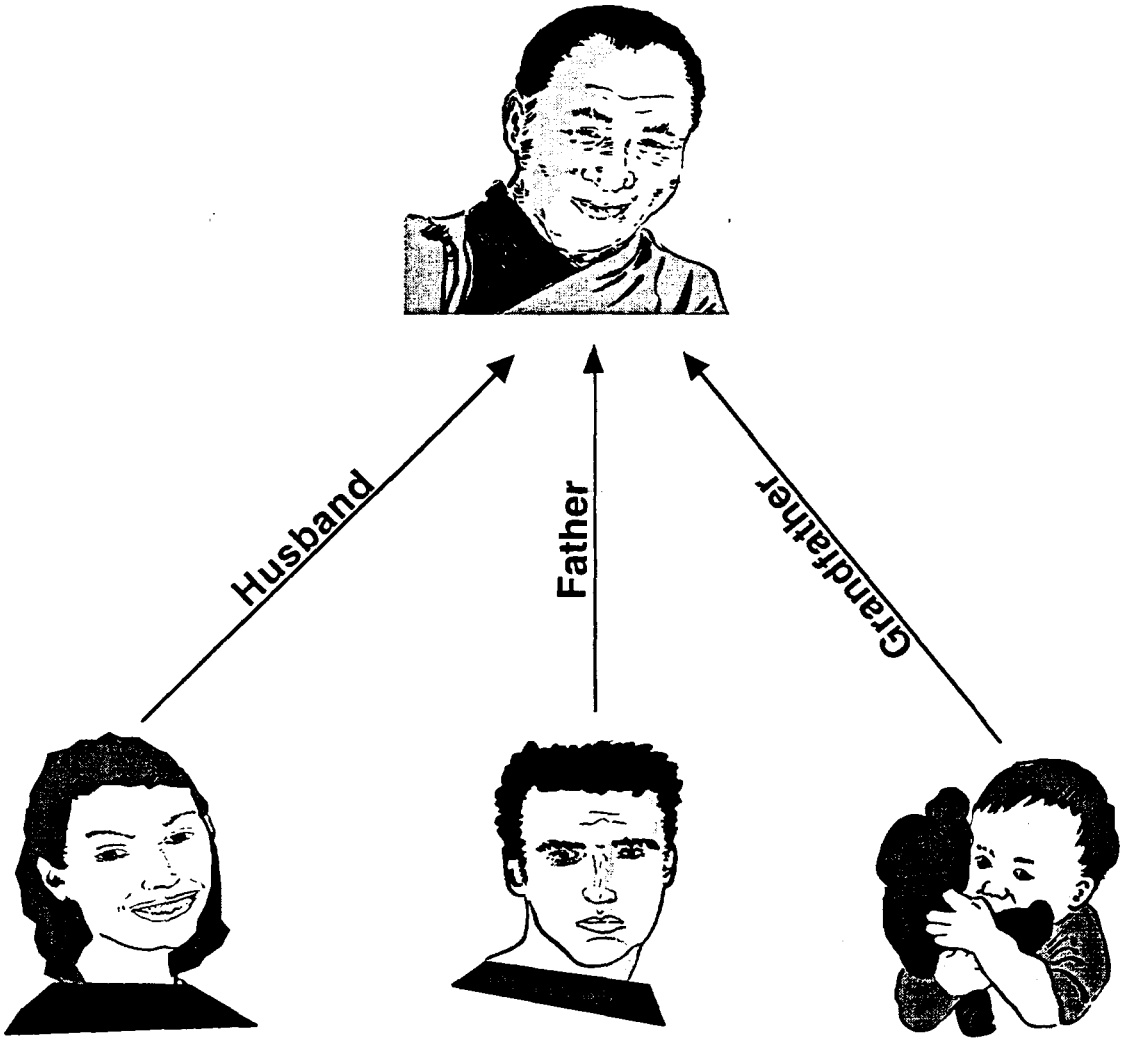


4) Four congruent angles

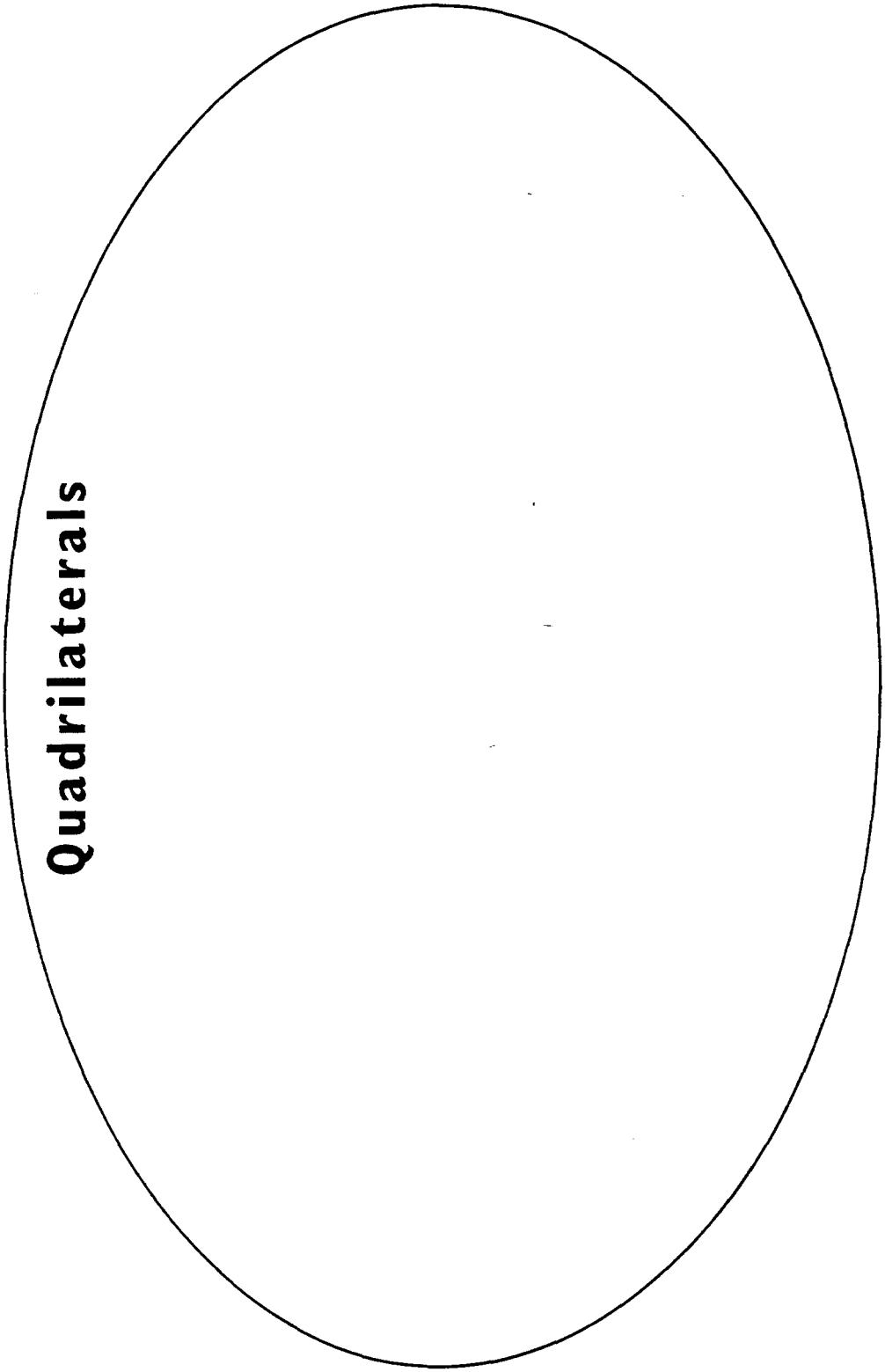
5) Four congruent sides

Family

This man is just one person in the family, but there are many family names for him.



Quadrilateral Venn Diagram

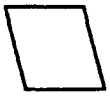


Name: _____

Quadrilateral Review

Parallelogram
Quadrilateral with 2 sets of parallel sides.
Opposite sides are equal in length.

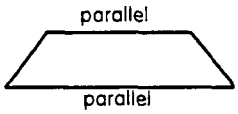
Rhombus
Parallelogram with all 4 sides equal in length.



Rectangle
Parallelogram with all 4 right angles.

Square
Rectangle with all 4 sides equal.

Trapezoid
Only set of sides parallel.

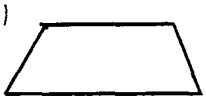


Write as many quadrilateral names as you can for each figure.

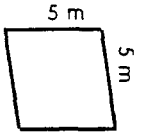
1)

2)

3)



4)



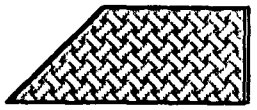
_____ _____ _____ _____

_____ _____ _____ _____

Circle the correct answer.

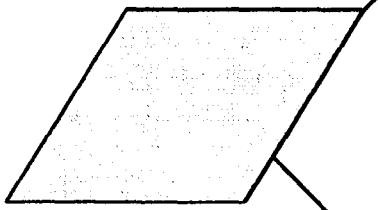
1) Only one set of sides is parallel.	a) square	b) trapezoid	c) parallelogram
2) Opposite sides are parallel.	a) trapezoid	b) quadrilateral	c) parallelogram
3) All four angles are right angles.	a) rectangle	b) rhombus	c) trapezoid
4) All four sides are equal and opposite sides are parallel.	a) trapezoid	b) rhombus	c) rectangle
5) All four sides are equal. All angles are right angles.	a) rectangle	b) rhombus	c) square

Quadrilateral Diagram

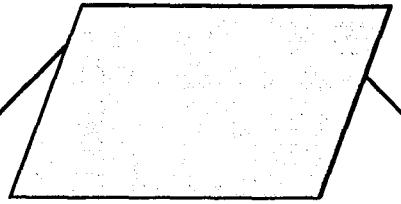


• polygon with 1 set of parallel lines

Why is the trapezoid not in the group with the other quadrilaterals?

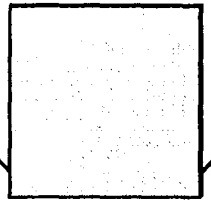


• parallelogram with all sides equal



• quadrilateral with 2 sets of parallel lines

• parallelogram with 4 right angles and 2 sets of parallel lines



• parallelogram with 4 equal sides and 4 right angles

Name _____

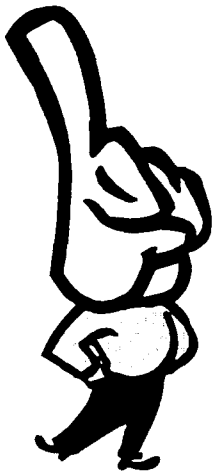
Polygon Vocabulary Practice

Fill in the boxes in the following chart. For the box that says "phrases," write a few words that describe or define the vocabulary term. Think of as many ways as you can to explain the meaning of the vocabulary word. In the box that says "illustrations," draw a picture or pictures that represent the meaning of the term.

Vocabulary Term	Phrases	Illustrations
Polygon	• a closed figure • sides are all line segments	
Triangle		
Quadrilateral		
Pentagon		
Hexagon		
Septagon		
Octagon		
Nonagon		
Decagon		

Vocabulary Practice

Vocabulary Term	Phrases	Illustrations
Parallelogram		
Square		
Rectangle		
Rhombus		
Trapezoid		



Name _____

Vocabulary Challenge

Part I. Analogies are comparisons between similar objects. In an analogy, you compare one pair of objects with another pair. Look at the following example:

wing : airplane as pages : book

The relationship between the examples shows that a wing is part of an airplane in a similar way that pages are part of a book.

Complete each comparison below using the math terms you learned in this lesson. Make sure that the first pair shows the same relationship as the second pair.

1. decagon : ten as quadrilateral : _____

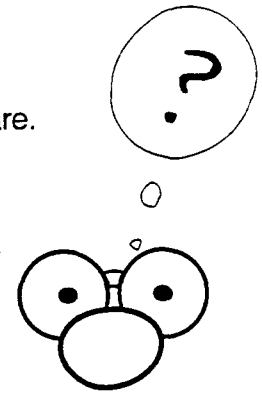
2. three : triangle as five : _____

3. regular polygon : equal as _____ : unequal

4. _____ : septagon as six : hexagon

5. octagon : eight as _____ : nine

Part II. True or False. Read the following sentences carefully and decide if they are true or false. Write T on the line if it is true; write F if it is false.



_____1. A square is a parallelogram.

_____2. A square is also a rectangle, but a rectangle is not a square.

_____3. A trapezoid is a parallelogram.

_____4. All quadrilaterals have two sets of parallel lines.

_____5. A rectangle is not a parallelogram.

_____6. All parallelograms are rectangles.

_____7. A rhombus is never a trapezoid.

Answer Key
Objective 42

Name That Polygon! p. 7

1) quadrilateral
2) nonagon
3) triangle
4) octagon
5) decagon
6) septagon
7) hexagon
8) pentagon
9) triangle

Drawing Regular and Irregular Polygons

p. 10

Answers will vary.

Practice With Polygons p. 11

1) closed or open
2) polygons
3) sides and angles
4) equal, regular

Chart:

1) Triangle, 3, 3, 
2) Quadrilateral, 4, 4, 
3) Hexagon, 6, 6, 
4) Octagon, 8,8,
5) Nonagon, 9,9
6) Decagon, 10,10,

Name a polygon:

1) triangle
2) pentagon
3) octagon
4) quadrilateral

Practice With Polygons con't p.12

Name these polygons:

1) quadrilateral - regular
2) triangle - irregular
3) pentagon - irregular
4) hexagon - regular
5) quadrilateral - irregular
6) hexagon - irregular
7) triangle - regular
8) octagon - irregular

Use a geoboard or dot paper to make ...

Answers will vary.

True or False:

1) False
2) True
3) False
4) False
5) True

Learning About Quadrilaterals p. 14

1) rectangle
2) trapezoid
3) square
4) parallelogram
5) rhombus

Which of the polygons are...

squares = 6,8

trapezoids = 1,7

rectangles = 2, 4, 6,8

parallelograms = 3 , 2, 4, 5, 6, 8

Chart:

Figure	Name	Sides	Angles
	Parallelogram	2 sets // sides opp.sides same length	4 right angles
	Rectangle	2 sets // sides opp.sides same length	4 right angles
	Square	4 sides equal	4 right angles
	Rhombus	4 sides equal	2 acute 2 obtuse
	Trapezoid	1 set // sides	2 acute 2 obtuse

Answer Key
Objective 42

Learning About Quadrilaterals p.15

Use your geoboard.....

Answers will vary.

Draw and name the quadrilateral.

1) rhombus 

2) trapezoid 

3) square 

4) rectangle  or 

Circle the correct quadrilateral:

1) square (#1)
2) trapezoid (#3)
3) rectangle (#2)
4) rectangle (#2)
5) square (#1)

Quadrilateral Review p.18

Write as many names for each figure.....

1) rectangle, parallelogram,
2) square, rectangle, parallelogram
3) trapezoid
4) rhombus, parallelogram

Circle the correct answer:

1) trapezoid
2) parallelogram
3) rectangle
4) rhombus
5) square

Vocabulary Practice - Analogies

Part I

1) four
2) pentagon
3) irregular
4) seven
5) nonagon

Part II

1) T
2) T
3) F
4) F
5) F
6) F
7) T

Objective 43: Develop the strategies for and find the sum of the interior angles of any polygon.

Vocabulary

regular triangle
interior angle
equilateral triangle
diagonal

Materials

scissors
glue
grid paper (included in lesson)

Transparencies

- Triangle Warm-Up
- Exploring the Sum of the Interior Angles of a Triangle
- The Sum of the Interior Angles of Polygons
- Hexagon

Student Copies

- Triangle Warm-Up
- Exploring the Sum of the Interior Angles of a Triangle
- Interior Angles of Triangles
- The Sum of the Interior Angles of Polygons
- Practice with Interior Angles of Polygons
- Writing about Polygons and Angles

Language Foundation

1. Review types of triangles such as equilateral, isosceles, right, and scalene.

2. Have students name the polygons they learned in the previous lessons. Review number prefixes they learned.

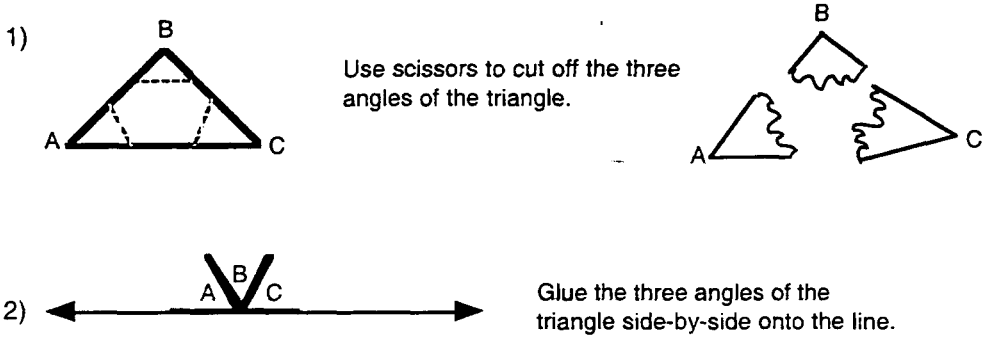
3. Teach students the prefix in to help them understand the word interior . Explain that the prefix *in* means "in" or "into." Brainstorm words with the class that have the same prefix such as inside, inner, insert, and internal. Ask students if they can tell you where the **interior angles** of a polygon are located. Continue your discussion as you begin the Mathematics Component with the Triangle Warm-Up activity.

4. Draw a box on the board or on a transparency. Ask a student to come up and draw a **diagonal** line across the box. Talk about the parking spaces in the parking lot at your school. Ask if the spaces are straight or **diagonal**. Tell students that in this lesson, they will be drawing **diagonal** lines inside polygons to form triangles.

Mathematics Component

1. Explore the sum of the interior angles of a triangle

   - Distribute student copies of the activity sheets Triangle Warm-Up and Exploring the Sum of the Interior Angles of a Triangle.
   - Using a transparency copy of Triangle Warm-Up , review that this is a **regular triangle**. (All sides the same length, all angles the same measure.) Remind students that a triangle with equal sides and equal angles is called an **equilateral triangle**.
   - Discuss **interior angles** as the angles inside the polygon. Point out interior angles A, B, and C on the transparency.
   - Model and have students follow directions given for cutting out the three angles of the triangle on the Triangle Warm-Up sheet. Model by placing the angles on a transparency copy of the activity page. Have students glue the angles onto the line provided on the activity page Exploring the Sum of the Interior Angles of a Triangle.



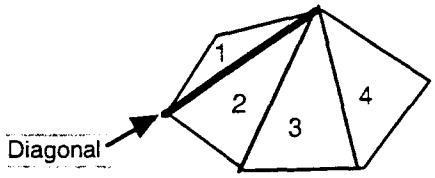
   - Ask students what they notice about the angles. (The angles fit together to cover the same measure as a straight line.)
   - Demonstrate that the three angles form half of a circle by drawing a semicircle around the angles as shown.

   - Remind students that a circle has 360°. Say, "How many degrees are in a straight line? Why?" (There are 180 degrees in a straight line since it is half of a circle. 180° is called a straight angle.)
   - Point to the statement at the bottom of Exploring the Sum of the Interior Angles of a Polygon. It says "There are 180 degrees in a **triangle** because....."
     Have students give an oral response to complete the statement as you write on the transparency. (There are 180° in a triangle because the three angles form a straight angle or half a circle when put together.) Ask students to finish writing the statement on their papers.

- The activity sheet Interior Angles of Triangles is provided for further practice. Point out to students that if they know that the interior angles of a triangle always measure 180°, they can easily find the measure of missing angles in a triangle - just add the two given angles and subtract from 180°. Work through the first problem with students and have them complete the page.

2. Find the sum of the interior angles of any polygon.

- Distribute copies of The Sum of Interior Angles of Polygons and grid paper to students.
- Read "Task" and "Directions" aloud. Using a transparency copy, review the information for the first polygon (a triangle) on the worksheet. (triangle, 3 sides,1 triangle,180°)
- Point out to students that every polygon can be broken up into several triangles, each with 180°. This will help them find the sums of the interior angles without having to measure with a protractor.
- Model dividing into triangles with the Hexagon transparency. Tell students that you can draw **diagonals** inside a polygon to form triangles. Explain that a **diagonal of a polygon** is a line segment with endpoints that are on vertices that are not next to each other.
- Work through the 4 steps to find the sum of the interior angles with students. Begin by picking a vertex and drawing diagonals.



1) There are 4 triangles in the hexagon.
2) The sum of the angles in each triangle is 180°.
3) 4 x 180° = 720°
4) The sum of the angles in the hexagon is 720°.

- To practice this fact, have students draw a hexagon on their grid paper. Students should come up with several different hexagon shapes.
- Tell them to divide the hexagon into triangles with a ruler.
- Ask a student to come up and draw his/her hexagon and divide it into triangles.
- Ask the class how many triangles can be formed inside the hexagon no matter what its shape. (4)
- Ask students how many degrees will be in each triangle. (180°) Ask students if they can find the sum of all the interior angles in any hexagon using what they know about triangles. (180° in each triangle x 4 = 720°)
- Allow students to work in pairs to complete the rest of the worksheet. Each student should work on his/her own grid.

Note to Teacher:

- To sketch all 8 polygons on the chart and other polygons in the exercises on p. 2 of the activity sheet, two or more grid sheets are needed.
- Using different colored pencils to outline each polygon helps define the shape.
- Labeling each polygon reinforces geometric vocabulary.

3. Find a pattern for the sum of the interior angles of any polygon. Write an algebraic expression to represent this pattern.

- After students have completed The Sum of Interior Angles of Polygons chart, discuss patterns students observe on the worksheet. How many degrees were added with each new polygon? (When a side is added to a polygon, the sum of its angles increases by 180°.)
- Tell students that 180° is a constant. Have students fill in this information on p.2 - A Pattern. Fill in the bottom line of the column "Degrees Added" on The Sum of Interior Angles chart with **180°.**
- Ask students what they notice about the number of sides and the number of triangles. Is there a pattern between the number of sides and the number of triangles? (The number of triangles is always 2 less than the number of sides.) Tell students that 2 is a constant. Have students fill in this information on p. 2 - A Pattern. Fill in the bottom line of the column "Number of Triangles" on the chart with **n-2.** Explain that **n** represents the number of sides.
- Ask students if they can think of a way to find the number of degrees in ANY polygon if they know the number of sides. Point out to students that the number of sides is a variable.
- Share ideas and then lead students to see that earlier they multiplied the number of triangles by 180°. (Number of △) 180°

  On the chart, the number of triangles is always 2 less than the number of sides.
  This fact can be substituted into the expression.

  (Number of △) 180°

  △ = number (n) of sides - 2

  △ = n - 2

  SO

  (Number of △) 180°

  △ = (n - 2)

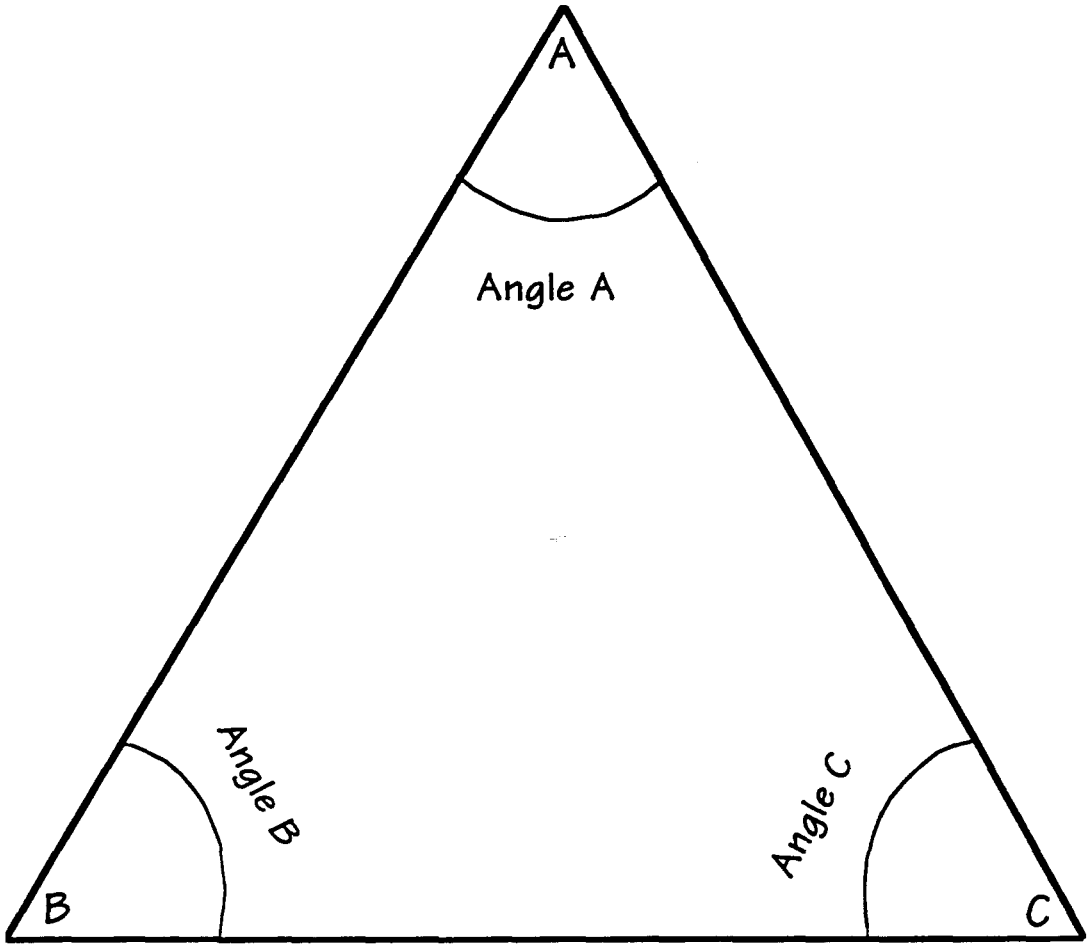
  The algebraic expression to describe the sum of the interior angles of any polygon is the number of sides minus 2, times 180° ⟶ (n-2) 180°
- Have students record the expression **[(n-2) 180]** under the section labeled "Next Step." Fill in the bottom line of the column " Sum of Interior Angles" with **(n - 2)180°**
  The completed bottom line of the chart will allow the students to see the relationship between the parts of the algebraic expression or formula.
- A section entitled "Challenge" is provided on the activity sheet so that students can try out their formula on larger polygons. For the 50 sided polygon, students should come to the realization that it is easier to use the formula than to draw a 50 sided polygon and divide it into triangles.
- Additional practice is provided on the activity sheets Practice with Interior Angles of Polygons.

Language Development Activities

- The activity page Writing About Polygons and Angles will give students the opportunity to use math vocabulary and their knowledge of polygons to write explanations and solution steps to extend their thinking about material presented in the lesson.

Name:_____ Transparency/Student Copy

Triangle Warm-Up



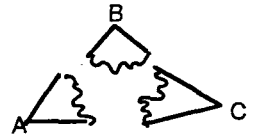
Equilateral Triangle

Name:_____ Transparency/Student Copy

Exploring the Sum of the Interior Angles of a Triangle

Directions:

- Cut out the three angles from the triangle.
- Glue your angles side-by-side onto the line below.
- What do you notice?



A straight angle has 180° or half of a 360° circle.

There are 180 degrees in a triangle because

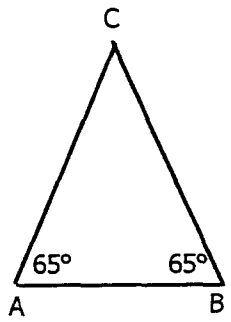
_____

_____

_____

Name:_____

Interior Angles of Triangles

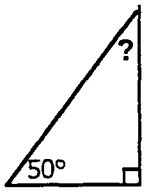


1) What kind of triangle is this?

a) equilateral b) isosceles c) scalene d) right

2) What are the total number of degrees inside the triangle? ____

3) What is the measure of <C ? _____

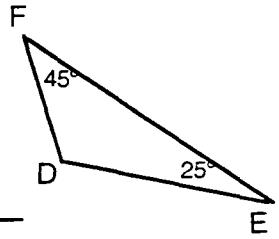


4) What kind of triangle is this?

a) equilateral b) isosceles c) scalene d) right

5) What are the total number of degrees inside the triangle? ____

6) What is the measure of < ? _____



7) What kind of triangle is this?

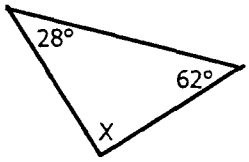
a) equilateral b) isosceles c) scalene d) right

8) What are the total number of degrees inside the triangle? ____

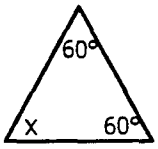
9) What is the measure of <D ? _____

Find the measure of the missing angles.

10)



11)



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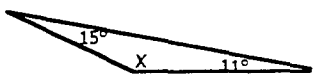
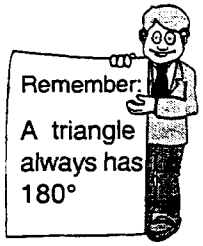


Name:_____ Transparency/Student Copy

The Sum of Interior Angles of Polygons

Task: Find a pattern for the sum of the interior angles of any polygon.



Directions:

1) **Draw** each polygon from the chart on the grid sheet.
2) **Outline** the polygon with a **colored pencil** on the grid sheet and **label** with its name.
3) **Draw diagonals to form triangles** in the interior of the polygon. Record how many triangles.
4) Find the **sum of all the interior angles** in the polygon. Record the sum on the chart.

Sums of Interior Angles of Polygons Chart

Polygon Name	Number of Sides	Number of Triangles	Sum of Interior Angles	Degrees Added
Triangle	**3**	**1**	**180°**	———
	4			
	5			
	6			
	7			
	8			
	9			
	10			
n-agon	**n**			

Now, look for a pattern in the angle sums of the polygons.

A Pattern: When you add a side to a polygon, the sum of its angles increases by _____.

A Pattern: The difference between the number of sides and the number of triangles is always _____.

Next step ⟶ Write an algebraic expression to describe the number of degrees in ANY polygon having ***n*** sides. _____

Challenge : Try out your expression with:

- A 13 sided irregular polygon

  1) Use the formula to calculate the angle sum.

  2) Check by drawing the polygon on a grid.
  3) Add up the interior angles using triangles.

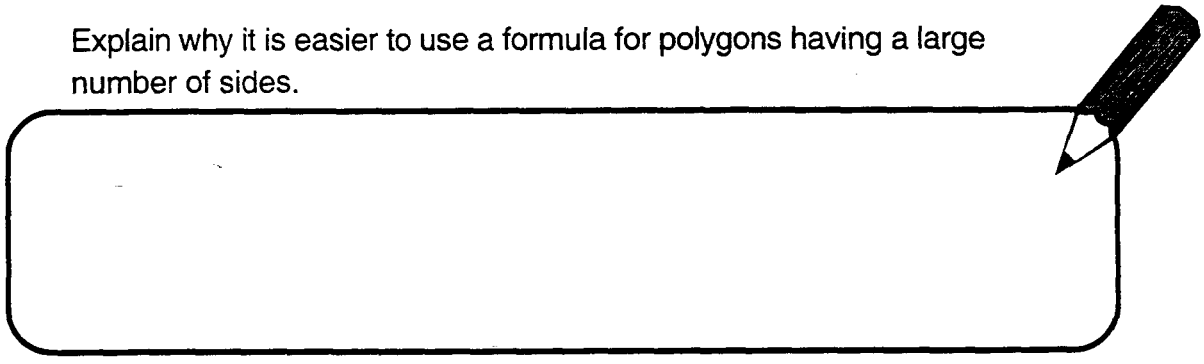
- A 50 sided regular polygon

  Are you going to draw this polygon? _____

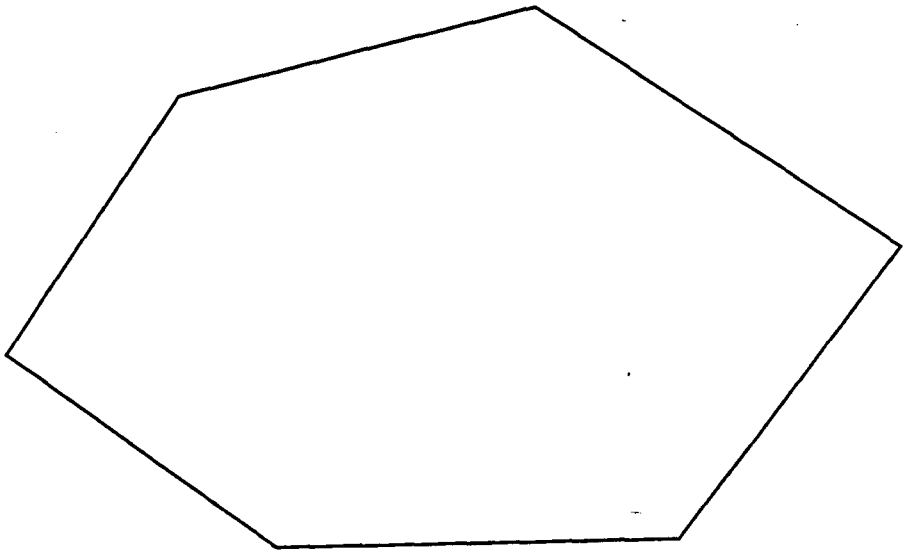
  What will you do to find the sum of the angles? _____

  What is the sum of the angles?

Explain why it is easier to use a formula for polygons having a large number of sides.



Hexagon



1) There are _____ triangles in the hexagon.

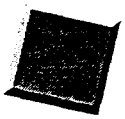
2) The sum of the angles in each triangle is _____.

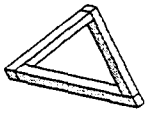
3) To find the sum of the interior angles :

_____ X _____

4) _____ X _____ = _____ °

Name: _____



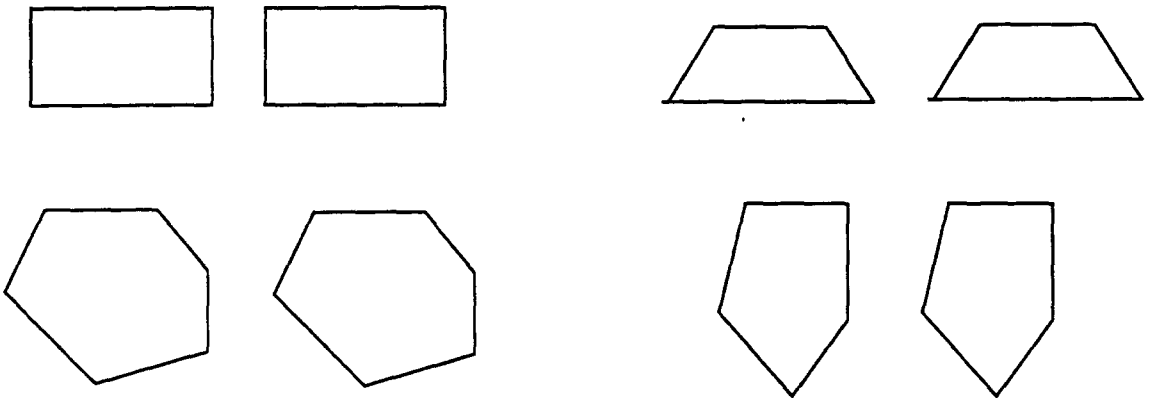


Practice with Interior Angles of Polygons

Review of Diagonals

A diagonal is a line connecting 2 vertices that are not next to each other.

Draw diagonals for these polygons. Show different possibilities for each shape.



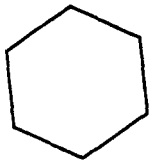
Draw a diagram of each polygon and complete the information.

Example :
Quadrilateral  2 triangles $2 \times 180° = 360°$

Polygon Shape	Diagram	# of Triangles	Sum of the Measure of the Interior Angles
1) Pentagon			
2) Dodecagon (12 sides)			
3) Septagon			

Practice with Interior Angles of Polygons

Name each polygon. Find the sum of the measure of the angles using the formula.



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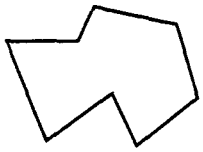

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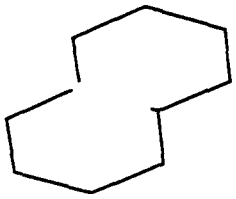
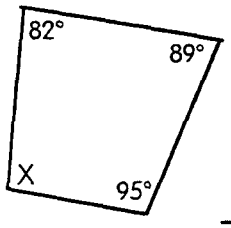


_____

Find the measure of angle X in each polygon. Show your work.

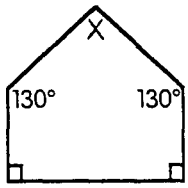
1)



Hint: What kind of polygon is it?
How many total degrees are there in the polygon?

_____

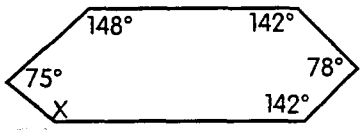
2)



_____

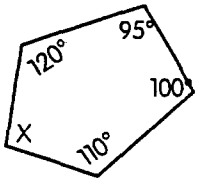
Practice with Interior Angles of Polygons

3)



_____

4)



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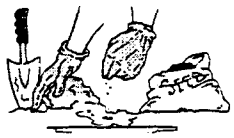
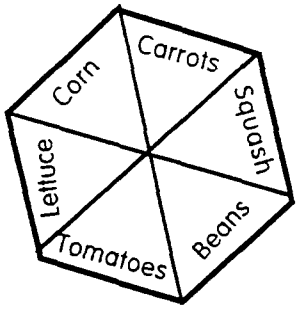
Problem Solving

Mai Li is planting a vegetable garden in the shape of a regular hexagon. Look at the diagram of her garden and answer the questions.



Mai Li's Garden



1) What is the shape of each section of vegetables?

_____

2) What is the shape of the part of the garden planted with carrots, corn, and lettuce?

_____

3) What is the shape of the part of the garden planted with tomatoes and beans? _____

4) What is the measure of the interior angle at each of the six corners of the garden? _____

5) What is the measure of each interior angle in the triangles? _____

Name: _____

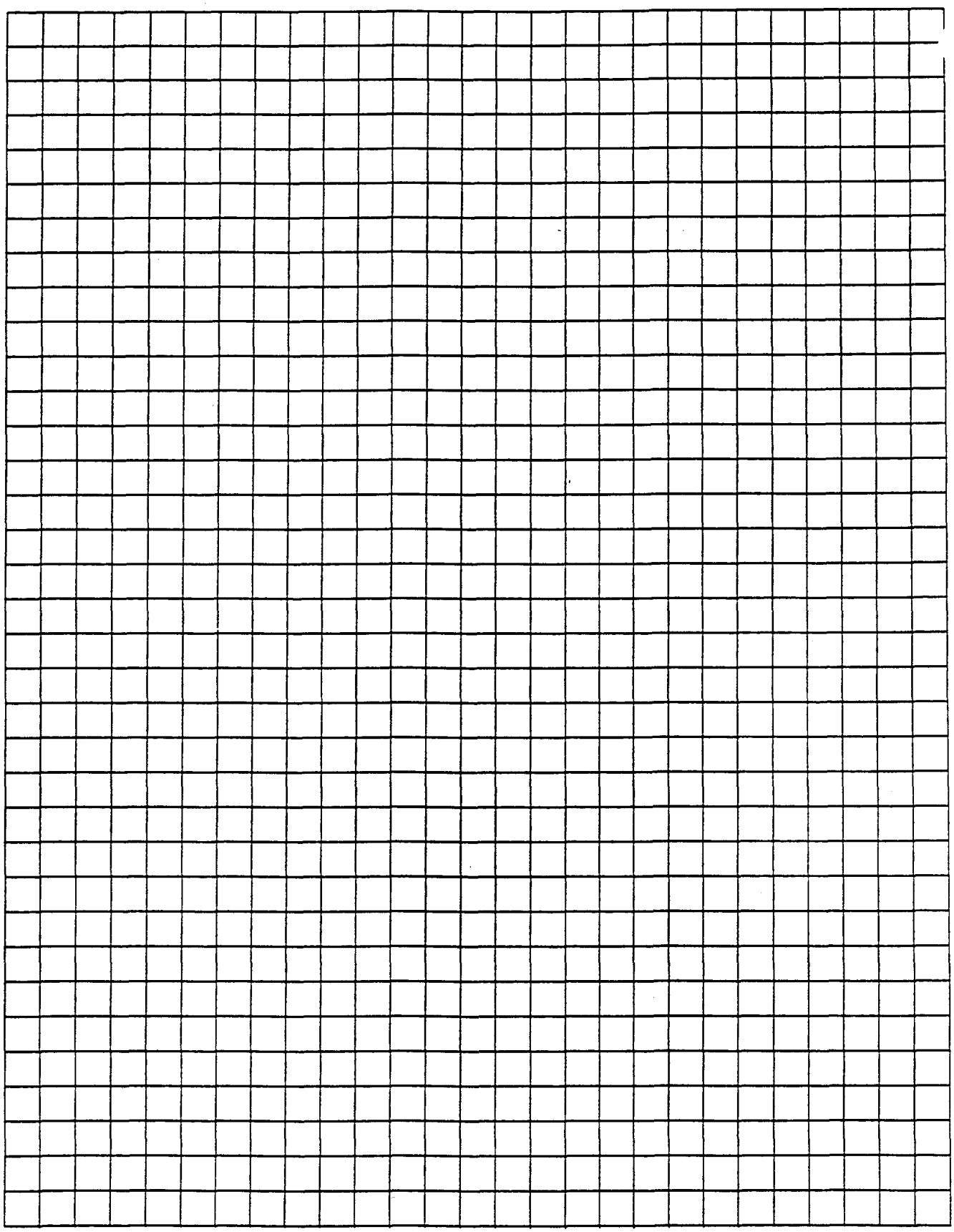
Writing About Polygons and Angles

As the number of sides of a polygon increases, how does the measure of the interior angles change?

Describe a way to find the measure of <u>one</u> interior angle of a regular 32 sided polygon.

The rock garden and the flower garden are both quadrilaterals.
Use math vocabulary to describe how the shape of the rock garden and the shape of the flower garden are different.

Grid Paper



Answer Key
Obj. 43

Interior Angles of Triangles (p.7)

1) b	4) d	7) c	10) 90°
2) 180°	5) 180°	8) 180°	11) 60°
3) 50°	6) 40°	9) 110°	12) 154°

Sum of Interior Angles of Polygon Chart (p.8)

Polygon Name	Number of Sides	Number of Triangles	Sum of Interior Angles	Degrees Added
Triangle	3	1	180°	———
Quadrilateral	4	2	360°	180°
Pentagon	5	3	540°	180°
Hexagon	6	4	720°	180°
Septagon	7	5	900°	180°
Octagon	8	6	1080°	180°
Nonagon	9	7	1260°	180°
Decagon	10	8	1440°	180°
n-agon	n	n - 2	(n - 2)180°	180°

A Pattern ..... sum of its angles increases 180°

A Pattern ..... is always 2.

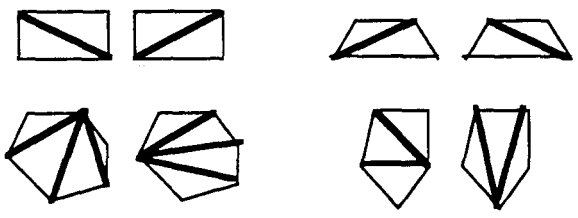
Next Step .... (n-2) 180°

Challenge 13 sided polygon (13 -2)180° = (11)180° = 1980°

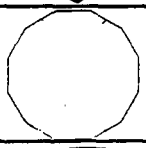
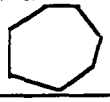
50 sided polygon No; Use the formula; (n-2)180° = (50-2)180° = (48)180° = 1980°

Too hard or impossible to draw a polygon with so many sides. It would be impractical to try and draw all the triangles, too. It is much simpler to use the formula.

Practice with Interior Angles of Polygons (p.11)



Practice with Interior Angles of Polygons (p.11) con't

Polygon Shape	Diagram	# of Triangles	Sum of the Measure of the Interior Angles
1) Pentagon		3	3 x 180° = 540°
2) Dodecagon (12 sides)		10	10 x 180° = 1800°
3) Septagon		5	5 x 180° = 900°

Practice with Interior Angles of Polygons (p.12)

hexagon (4 x 180° = 720°)
octagon (6 x 180° = 1080°)
decagon (8 x 180° = 1440°)

1) quadrilateral	82 + 89 + 95 = 266	360° - 266° = 94°
2) pentagon	90 + 90 + 130 + 130 = 444	540° - 440° = 100°
3) hexagon	75 + 148 + 142 + 78 + 142 = 585	720° - 585° = 135°
4) pentagon	120 + 95 + 100 + 110 = 425	540° - 425° = 115°

Problem Solving

1) equilateral triangle
2) trapezoid , quadrilateral
3) quadrilateral, parallelogram
4) 4 x 180° = 720 720÷6= 120°
5) 60°

Writing About Polygons and Angles (p.14)

1) When a side is added, the sum of the interior angles increases by 180°.
2) Use the formula (n-2)180° to find the total measure of all the interior angles.
   Then, divide by 32 to find the measure of one interior angle.
   (32 - 2) 180° = (30)180° = 5400°
   5400÷ 32 = **168.75°**
3) Both are 4 sided figures with two pair of parallel sides.
   Their angles are different. The flower garden has four 90° angles. The rock garden has 2 obtuse angles and 2 acute angles. The rock garden also has two diagonal parallel lines.

Objective 44: Investigate the properties of symmetry and transformation including: slide/translation, flip/reflection, and turn/rotation.

Vocabulary

transformation
slide (translation)
flip (reflection)
turn (rotation)
symmetry
line of symmetry
line of reflection
clockwise
counterclockwise

Materials

scissors
white paper
dark markers

Transparencies

- Visualizing a Slide, Flip, and Turn
- Symmetry All Around Us
- Slide/Translation
- Grid Paper
- Creating a Reflection
- Exploring Rotation

Student Copies

- Lines of Symmetry
- Symmetry Practice
- Can You Draw a Translation?
- Grid Paper
- Let's Rotate
- Practice with Slides, Flips, and Turns
- Visual Thinking with Transformations
- Vocabulary Match
- Let's Talk Transformations and Symmetry

Language Foundation

1. Explain to students that you can **transform** or change a figure by moving it in some way. Point out that in math there are three ways you can change a figure. You can move a figure by sliding, flipping, or turning it. When you move a figure, you make a **transformation** or a change in the figure. Tell students that **transform** is the verb and **transformation** is the noun.

2. To facilitate students' understanding of the concepts in this lesson, it would help them to visualize the meaning of the terms **slide** (translation), **flip** (reflection), and **turn** (rotation).

   Ask students to think of winter sports they might have seen on television or in a movie. Use the transparency Visualizing a Slide, Flip, and Turn to help students picture these sports. Ask students to think about the movement of a person on skis. Explain that this movement is like a **slide** in math. A **slide** means that every point in a figure moves the same distance in the same direction. The concept of a slide can also be demonstrated by the dance step the "electric slide."

   Have students look at the skier doing a "flip." Explain that in math a figure can be **flipped** over a line. The flipped figure will look like the image reflected in a mirror. The concept of a flip can also be demonstrated by picturing a gymnast making a flip or someone doing a flip off the diving board at a swimming pool.

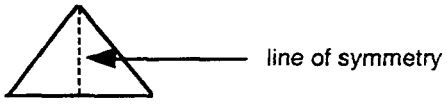
   For **turn**, have students look at the figure skater making a turn. Explain that in math a **turn** means that a figure moves around a center point. When you turn a figure, you can describe the movement by giving the direction the figure moved, the amount of the turn, and the center point that doesn't turn. Talk about the point of the skate where the skater makes her turn and have students describe turns the skater could make.

3. Explain to students that they will learn more about these **transformations** as the lesson develops.

Mathematics Component

1. *Investigate the property of symmetry.*
   - Write the word "symmetry" on the board.
   - Using the transparency Symmetry All Around Us, show students that some objects in everyday life are examples of **symmetry**.
   - Explain that if you can fold a figure so that it has two parts that match exactly, the figure has **symmetry**. The line where the figure can be folded so that one side matches exactly with the other side is called the **line of symmetry**. Point to the line of symmetry on each of the objects.
   - Distribute individual copies of the activity sheet Lines of Symmetry.
   - Review the names of each of the polygons. Triangles classified by the length of the sides were introduced in Volume I. You may need to review these briefly.

     isosceles triangle (two equal sides)
     equilateral triangle (three equal sides)
     scalene triangle (no equal sides)
     trapezoid
     parallelogram
     rectangle
     rhombus
     square
   - Have students cut out the isosceles triangle.
   - Remind them that a **line of symmetry** is the line where a figure can be folded in half and one side matches the other exactly. Ask students to fold the triangle as many ways as possible to find a line of symmetry. Ask how many lines of symmetry they can find. (1 line of symmetry)
   - Have a student come up and draw the line of symmetry on a transparency copy of the activity sheet.



isosceles triangle

   - Ask students if they think there will also be one line of symmetry on an equilateral triangle.
   - Have students cut out the equilateral triangle, fold to find as many lines of symmetry as possible, and then draw them in on the transparency. (3 lines of symmetry)

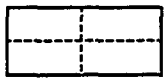
equilateral triangle

   - Repeat the same procedure for the other polygons, drawing the line(s) of symmetry for each on the transparency after folding and experimenting with paper.

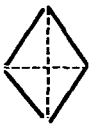
scalene triangle (0)

trapezoid (1)

parallelogram (0)



rectangle (2)



rhombus (2)

square (4)

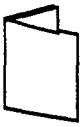
- Review the concept of symmetry by discussing the following ideas:
  - What is symmetry? (Symmetry is when you can fold a figure and one half matches the other half exactly.)
  - What is a line of symmetry? (The line along which a figure can be folded so that both halves match exactly.)
  - Do all geometric figures have a line of symmetry? (No)
  - Name/draw two geometric figures that do have at least one line of symmetry. (Answers will vary, but may include triangle, trapezoid, rectangle, rhombus, or square.)
  - Name/draw two geometric figures that do not have a line of symmetry. (Answers will vary, but may include scalene triangle or parallelogram.)
- The activity sheet Symmetry Practice can be assigned for further practice.

2. Explore the concept of a translation (slide).
   - Refer back to the language foundation to review the word **transformation**. Remind students that a transformation is a "change" such as a slide, a flip, or a turn.
   - Explain that students will learn special math words for these changes.
   - Write the word **translation** on the board. Tell students that this is a math word which means the same thing as the word "slide."
   - Place a clean piece of white paper over the transparency Slide/Translation. Trace over the triangle and the rectangle shown on the transparency. Cut the two traced pieces from the white paper.
   - Place the transparency copy of Slide/Translation on the overhead.
   - Read the information at the top of the page aloud, emphasizing that every point in the figure must slide the same distance and in the same direction for a translation.
   - Place the paper tracing of the triangle over the original triangle on the transparency. Model "sliding" the triangle over into its new position inside the dotted lines. Tie the concept of translation back to the language foundation by telling students that the figure slides in the same way that a skier slides across the snow.
   - Talk about the **translation arrow**. Show students that this arrow identifies matching points in each figure.
   - Read the information in the box underneath the triangles aloud.
   - Discuss the direction(s) a figure can move. (right, left, up, down) It is helpful if students learn to check a translation for right or left movement first, and then up or down movement.
   - To reinforce the concept that all points of the figure must move the same distance, start at each point of the original triangle and count 6 units to the right. Say, "Each point of the triangle moved 6 units to the right. This is called a **translation**."
   - Reposition the paper triangle so that it is slanted in a different direction from the original triangle.

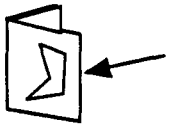
Ask students to explain why this is or is not a translation. (It is not a translation because each point did not move the same distance in the same direction.)

- Distribute individual copies of the activity sheet Can You Draw a Translation? Allow time for students to draw a second translation figure for each of the polygons.
- Have students share their translations with a partner. Ask them to talk with their partner about:
  1) the direction(s) each figure moved (right - left - up - down)
  2) the number of units each figure moved
- Choose a couple of students to draw their translations on the overhead. Discuss the movements of each figure.

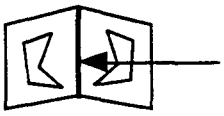
3. Explore the concept of reflection (flip).
   - Distribute a piece of white paper and a dark marker to each student.
   - Have students fold the paper in half as shown below.



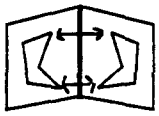
   - Review the definition of a polygon. Without unfolding the paper, ask them to use a ruler to draw any polygon on one side of the paper. Be sure they use a dark marker.



   - Tell students to turn over their folded paper and trace the polygon.
   - Have students unfold their papers and draw a line along the fold.

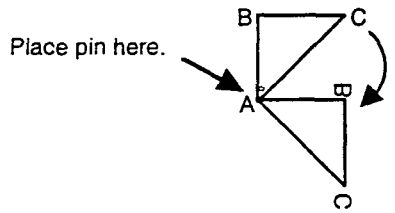


   - Ask students if they notice anything about the two figures. Lead them to understand that the figures are like looking into a mirror. A mirror produces a reflection of our faces.
   - Explain that in geometry, a **reflection** is a **transformation** where a figure is flipped over a line. That is why it is also called a "flip." It is like picking up a figure and turning it upside down on the other side of the line. The line is called the **line of reflection**.
   - Say, "In a reflection, each matching point is the same distance from the line reflection. Have students measure the distance from the line of reflection to matching points on the two polygons they have drawn.



- Place the transparency Grid Paper on the overhead. Draw a line of reflection down the middle.
- Cut any polygon from a piece of construction paper.
- Place the polygon onto the transparency on one side of the line of reflection.
- Trace around the polygon.
- Model "flipping" the polygon and carrying it to the other side of the line of reflection. Tie this concept back to the language foundation by explaining that you flip the figure in the same way that a skier might do a flip.
- Place the polygon so that matching points are the same distance from the line of reflection. Say, "A **reflection** is created when you "flip" a figure over a line. Matching points are the same distance from the line."
- Place the directions Creating a Reflection on the overhead and read aloud as you review the steps taken above to create a reflection.
- Provide grid paper and materials for students to repeat these steps to create their own reflections.

3. Explore the concept of rotation (turn).
   - Briefly review the two types of transformations introduced so far. (A **translation** is the same as a slide. A **reflection** is the same as a flip.)
   - Tie the word rotation back to the language foundation by telling students that the third type of transformation is like an ice skater turning around on the ice. Write the word "rotation" on the board. Say, "**Rotation** is the same thing as a turn."
   - Trace the triangle on the transparency onto a clean piece of paper. Cut the paper triangle out.
   - Place the transparency Exploring Rotation on the overhead.
   - Read the information at the top of the page aloud.
   - Place the paper triangle over the the one on the transparency. Place a straight pin through both triangles at point A.
   - Slowly rotate the paper triangle around point A until it reaches a quarter turn (90°). Point A is the only point that will not move.

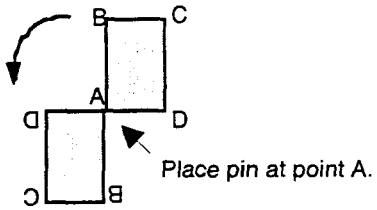


   - Tell students that this is a **rotation**. Explain that Point A is the **center of rotation**. It is the point that does not turn.
   - Ask students if the triangle changed size when it turned. (no) Shape? (no)
   - Discuss what information students could give to describe the rotation. Lead students to

understand that there are three ways to describe the rotation:

1) the amount of the turn (as a fraction or as degrees)
2) the direction of the turn (**clockwise** or **counterclockwise**)
3) the location of the center of rotation

- Have students help describe these three things about the rotation of the triangle on the transparency. (The triangle rotated 1/4 turn (90°). It moved like a clock - clockwise. The center of rotation was point A.)
- Repeat the same steps with the rectangle, rotating it 90° around point A as shown below.



- Have students describe the rotation. (The rectangle turned halfway (180°). It moved opposite of a clock - counterclockwise. The center of rotation was point A.)
- Additional practice with rotation is is provided on the activity sheet Let's Rotate.
- The activity sheets Practice with Slides, Flips, and Turns and Visual Thinking with Transformations can be used for further review of the three transformations learned in this unit.

Language Development Activities

- The teacher is encouraged to spend time working with students on the vocabulary in this objective as Obj. 44 is quite language intensive. Working together with the students as they add these words to their vocabulary notebook is a good strategy for vocabulary instruction and retention. The activity sheet Vocabulary Matching will give students an opportunity to review the meaning and usage of this geometric vocabulary.
- Let's Talk Transformations and Symmetry provides students with the opportunity to think and write about symmetry and transformations in a more creative way, using geometric vocabulary to explain and answer questions.