Objective 27: Identify the relationship between sets of data using a ratio model.

Vocabulary

data
ratio
proportion
equal
compare
colon (:)

Materials

two color counters
overhead counters
small brown bags containing
    overhead manipulatives
    other items (for students)

<u>Using Ratios to Compare</u> activity sheet
<u>Ratios and Proportions</u> activity sheet
    student copies

Language Foundation

1. You may need to review the word **compare** meaning how things are the same and how they are different. In this lesson, we will use **ratio** to compare the number of things in two groups. Give an example comparing the number of girls to the number of boys in the class. Set up the ratio using the colon (g:b) and explain that this symbol is used to separate the two groups.

2. Point out that when two ratios are equal, they form a **proportion.** Say that if in your class, half of the students wear tennis shoes to school (8:16) and in Mrs. Turner's class, half wear tennis shoes to school (12:24). This is a proportion because both ratios equal one half. Further, explain that even though the number of students in each class is different, the ratio is the same.

Mathematics Component

1. Place three yellow transparent counters and 2 two red transparent counters on the overhead.
   - Point first to the yellow counters and say: "We are going to compare this number of yellow counters to this number of red counters." On the board write each of the following, saying "three yellow to two red" each time:

3 yellow to 2 red

3 to 2

3:2 ← Point out the "colon"

3/2

$\frac{3}{2}$

   - Each of these is called a **ratio**. A ratio compares two numbers.
   - Ask students, "When I compared the number of yellow counters to the number of red counters, which number did I write first?" (Yellow)
   - Now, using the same counters, point first to the red counters and say: "We are now going to compare this number of red counters to this number of yellow counters."
   - On the board write each of the following, saying " two red to three yellow" each time:

2 red to 3 yellow

2 to 3

2:3

2/3

$\frac{2}{3}$

   - Each of these is also a ratio. Ask students, "When I compared the number of red counters to the number of yellow counters, which number did I write first?" (Red)
   - Ask students why the fraction 2/3 is listed as a ratio. (A fraction is a ratio which compares two things.)
   - Point to the numbers in the first group of ratios above as you say: " The order of numbers in a ratio is important." Explain to students that if they are comparing yellow to red , then they need to write the number of yellow counters first. Point to the second group of ratios and ask: "If you are comparing red counters to yellow, which number do you write first? (Number of red) Second?" (Number of yellow)

2. Now, pointing again, say: "We may also compare the number of yellow counters to the whole group of counters." On the board write:

3 (yellow) to 5 (whole group)

- Ask students to come up and show other ways to write the same ratio.
  (3 to 5 or 3:5 or 3/5 or $\frac{3}{5}$)
- Ask students what the ratio would be if they compared the number of red counters to the whole group. (2 (red) to 5 (whole group) or 2 to 5 or or 2:5 or 2/5 or $\frac{2}{5}$)
- Ask students if the number of red and yellow counters on the overhead has changed. (No) Ask students what did change as they wrote the ratios. (The order of the numbers)
- Remind students that **order** means which comes first, second, third, etc. Also remind students that different kinds of things can be compared such as: yellow to red, red to yellow, yellow to whole group, red to whole group...)

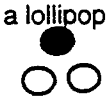
3. Take out a small brown lunch bag which has been previously filled with two different kinds of items and marked on the outside with a large "1." (Items might be overhead pattern blocks such as 4 circles and 3 squares, or overhead coins such as 5 pennies and 2 dimes, or overhead lab gear pieces...)
   - Have students work in groups of 3 or 4 and give one copy of the activity sheet Using Ratios to Compare to each group.
   - As you place the items in bag "1" on the overhead, explain to students that we need to write four different ratios comparing the items in this bag.
   - Ask students to volunteer suggestions. Using overhead of activity sheet, model how answers will be written in the four boxes beside number 1. Use words to describe what is being compared and colons to show the ratio.

   Example (with 4 circles and 3 squares):

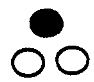
1.	circles to squares 4:3	squares to whole group 3:7	squares to circles 3:4	circles to whole group 4:7

   - Bring out remaining bags (2-7) which have been previously filled with items.
   - Give one bag to each group. Ask students to look at the number on the outside of the bag and work as a group to write four different ratios beside the appropriate number on their activity sheet for the items in their bag. Once all groups have finished the four ratios, ask students to place the items back in the bag.
   - Rotate bags and repeat so that each group completes ratios for bags 2-7.
   - Use transparency to discuss and review answers.

4. Place one red transparent counter on the overhead with two yellow counters as shown below.
   - Explain that the red counter is a lollipop and each yellow counter is 1¢.



- Ask: "What is the ratio of lollipops to cost?" (1:2)
- Now show a model for two lollipops and ask what the ratio of lollipops to cost is.

one lollipop   two lollipops

1:2 2:4

- Ask what the ratio would be for three lollipops, four lollipops... (3:6) (4:8)...
- Explain that these are called **equal ratios** because the relationship 1:2 remains the same - for every one lollipop there are two pennies.
- Ask students if there is a way to build equal ratios for 1:2 without using the counters. (Multiplication can be used.)
- What would we have to remember? (To multiply both parts of the original ratio by the same number.)
- Ask students to name equal ratios for each of the following: 2:3, 5:4, 3:1.

2:3	(4:6) (6:9) (8:12)
5:4	(10:8) (15:12) (20:16)
3:1	(6:2) (9:3) (12:4)

- Write the same ratios as fractions and ask students if they notice anything. (Equivalent fractions are equal ratios.)

2/3 = 4/6 = 6/9 = 8/12
5/4 =10/8 = 15/12 = 20/16
3/1 = 6/2 = 9/3 = 12/4

- When two equal ratios (or fractions) are written with an equal sign, they are called a **proportion**.
- Use the equivalent ratios above to set up proportions. (2/3 = 4/6, 5/4 = 15/12, 3/1 = 9/3...)
- Have students complete the activity sheet <u>Ratios and Proportions</u>.

Names ________________

____

Using Ratios to Compare

Write four **different** ratios for each numbered bag of items. Use **words** to describe what you are comparing and **colons** to show each ratio.

1.				
2.				
3.				
4.				
5.				
6.				
7.				

Name ____________

Date ________

Ratios and Proportions

1. Write the ratio **3 wins to 5 losses** 4 different ways. ________
____
____
____

In Mr. Jones' class there are 13 boys, 14 girls and 2 teachers. Use this information to write the following ratios:

2. boys to girls ______________
3. girls to students ______________
4. teachers to students ______________
5. boys to teachers ______________

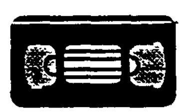
Write three equivalent ratios to the given ratio.

6. 3:4 ______ ______ ______ ______
7. 2:7 ______ ______ ______ ______
8. 1:1 ______ ______ ______ ______

Use the following advertisement to write the following ratios.

Movie Rentals!!!!

Rent movies for
$3.50 per movie.



9. Write a ratio to represent one movie rental. ________
10. Write a ratio to represent two movie rentals. ________
11. Write a ratio to represent five movie rental. ________

Answer Key
Obj. 27

Using Ratios to Compare
Answers will vary

Ratios and Proportions

1. Write the ratio **3 wins to 5 losses** 4 different ways. **3:5 3/5 3 to 5** $\mathbf{\frac{3}{5}}$

2.	boys to girls	**13:14**
3.	girls to students	**14:27**
4.	teachers to students	**2:27**
5.	boys to teachers	**13:2**

6. 3:4 **6:8 9:12 15:20 30:40** (answers will vary)
7. 2:7 **1:3.5 4:14 6:21 20:70** (answers will vary)
8. 1:1 **2:2 50:50 9:9 100:100** (answers will vary)

9.	Write a ratio to represent one movie rental.	**1 : $3.50**
10.	Write a ratio to represent two movie rentals.	**2 : $7.00**
11.	Write a ratio to represent five movie rental.	**5 : $17.50**

Objective 28: Solve proportions to find the unknown.

Vocabulary

ratio
fraction
expanded fraction
equivalent
proportion
cross-products

Materials

two colors of overhead counters

Solving Proportions Practice Sheet
student copies

Language Foundation

1. Tell students that **expand** means to get bigger. A balloon expands when you blow it up. A rubber band expands. Have students give other examples of things that expand. (elastic waistbands, the size of the class when new students enter, a sponge when it gets wet.)

2. Break the word **equivalent** apart. (equal + value) Equivalent means of equal value. For example, four quarters are equivalent to one dollar. Have students give other examples of equivalences.

3. Show students the symbol "X" and ask them what the two lines are doing. (Crossing) Discuss other things you can cross- going from one place to another.

Mathematics Component

1. Using two colors of overhead counters (red and yellow, for instance), place three yellow and five red counters on the overhead.
   - Write the fraction 3/5, reminding students that the amount of yellow counters can be compared to the amount of red counters by using a fraction.
   - Now show students 6 yellow and 10 red counters, slightly separating the yellows into two groups of 3 each, and the reds into two groups of five each.
   - Now write the fraction 6/10. For every 3 yellow counters, we have 5 red ones. If we have 6 yellow counters (2 groups of 3) we will need 10 red counters (2 groups of 5) to keep the same relationship of yellow to red.
   - Remember that 3/5 and 6/10 are **equivalent** fractions or **equivalent** ratios. Two equivalent ratios are called a **proportion**. Ask students how many red counters would be needed for 12 yellow ones. (20) We can write this:

$$\frac{3}{5} = \frac{12}{(20)}$$

Ask students how they arrived at the answer. (Four groups of 3; therefore 4 groups of 5.)

   - Ask students how many yellow counters would be needed for 30 red ones. (18)

$$\frac{3}{5} = \frac{(18)}{30}$$

Six groups of 5; therefore six groups of 3 would be needed.

   - Next, put several sets of fractions on the board and have students decide whether they are proportions:

$$\frac{4}{5} = \frac{12}{15}$$

3 groups of 4 make 12; 3 groups of 5 make 15; this is a proportion

$$\frac{3}{4} = \frac{12}{16}$$

4 groups of 3 make 12; 4 groups of 4 make 16; this is a proportion

$$\frac{2}{5} = \frac{6}{10}$$

3 groups of 2 make 6; 3 groups of 5 make 15, not 10. <u>This is not a proportion and the = should be replaced with a ≠.</u>

   - Checking to see that the pairs are equivalent fractions is one way to tell if a set of fractions forms a proportion. There is another way to tell if the fractions form a proportion.
   - Put an oval around 4 and 15 in the first proportion. Now write 4 • 15 = 60. Next, put an oval around 5 and 12 in the first proportion. Write 5 • 12 = 60. These two multiplications are called **cross-products.** <u>In proportions, cross-products are equal</u>. We can

tell if a proportion is correct if its cross-products are equal.

- Now put an oval around 3 and 16 in the second example: $3 \cdot 16 = 48$
- Put an oval around 4 and 12: $4 \cdot 12 = 48$ The cross-products are equal and the two sets of fractions form a proportion.
- In the third example, $2 \cdot 10 = 20$, but $5 \cdot 6 = 30$. The cross-products are not equal, so the two fractions **do not** form a proportion.

2. In addition to checking to see if two ratios form a proportion, we can also find the missing number, or solve a proportion by using the fact that cross-products are equal.

- Here's an example: $\frac{2}{3} = \frac{x}{15}$ We can solve this proportion, or find the missing number, in two different ways. As we did before, we can say that 5 groups of 3 produced 15; therefore we would need 5 groups of 2 to have a proportion; $5 * 2 = 10$ 10 is the missing number.

- Or, we can use the cross-products to form an equation:

$$3 \cdot x = 2 \cdot 15$$

$$3x = 30$$

$$\frac{3x}{3} = \frac{30}{3} \quad \text{divide both sides by 3}$$

$$x = 10 \quad \text{10 is the missing number in the proportion.}$$

- Here's another one:

$$\frac{2}{8} = \frac{5}{x}$$

$$2 \cdot x = 5 \cdot 8$$

$$2x = 40$$

$$\frac{2x}{2} = \frac{40}{2} \quad \text{divide both sides by 2}$$

$$x = 20 \quad \textbf{20} \text{ is the missing number in the proportion}$$

- Some proportions produce decimals for the missing number:

$$\frac{3}{5} = \frac{x}{18}$$

$$5 \bullet x = 3 * 18$$

$$5x = 54$$

$$\frac{5x}{5} = \frac{54}{5}$$ divide both sides by 5

$$x = 10.8$$ The missing number is 10.8.

- Distribute Solving Proportions Activity Sheet.

Name________________________________

Solving Proportions Practice Sheet

Write the cross-products of the terms.

1. $\frac{20}{24} = \frac{5}{6}$
2. $\frac{32}{8} = \frac{20}{5}$
3. $\frac{25}{40} = \frac{n}{8}$

Tell whether each proportion is *True* or *False*.

4. $\frac{21}{24} = \frac{7}{8}$
5. $\frac{10}{12} = \frac{3}{4}$
6. $\frac{8}{6} = \frac{6}{8}$
7. $\frac{25}{6} = \frac{8}{2}$
8. $\frac{1.5}{3} = \frac{10}{20}$
9. $\frac{50}{100} = \frac{3.2}{6.4}$

Solve each proportion.

10. $\frac{12}{c} = \frac{4}{6}$
11. $\frac{22}{10} = \frac{m}{5}$
12. $\frac{a}{5} = \frac{9}{3}$
13. $\frac{1.2}{1.5} = \frac{y}{10}$
14. $\frac{2.5}{5} = \frac{c}{8}$
15. $\frac{6}{t} = \frac{4}{6}$
16. $\frac{2n}{8} = \frac{7}{10}$
17. $\frac{15}{4} = \frac{9}{2t}$
18. $\frac{9}{10} = \frac{3x}{5}$

Create a proportion using the following ratios:

19. $\frac{9}{11} =$
20. $\frac{7.5}{8.2} =$
21. $\frac{5}{8} =$

Answer Key
Obj. 29

Solving Proportions Practice Sheet

1.	20 * 6	5 * 24	2.	32 * 5	8 * 20
3.	25 * 8	40 * n	4.	yes	
5.	no		6.	no	
7.	no		8.	yes	
9.	yes		10.	18	
11.	11		12.	15	
13.	8		14.	4	
15.	9		16.	2.8	
17.	1.2		18.	1.5	

19-21. Answers will vary

Objective 29: Estimate the percent of a number using the percent bar model.

Vocabulary

%
percent
percentage
equivalent
reasonable estimate
part/whole

Materials

counters
base ten blocks
blank transparency

<u>Percent Bar Model</u>
transparency

<u>Estimating with Percent Bars</u>
<u>Create A Problem</u>
student copies

Language Foundation

1. Break the word **percent** into 2 parts: per = out of, cent = hundredth. Explain that **percent** means "out of one hundred" and tells us how many parts out of 100 we are talking about. Use money for an example. Show students one dollar (100 cents). Tell them that they will need to spend 75 out of the 100 cents to buy a soda. They will be using 75 **parts** of the **whole** dollar to buy the soda. This is the same as 75 percent.

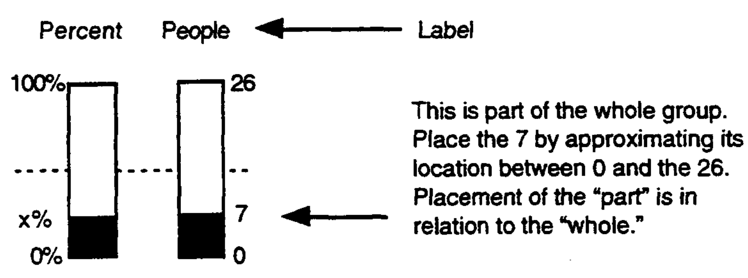
2. **Equivalent** can be reviewed in terms of equal parts. Use the base ten blocks to show how ten units equal one rod; ten rods equal one flat. These amounts are equivalent. Let students make up other examples of equivalency by using objects in the classroom which show same size, same height, etc.

3. You can help students understand the idea of **reasonable estimate** by explaining that an estimate is not an exact answer, but that it is close. It is reasonable if it "makes sense" in terms of the what the problem is asking for. Tell students to ask themselves, "Does this look right?" or "Does this make sense?" For example, if the problems involves numbers in the thousands, it would not be reasonable to end up with an answer in the hundreds. Further, show the following on the overhead: 4000 + 3000 = 700; 4000 + 3000 = 7000. Ask the students to explain which answer is reasonable and why.

Mathematics Component

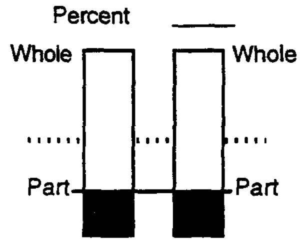
1. Tell students that percent bars are a way to see how part of a whole group relates to percent.
   - Place the Percent Bar Model Transparency on the overhead and explain that two bars will be used to show this relationship.
   - Show that each bar has a label to show what it represents. Percent is on the left bar and the number being described is on the right bar.
   - Say, "The percent bar starts with 0% on the bottom and rises to the top with a whole group which is 100%. The right bar starts with 0 a the bottom and rises to the total amount of the number being described."
   - Ask students what they think the dotted line represents. (Elicit an understanding that the dotted line is marking one-half of each bar.)
   - Ask what the one-half mark on the percent bar should be called. (Have students explain why 50% would be an appropriate name for this mark.)
   - Ask students what the one-half mark on the right bar should be called. (Elicit a response that indicates understanding that the one-half mark can not be named unless we know the total amount at the bottom of the bar.)
   - Ask students what the one half mark would be called on this bar if 10 were the total amount, 80, 200, etc. (Fill in each total amount and one-half mark on the right bar as you give examples.)

2. Use counters, or another manipulative, as you explain the following situation.
   - "Suppose that 26 young people were waiting to get into a movie theater. (Show 26 counters.) They are told that there are only 7 seats left and only 7 people may go in. (Separate the 7 counters from the rest of the group.) Ask what percentage of the young people can get in?"
   - Allow some time for students to respond. Explain that percent bars can be used to estimate the percent. Use the Percent Bar Model Transparency to illustrate this relationship.

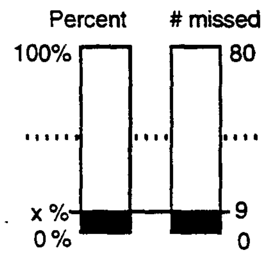


Reasonable estimates would be 20-30%.

   - Explain that the percentage estimate can be made by comparing the position of the "known" part on the right bar to the "unknown" percent (x%) on the left bar. (Students should be able to estimate x% based on its relationship to the 50% line on the percent bar.) **Be sure students understand** that the relationship on each of the **equivalent** bars is part to whole.



- Divide students into groups of 3-4. Ask students to work in groups to draw a percent bar model on a blank transparency and estimate the percent for the following situation. (Remind students that bars must be **equivalent.**) Say, "On a test with 80 questions, a student misses 9 questions. About what percent of the questions does the student miss?"



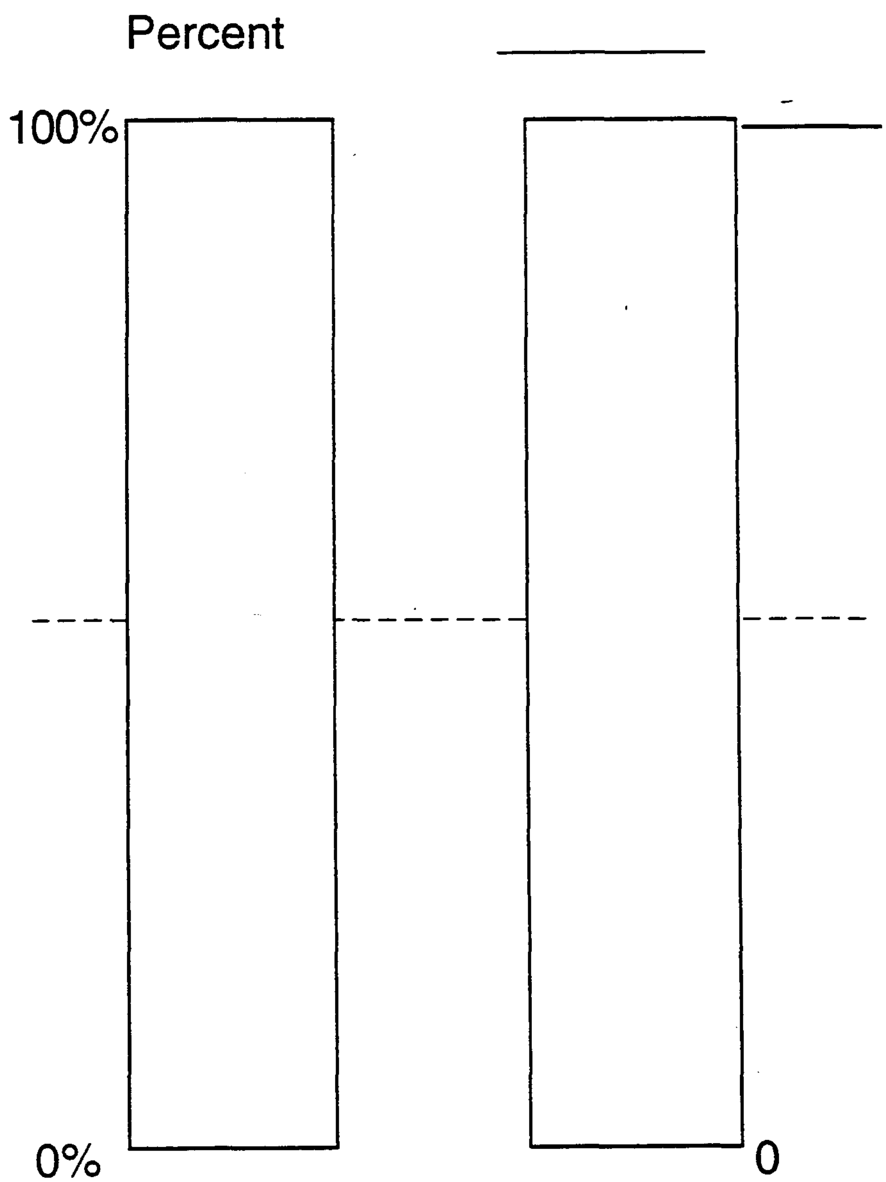
- Allow time for students to share their models and explain their reasoning. (Following the model given above, reasonable estimates would be 5-15%.)
- Have students work in pairs using percent bar models to complete the activity sheet Estimating With Percent Bars.

Additional Activities

Assessment - Create a Problem

Have each student create (telling or writing) a word problem, represent the information on a percent bar model, and make a reasonable estimate. A recording sheet is provided.

Percent Bar Model

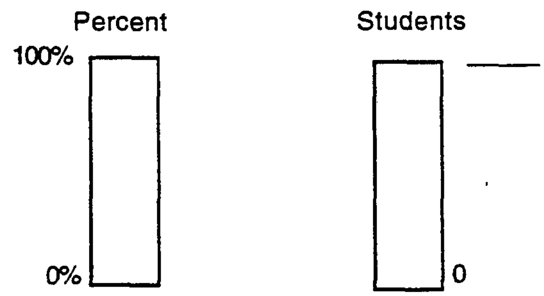


Names ____________________
____

Estimating With Percent Bars

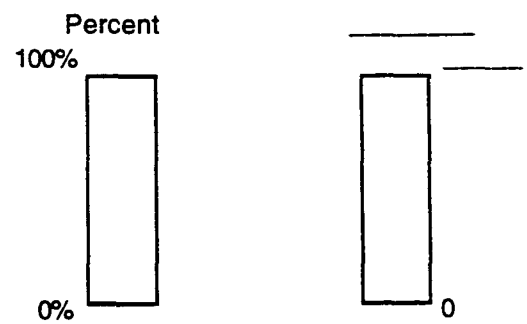
Fill in and label percent bars for each question. Then make a reasonable estimate.

1. Sixty students went to an amusement park. 21 of the students rode the roller coaster. Estimate the percent of students who rode on the roller coaster.



Estimate ______ %

2. The auditorium had 85 chairs. Each of the 60 students entering the room sat in one chair. Estimate the percent of chairs the students sat in.



Estimate ______ %

3. The swimming pool has 1500 gallons of water. Some water is taken out before winter so it will not freeze. 300 gallons of water is left in the pool. Estimate the percent of water taken out of the pool.

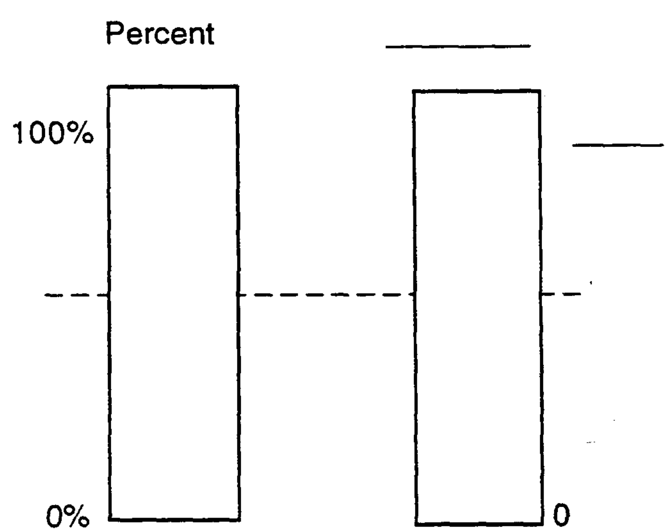
Draw bars, fill in, and label on the back of this paper. Then make a reasonable estimate. Think about how you will explain your thinking to the class.

Name ____________________

Date __________

Create a Problem

Write a word problem, including a question, which can be represented using a percent bar model. Fill in the bars and give a reasonable estimate which answers your question.



Estimate ________%

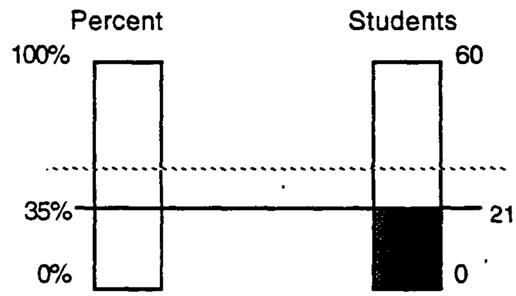
Answer Key

Obj.29

Estimating With Percent Bars

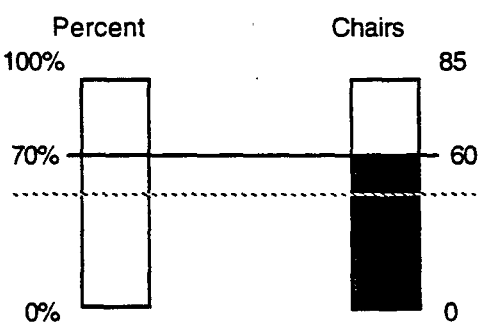
Fill in and label percent bars for each question. Then make a reasonable estimate.

1. Sixty students went to an amusement park. 21 of the students rode the roller coaster. Estimate the percent of students who rode on the roller coaster.



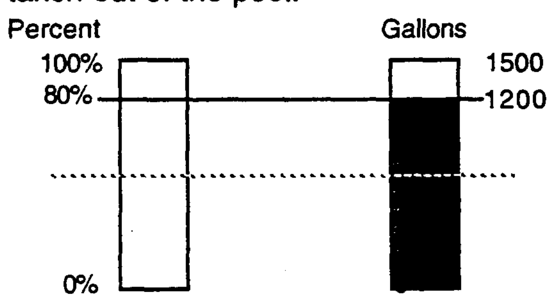
Estimate 35 %

2. The auditorium had 85 chairs. Each of the 60 students entering the room sat in one chair. Estimate the percent of chairs the students sat in.



Estimate 70 %

3. The swimming pool has 1500 gallons of water. Some water is taken out before winter so it will not freeze. 300 gallons of water is left in the pool. Estimate the percent of water taken out of the pool.



Estimate 80 %

Objective 30: Solve percent problems using a percent bar model

Vocabulary

compare
comparison
proportion
ratio
percent
equivalent
estimate

Materials

Percent Bar Model
transparency

Using Percent Bars and Proportion
student copies

Language Foundation

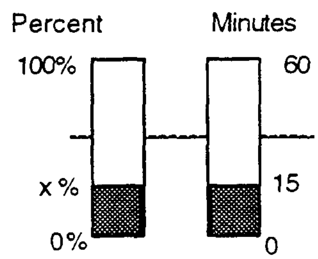
1. You may need to review the words **ratio, proportion, and percent** from previous lessons. Be sure to emphasize that these are all different ways of showing comparison.

2. Ask students to find the word equal in the word **equivalent** (equivalent). Equivalent is another way of saying equal. Remind them that equal means the same size, same number, etc.

Mathematics Component

1. Review the concept of percent.
   - Review the idea that **"per cent"** means the same as **out of 100**. Use the example that $0.12 is 12% of $1.00.
   - Review the concept of fraction, decimal and percent **equivalence** worked with in objective 14.
   - Remind students that in the last lesson they were estimating percent using percent bars.

1. Describe the following situation to students.
   During a 60 minute television program, 15 of the minutes are made up of commercials. Pose the following problem. What percent of the total program time is commercials? Ask students to explain what we are trying to find in this problem. (What percent of 60 is 15?) Ask how this can be set up using the Percent Bar Model Transparency. (See below.)



Be sure students understand the relationship between the bars which shows that 100% of the minutes is 60 and 0% of the minutes is 0.

- Ask students to make an **estimate** of the answer.
- Tell students that a **proportion** (pair of **equivalent** ratios) can also be set up to find the unknown percent.
- We can compare part to whole on each bar to set up two ratios because a ratio is a comparison of two things. Use the model above to point out the unknown percent (or part) on the percent bar which can be called "x" and the whole which is 100.
- Therefore, the ratio of part to whole on the percent bar is x:100 or $\frac{x}{100}$. (Set up the following proportion under the percent bars on the transparency as you illustrate.)
- "Part" on the minutes bar is 15 and the "whole" is 60. Therefore the ratio on the minutes bar is 15:60 or $\frac{15}{60}$. A proportion can be set up to find the unknown using this pair of ratios.

$$\frac{x}{100} = \frac{15}{60}$$

$60x = 1500$ ← Students should use cross multiplication to solve the proportion.

$$\frac{60x}{60} = \frac{1500}{60}$$

(Review the concept of cross-multiplying, if necessary.)

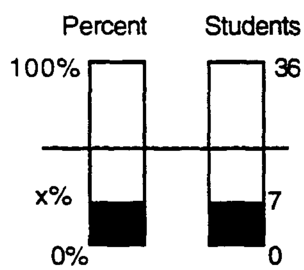
$$x = 25\%$$

- Discuss if this answer makes sense based on the student's estimates from the percent bar model.

- Repeat the above procedure assuming that during a 60 minute television show, 40 minutes were commercials. What percent of the total program time is commercials? (An accurate percentage is 66.66 = 67% of the total time.)

- Have students draw and work through the following examples on paper as you discuss them using the Percent Bar Model Transparency. First have student estimate an answer then use a proportion to solve each problem. Be sure to **compare** the estimate with the percent obtained through solving a proportion. (Examples may be made meaningful to your students.)

  Seven FAST Math students were absent today out of a total of 36 students. What percent of the FAST Math students were absent?



$$\frac{x}{100} = \frac{7}{36}$$

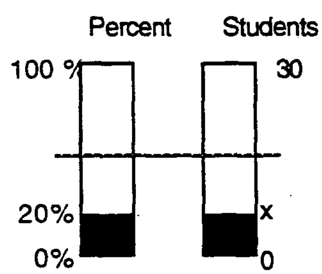
$$36x = 700$$

$$\frac{36x}{36} = \frac{700}{36}$$

$$\mathbf{x = 19.44 = 19\%}$$

2. Explain to students that this model can also be used if the **number** being described is the "unknown."
   - Work through the following problem together.

   Twenty percent of the students in FAST Math brought their lunch to school today. There are 30 students in the class. How many students brought their lunch?



$$\frac{20}{100} = \frac{x}{30}$$

$$100x = 600$$

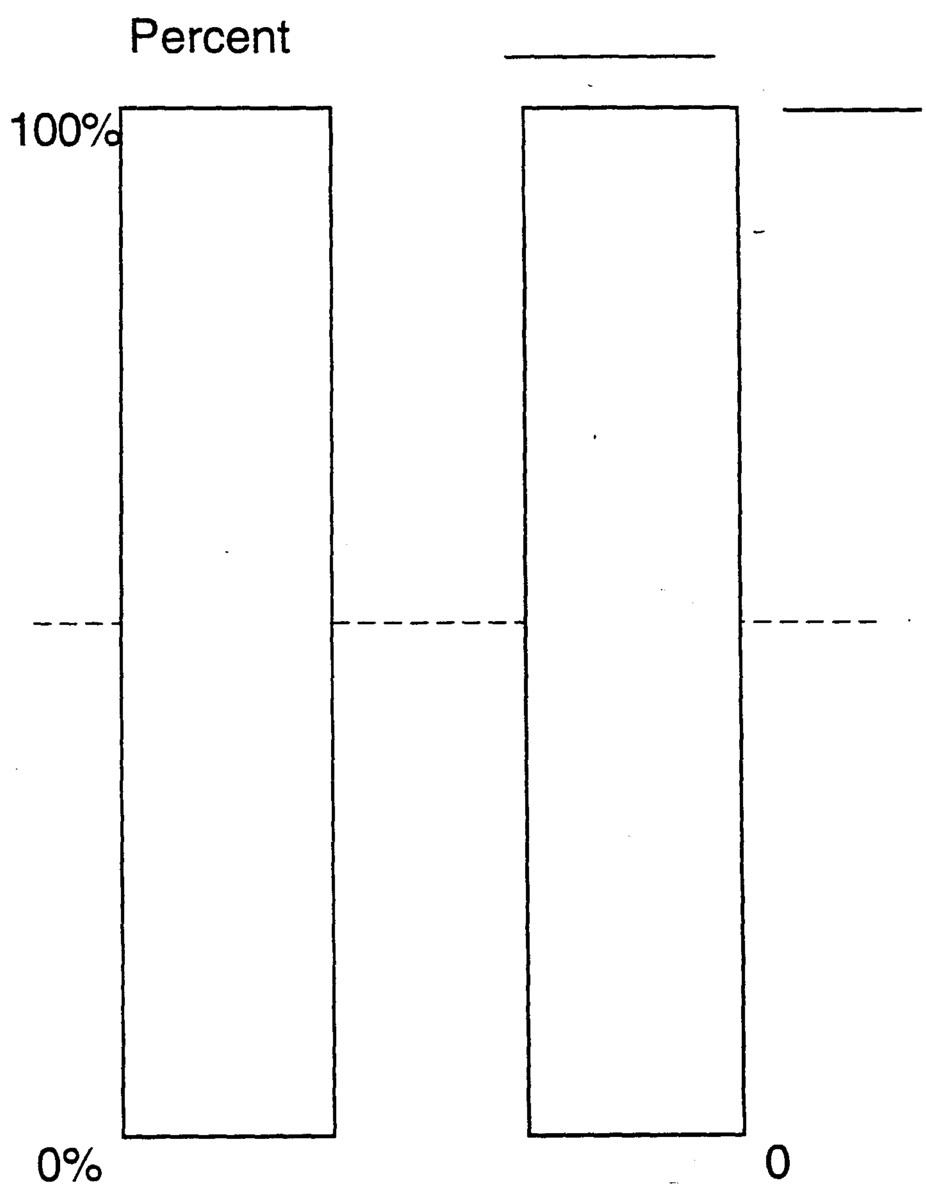
$$\frac{100x}{100} \quad \frac{600}{100}$$

$$\mathbf{x = 6\ students}$$

- Have students work in pairs to complete the Using Percent Bars and Proportions activity sheet.

Transparency

Percent Bar Model



Names ____________________

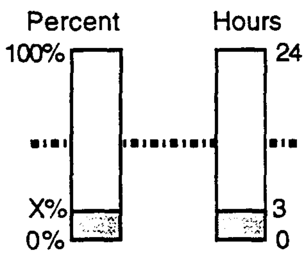
____

Using Percent Bars and Proportions

Write what you are trying to find in each problem. Then use a percent bar model and a proportion to find the "unknown" in each problem. **Compare** your two answers to see if they make sense.

EXAMPLE: Juan spends some of his spare time watching television. Out of a 24 hour day, he watches television about 3 hours each day. What percent of each day does Juan watch television?

We need to find: <u>What percent of 24 is 3?</u>



Estimate the answer <u>12%</u>

$$\frac{x}{100} = \frac{3}{24}$$

$$24x = 300$$

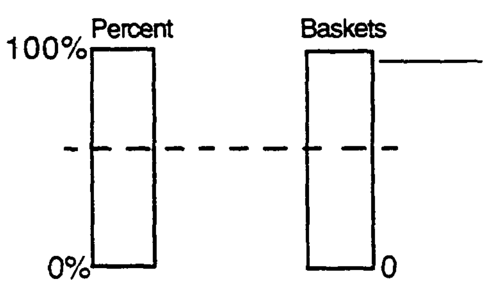
$$\frac{24x}{24} = \frac{300}{24}$$

$$x = 12.5\%$$

Was your estimate reasonable? <u>Yes, because the estimate of 12% and the real answer of 12.5% are almost the same number.</u>

1. Lin was the top scorer on her high school basket ball team. She made 80% of all her shots. If she shot 60 baskets, how many times did she score?

   We need to find: ________________________________



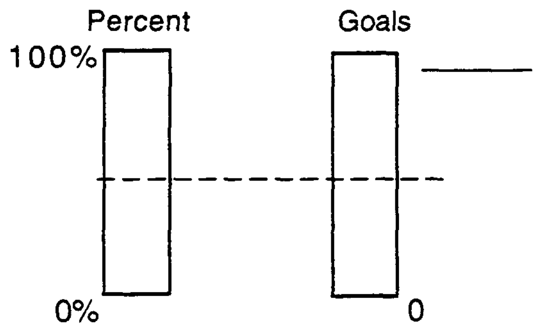
<u>Proportion</u>

Estimate the answer _____%

Was your estimate reasonable? Why or why not? ______________

2. Tran is on his high school soccer team. During the game on Friday, he scored 2 goals out of 6 tries. What percentage of his tries were successful?

We need to find: ______________________________

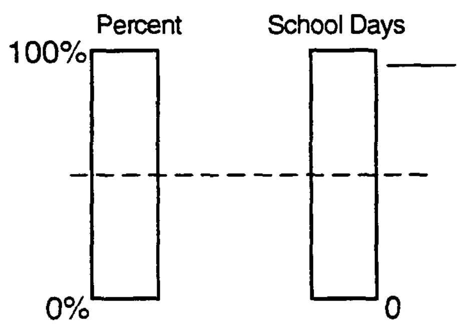


Proportion

Estimate the answer _____%

Was your estimate reasonable? ______________________________
____

3. Nhen was sick last year and missed several days of school. She was absent 19 days out of the 180 school days. What percentage of the time was Nhen absent?



Proportion

Estimate the answer _____%

Was your estimate reasonable? ______________________________
____

Which numbers, the estimates or the proportions, are more **accurate**? **Why?**

____
____
____

Answer Key
Objective 30

1. $\frac{80}{100} = \frac{x}{60}$ x = 48 baskets

2. $\frac{2}{6} = \frac{x}{100}$ x = 33 1/3%

3. $\frac{19}{180} = \frac{x}{100}$ x = 10.6%

The proportions are more accurate because they show exact number or percent mathematically.

Objective 31: To find the sale and original price when given a discount

Vocabulary

original
price
sale
discount
amount
mark down
working backwards
percent
proportion
reasonable
whole
exact

Materials

sales ads from catalogs or newspapers

<u>Percent Bar Model</u>
transparency (Obj. 30)

<u>Working with Discounts</u>
student copies

Language Foundation

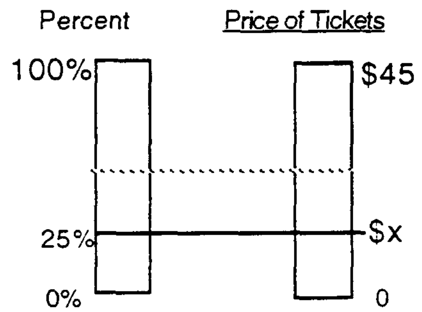
1. Help students understand terminology used in sales advertisements by showing them an example from a catalog or newspaper. Explain that the **original price** is the amount of money that was first assigned to the item. It is the larger amount. Show them the **sale price**, which is a smaller amount. Ask the students to find the difference in these two prices. Explain that this is the amount of money the buyer would save on this item. It is called the **discount.** Further, explain that the **mark down** is the same as the discount.

2. Problems that require the use of the **working backwards strategy** are special because students must begin by working with data that they have at the end of the problem, perform a series of calculations and end with data that would have been at the beginning of the problem. For example, we know the sale price of an item and the amount of discount (or savings), but we don't know the original price of the item. In this case, we are going to start with the ending price and work backwards to find the beginning price.

Mathematics Component

1. Discuss the meaning of the word "**discount**."
   - Use the "percent bar model" transparency master to help students think about the following problem. Fill in the correct information on the transparency as you talk about the problem.

     Tickets to a concert are regularly $45 dollars each. They are marked down 25%. How much money would you save if you buy the discounted tickets? (Explain that you can also say, "What is the discount?)



Ask students to explain what this bar represents (price of tickets) and label it on the transparency.

$45 is normally the whole cost of one ticket, so it goes at the top.

   - Ask students to think about how we might estimate the amount of discount using this model. Guide them to understand that the discount can be estimated based on the location of the 25% line on the percent bar or 1/4 of the way.
   - Locate approximately where 25% would be located on this bar and draw a line across the two bars. (A reasonable estimate would be $10 - $12 because this is about 1/4 of 45.)
   - Ask students how we could find the exact amount of the discount? (Setting up a proportion and cross multiplying will give an exact answer.) Set up the following proportion below the percent bar model and solve.

$$\frac{25}{100} = \frac{x}{45}$$

$$100x = 1125$$

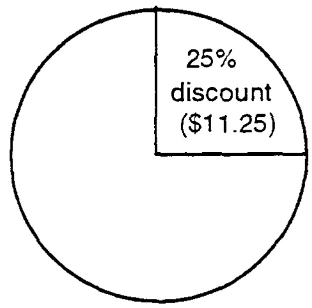
$$\frac{100x}{100} = \frac{1125}{100}$$

$$x = \$11.25$$

2. Ask students to think about the numbers in this problem to determine the "**sale price**." (Subtracting the discount from the original price results in the sale price.)

   Original price: $45
   Discount : $11.25 (Discount % = 25%)
   Sale price: $33.75

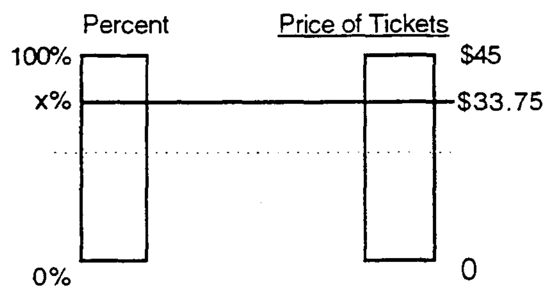
- A circle graph might also be a good way for students to visualize this process:



The whole circle represents 100% of the original price of $45. 25% of the circle is the discount of $11.25.

- Ask what percentage of the circle is **not** the discount. (75%)
- What dollar amount is that equal to? ($33.75)
- If students have difficulty understanding that the sale price of $33.75 must be 75% of the original price, restate the problem as follows and use the percent bar model transparency to illustrate:

$33.75 is what percent of $45?



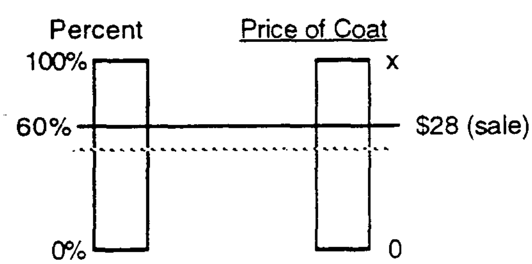
$$\frac{x}{100} = \frac{33.75}{45}$$
$$45x = 3375$$
$$\frac{45x}{45} = \frac{3375}{45}$$
$$x = 75\%$$

- Lead students to understand that to find the original price they would use the "**working backward**" problem solving strategy.

3. Choose a student to fill in the percent bar model transparency on the overhead as the class discusses the following problem:

   A coat is sale-priced at $28 after a 40% mark down. What was the original price?

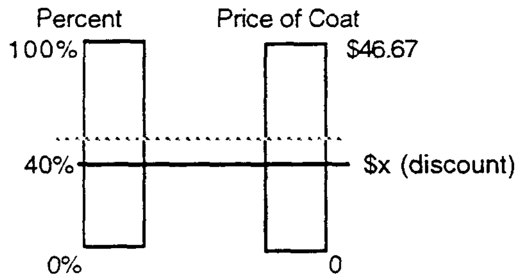
   - Lead students to understand that the original price is the "unknown." The original price would represent 100% of the whole. The sale price is 100% minus a 40% discount (100% - 40% = 60%). Therefore, the **sale price represents 60%** of the original price. This could be illustrated as follows:



Proportion

$$\frac{60}{100} = \frac{\$28}{x}$$
$$60x = 2800$$
$$\frac{60x}{60} = \frac{2800}{60}$$
$$x = \$46.66 = \mathbf{\$46.67}$$

- Ask students if this amount would be **"reasonable"** as the original price. (They might reason that a 40% discount would be a little less than "half" the original price. Half of $46.67 would be about $23. If you subtract <u>a little less</u> than $23 from $46.67, you would get about $28 so this **is** a reasonable answer.)
- Lead students to understand that this answer can be checked by **"working backwards."** Using $46.67 as the original price, a proportion can be set up to find the discount as follows:



$$\frac{40}{100} = \frac{x}{46.67}$$

$$100x = 1866.8$$

$$\frac{100x}{100} = \frac{1866.8}{100}$$

$$x = \$18.668 = \mathbf{\$18.67}$$

With a discount of $18.67, the **sale price would be $46.67 - $18.67, = $28.**

4. Ask students to work in pairs to complete the activity sheet <u>Working with Discounts.</u>
In these problems the students will sometimes be finding the discount **and other times** they will be asked to find the original price or sale price. You may want to write the following steps on the board and read them together.

   - Read the problem carefully before beginning.
   - Set up a percent bar model to represent the information given.
   - Solve each problem using a proportion.
   - Fill in all of the information below each problem.
   - Think about whether your information is **reasonable**.

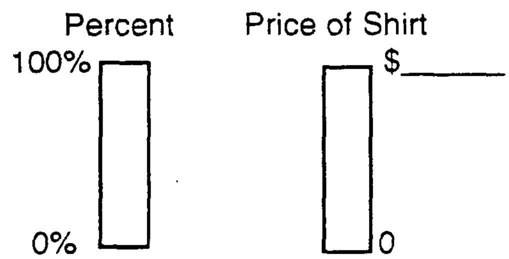
Names ________________ and ________________

Working with Discounts

Read each problem carefully. Be sure to think about what the problem is looking for.
Remember:

- The **% of discount** is the percent something has been marked down.
- The **dollar discount** is the amount of money that is being saved.
- The **sale price** is the amount of money you pay after you have subtracted the dollar discount from the original price.

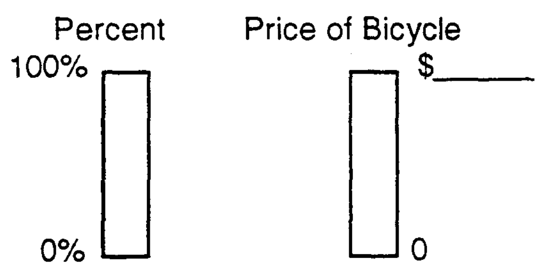
1. A shirt is on sale for $12 after a 40% mark down. What was the **original price**?



Proportion

% Discount:
Sale Price: ________
Original Price: ________
____

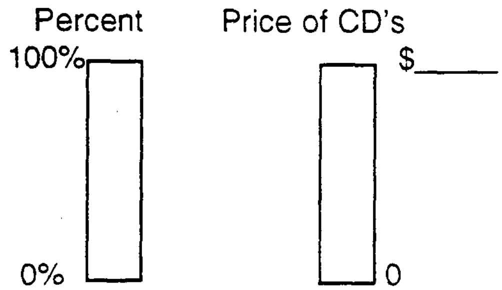
2. The original price of a bicycle was $112. It has been marked down 25%. What is the **sale price**?



Proportion

$ Discount: ________
Sale Price: ________
Original Price: ________

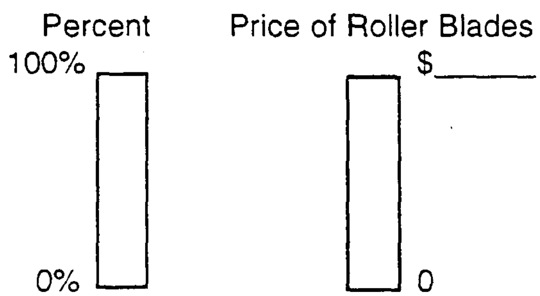
3. The original price of a set of CD's was $25. They are marked down 20%. What is the dollar amount of **discount**?



<u>Proportion</u>

$ Discount: ______
Sale Price: ______
Original Price: ______

4. The original price of a set of roller blades is $88. They are now on sale for $22. What is the **% of discount**?



<u>Proportion</u>

% of Discount: ______
Discount: ______
Sale Price: ______
Original Price: ______

How were you able to find the % of discount? ______________________________

____

____

Answer Key
Obj. 31

1.	Discount:	40%
	Sale Price:	12
	Original Price:	20
2.	Discount:	$28
	Sale Price:	**$84**
	Original:	$112
3.	**Discount:**	**$5**
	Sale Price:	$20
	Original Price:	25
4.	**% of Discount**	**75%**
	Discount:	$66
	Sale Price:	$22
	Original:	88

How were you able to find the % of discount?

You need to subtract the 25% from 100% to get the percent of discount. (75%)

Objective 32: Determine the truth in statistics or advertising and adjust the misleading information

Vocabulary

advertising
advertisements
adjust
misleading

Materials

ads from newspapers or magazines (in a folder)
calculators
scissors
glue
markers
Poster board
1 per group

What's wrong here?
transparency

Truth in Advertising
2 per group

Language Foundation

1. Most students will recognize **advertisements** (ads) in the newspaper or on TV. Discuss the purpose of **advertising** and let students tell about some ads (commercials) they have seen or read recently.

2. Ask students to think about their first experiences coming to the U. S. or to a new school. What changes did they need to make in order to get used to living in the U. S. or going to a new school? The changes they made helped them to **adjust** to a new environment.

3. Tell students that in this lesson, **misleading** information is information that is not totally true, but will lead the reader to believe that it is.

Mathematics Component

This lesson asks students to use what they have learned about percentage to determine truth in advertising and adjust misleading information.

1. Show students advertisements taken from a newspaper or magazine which show sale prices and discounts.
   - Use these to open a discussion about advertising practices.
   - Ask students whether they have ever bought something because of an advertisement they have seen.
   - After sharing responses, ask them if they feel that all advertisements are "truthful?"

2. Use the What's wrong here? transparency master to allow students to investigate this question.
   - After looking at the first problem together, allow students time to work in groups of 2-3 students to determine what's wrong with the advertisement.
   - Encourage them to use what they have learned about percentage along with calculators and/or paper and pencil.
   - When students have had sufficient time to work on the problem, ask them to share their findings with the class.
   - Lead students to understand that a proportion can be set up to determine the "unknown" discount:

$$\frac{30}{100} = \frac{x}{\$12}$$

$$100x = 360$$

$$\frac{100x}{100} = \frac{360}{100}$$

$$x = \$3.60 \text{ (discount)}$$

With a discount of $3.60, the **sale price** would be $12.00 - $3.60 = **$8.40**

   - Ask students how the store might have included information that is not exactly the truth. (Suggestions should include the fact that often stores will "round" a discount.)
   - Ask students how they would "rewrite" the advertisement to make it more accurate. (Correct responses would include changing the sale price to $8.40 or changing the **% off** to 26%.)
   - Repeat the same procedure for the second problem and allow students an opportunity to explain how the information is misleading and what they would do to make sure the advertisement reflects accurate information. Students should be encouraged to use their understanding of proportions to verify the information in the advertisement as follows:

$$\frac{20}{100} = \frac{x}{2000}$$

$$100\,x = 40000$$

$$\frac{100\,x}{100} = \frac{40000}{100}$$

$$x = \$400 = \text{(discount)}$$

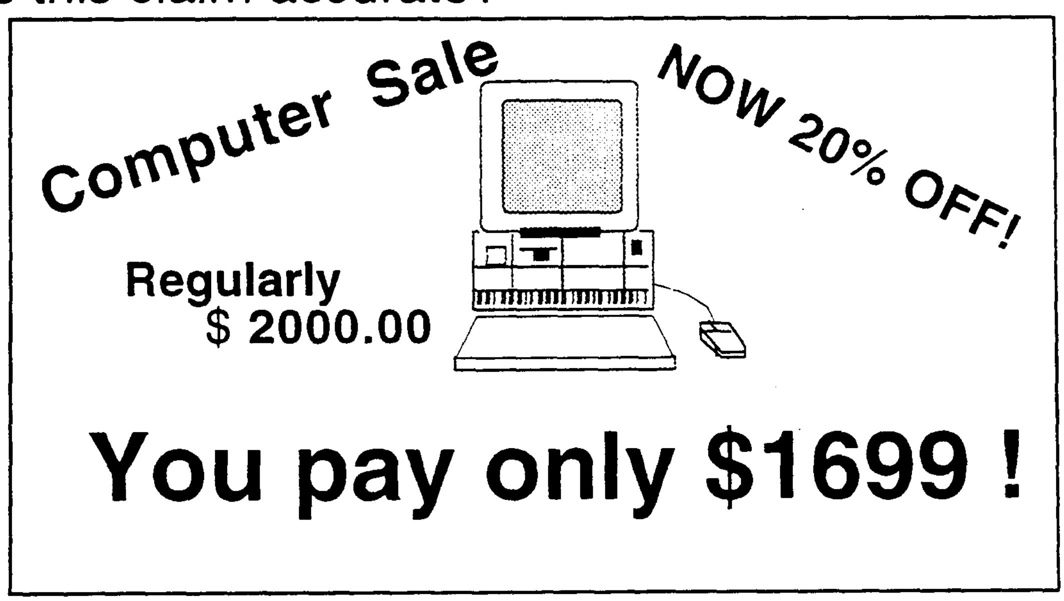
- Therefore the saleprice should be $1600 not $1699. Suggestions for reflecting more acccurate information in the advertisement should include correcting the line to read "Now ALMOST 20% off!"
- Elicit student reactions to what they have discovered about these advertisements and lead a discussion about what we can do, as "consumers" to make sure that we are making sound decisions based on accurate information. (Responses might include suggestions such as looking at advertisements more closely, checking their accuracy, or making decisions only after careful consideration of claims made in advertisements.)

3. Provide a folder with several advertisements which have been cut out from newspapers or magazines. (The ads should have been previously selected and should include inaccurate information which can be corrected by students through their understanding of percentage and their previous work with proportions.) Each group should also get two copies of the blank Truth in Advertising Activity Sheets.
   - Explain that students will work in pairs to check the accuracy of two different advertisements which they will select.
   - On each sheet, they will record the work they do to determine the accuracy of the ads and write a few sentences about each advertisement explaining their findings .
   - Each group will then use a poster board to create a display including: the original advertisements, their activity sheet (which can be cut in half, if desired), and a sketch showing how each might look if they were asked to create their **own** advertisement for this product showing accurate information.
   - After all projects have been completed, allow students an opportunity to share their work with classmates.

What's wrong here?

1. Is this advertisement telling the truth?

2. Is this claim accurate?



Truth in Advertising

Objective 33: Set up and solve proportions to find the unknown.

Vocabulary

proportion
cross products

Language Foundation

Before distributing Using Proportions Practice Sheet, it will be necessary to talk about any vocabulary contained in the problem questions that students may not understand.

Materials

Using Proportions Practice Sheet
Apartment Life
student copies

Mathematics Component

1. Pose a problem that can be solved by setting up and solving a **proportion.**
   - For example, a proportion can be used to find out the price of different quantities of items. If three donuts cost $1.18, how much will 5 donuts cost?
   - Help students make a connection between setting up proportions to solve percent problems (part/whole =part/whole) and setting up a proportion to solve this problem.
   - After setting up the proportion students should be able to tell you how to use **cross products** to find the answer.

$$\frac{3 \text{ donuts}}{\$1.18} = \frac{5 \text{ donuts}}{x}$$

$$3x = 5 * 1.18$$

$$\frac{3x}{3} = \frac{5.90}{3}$$

$$x = \$1.97$$

2. Work other examples together.
   - If it takes 6 gallons of gasoline for a 96 mile trip, how many gallons of gasoline would be needed for a 152 mile trip?

$$\frac{96 \text{ miles}}{6 \text{ gallons}} = \frac{152 \text{ miles}}{x \text{ gallons}}$$

$$96x = 6(152)$$

$$\frac{96x}{96} = \frac{912}{96}$$

$$x = 9.5$$

It would take **9.5 gallons** for a 152 mile trip.

- After you have explained the two examples to the students, point out the different ways to set up the problems correctly. For example:

$$\frac{\$1.18}{\text{3 donuts}} = \frac{x}{\text{5 donuts}}$$

This setup would still produce the same equation, $3x = 5(1.18)$

$$\frac{\text{6 gallons}}{\text{96 miles}} = \frac{x}{\text{152 miles}}$$

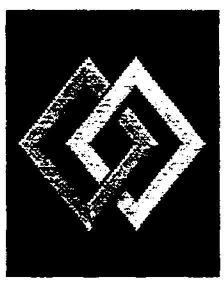
This setup would still produce the same equation, $96x = 6(152)$

- It is important to label the proportions clearly. In the next example, all labels are in centimeters, so we must use descriptions in our setup:

  A picture measuring 20 cm long is reduced on a copying machine to 15 cm long. If the width of the original picture is 16 cm, what is the width of the reduced copy?

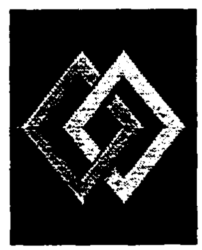
16 cm wide

20 cm long



x cm wide

15 cm long



$$\text{original length} \quad \frac{20 \text{ cm}}{15 \text{ cm}} = \frac{16 \text{ cm}}{x} \quad \text{original width}$$
$$\text{reduced length} \qquad\qquad\qquad \text{reduced width}$$

$$20x = 15(16)$$

$$\frac{20x}{20} = \frac{240}{20}$$

$$x = 12$$ The width is reduced to **12 cm.**

- Ask the students to set up the following problems:

1. If a 10-pound turkey takes 4 hours to cook, how long will it take a 14-pound turkey to cook?

$$\frac{10 \text{ pounds}}{14 \text{ pounds}} = \frac{4 \text{ hours}}{x \text{ hours}}$$

2. Cara worked five hours and earned $53.25. If she works 8 hours, how much does she earn?

$$\frac{5 \text{ hours}}{8 \text{ hours}} = \frac{\$53.25}{x}$$

- Distribute Using Proportions Practice Sheet.

Additional Activities

Problem Solving Activity

- Students use proportions to determine the actual dimensions of a variety of rooms.
- Distribute Apartment Life to each student

Name________________________________
Date ____________

Using Proportions Practice Sheet

Solve using a proportion:

1. Last week Mary drove her car 440 miles and used 20 gallons of gasoline. At the same rate, how many miles could she drive her car using 35 gallons of gasoline?

   $\frac{440 \text{ mi}}{x \text{ mi}} = \frac{20 \text{ gal}}{35 \text{ gal}}$ $\quad 20x = 35(440)$

   ____

   ____

2. The cost of three apples is $1.68. What is the cost of 7 apples?

3. A photocopy machine copied 50 pages in 1.5 minutes. At this rate, how long will the machine take to copy 80 pages?

4. Maria spent 3 hours writing addresses for 50 party invitations. At this rate, how long will it take Maria to write addresses for 90 invitations?

5. It costs $135 to rent a limousine for 3 hours. At this rate, how much will it cost to rent a limousine for 5 hours?

6. Two out of every 7 members of the school chorus are boys. If there are 112 members altogether, how many are boys?

7. The model of a truck is 4.75 inches wide. The scale of the model is 3 in.: 5 ft. What is the actual width of the truck?

8. Min Young pays $41.25 for 5 dancing lessons. At this rate, how much does she pay for 12 dancing lessons?

9. A fruit punch recipe calls for 3 parts of orange juice to 4 parts of cranberry juice. How many liters of cranberry juice should be added to 4.5 L of orange juice?

10. Five vests can be made from 2 1/2 yards of fabric. How many vests can be made from 8 yards of fabric?

11. How many cans of paint will be needed to cover 208 square meters of wall space, if five cans of paint will cover 130 square meters of wall space?

12. If two cans of tennis balls sell for $5.88, how much will 10 cans cost?

13. A boat travels 72 km in three hours. What is the boat's average speed per hour?

14. A seven-ounce box of crackers sells for $1.19. What is the cost of the crackers per ounce?

15. Five yards of upholstery fabric costs $80. What is the cost of fabric per yard?

16. A bottle of Golden Harvest Apple Juice contains 64 oz and costs 99 cents. Green Farms Apple Juice is available in bottles that contain 100 oz and cost $1.18. Which is the better buy?

Create a word problem and solve using a proportion.

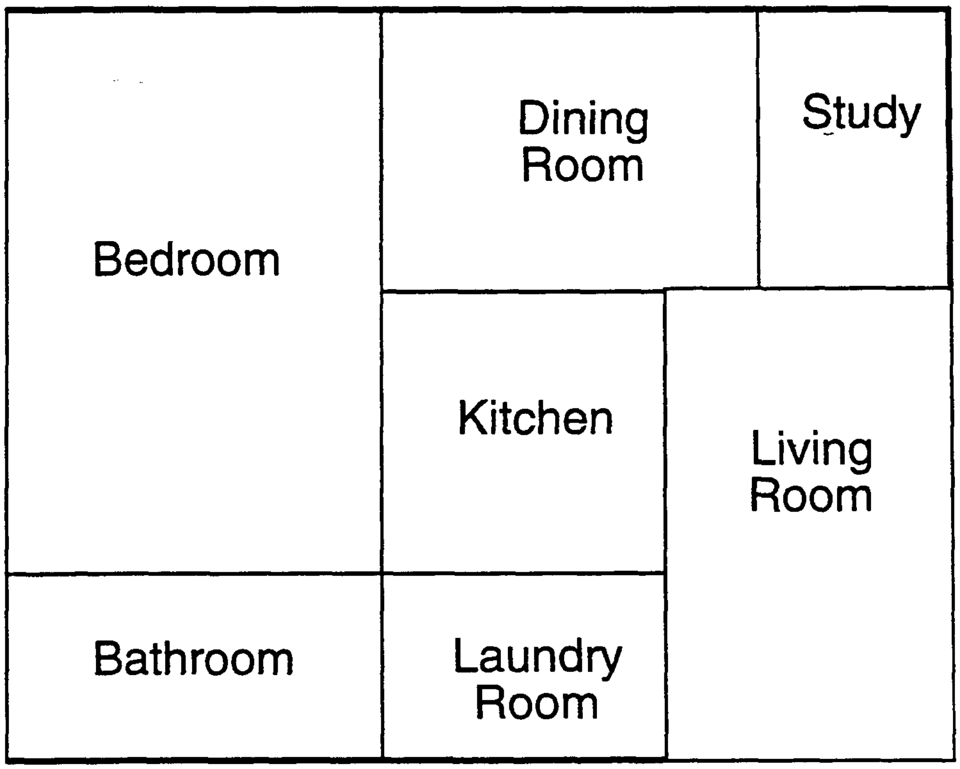
Name ____________________

Date _____________

Problem Solving Activity

Apartment Life

The apartment blueprint shown below is drawn to a scale of **1 inch to 8 feet.** Complete the table by first using a ruler to measure the dimensions in inches and then using the scale to find the actual dimensions in feet.



Room	**Scaled Dimensions**	**Actual Dimensions**
Bedroom	by	by
Bathroom	by	by
Laundry Room	by	by
Kitchen	by	by
Dining Room	by	by
Living Room	by	by
Study	by	by

What are the actual dimensions of the entire apartment? _________

Answer Key
Obj. 33

Using Proportions Practice Sheet

1.	770 miles	2.	$3.92	3.	2.4 minutes
4.	5.4 hours	5.	$225	6.	32 boys
7.	7.92 ft.	8.	$99	9.	6 liters
10.	16 vests	11.	8 cans	12.	$29.40
13.	24 km/hr	14.	$ .17 per oz	15.	$16 per yd
16.	Green Farms				

Create a word problem: Answers will vary

Problem Solving Activity

Room	Scaled Dimensions	Actual Dimensions
Bedroom	2 by 3	16 by 24
Bathroom	1 by 2	8 by 16
Laundry Room	1 by 1.5	8 by 12
Kitchen	1.5 by 1.5	12 by 12
Dining Room	1.5 by 2	12 by 16
Living Room	1.5 by 2.5	12 by 20
Study	1 by 1.5	8 by 12

What are the actual dimensions of the entire apartment? **40 feet by 40 feet**

Equations and Inequalities

Objective 34: Explore the concept of equations, including balance and working backwards

Vocabulary

equation
equal
balance
undo

Materials

number tiles
unit cubes
small baggies
Handling Equations
transparency
Equations
student copies

Optional:

wrapped present
kite
balancing scale

Language Foundation

1. Help students understand the importance of the equal sign in the equation. There must be a **balance** on both sides of the equal sign. The following examples might be helpful. To keep a kite balanced, sometimes streamers or tassels are added. The amount added to the right side must **equal** the amount added to the left side. Demonstrate this using a balancing scale, if available.

2. You may demonstrate "undo" with a box wrapped like a present. Explain (or model) to the students that first the box is covered with the wrapping paper. Then it is tied with ribbon or a bow is attached. Finally, a tag or a card is attached. The receiver of the present will **undo** the wrapping by first detaching the tag or card, then untying the ribbon or detaching the bow, and finally taking off the paper.

Mathematics Component

1. Using the transparency master Handling Equations, place a flat tile and 5 unit cubes into one of the transparent hands.
    - Place 9 unit cubes into the other hand. Explain to students that the flat tile is an "unknown."
    - The value of each unit cube is one. Therefore, one hand holds a tile plus 5 and the other hand holds 9.
    - Pose the following problem for students:

        "Both hands hold the **same total amount**. What is the value of the **flat tile**?"

    - Allow students an opportunity to share their ideas and solutions and then model by manipulating the tiles on the overhead as follows:

        "To find out how much the tile is worth, I can start to "undo" the problem by doing the opposite of what I did when I put cubes into the hands. I can take 5 unit cubes away from each hand. (Remind students that if the two hands were holding the same total amount to begin with and I remove 5 from each hand, they should still hold the same total amount.) One hand now has 4 and the other hand has only the tile. If the two hands are still holding the same amount, then the tile must be worth 4." Check by recreating the original problem and substituting 4 unit cubes for the flat tile.

2. Begin a second problem by placing a flat tile and 5 unit cubes into one of the overhead hands and 6 unit cubes into the other hand.
    - Repeat the process above, using student input to decide and model how to "undo" the problem. (Five unit cubes are removed from each hand and the the value of the tile is 1.)
    - Check by substituting 1 in the original equation.

3. A third problem could begin with 2 tiles and 10 in one hand and 18 in the second hand.
    - Ask students how we might find the value of **one tile** if two tiles plus 10 is the same as 18.
    - Allow students to discuss a solution.
    - Model taking 10 from each hand.
    - With two tiles remaining in one hand and eight in the other, ask students how much one tile would be worth. (Make two equal pairs in each hand and elicit an understanding that each tile must be worth 4.)
    - Check the solution.

4. Allow students to work in groups of 3-4.
    - Distribute small bags containing 2 tiles and 8 unit cubes to each group.
    - Ask each group to pretend that there is a set of hands on the desk. One hand contains 2 tiles and 1 unit cube and the other hand contains 7.

- Ask students to separate the cubes into two piles based on this information.
- Ask students to work in groups, using the tiles on their desks, to find the value of one tile.
- Ask for a volunteer to model a solution. (Remove one unit cube from each hand and then separate each hand into two even piles showing that each tile must be worth 3.)
- Place the tiles back into their original position on the transparency. (Two flat tiles and 1 unit cube in one hand and 7 unit cubes in the second hand.) Check the solution.
- Explain that we need a way to record or write down what is in the two hands.
- Ask students to suggest ways to record the amount and the **relationship** between the two hands. Write $2x + 1 = 7$ on the board.

5. Explain that this type of statement is called an equation. An **equation** is a statement that two quantities are equal, that the left-hand side is equal to the right-hand side. Equations can be number statements such as $4 + 3 = 7$, or they may have variables in them. What is the variable in $2x + 1 = 7$? (x)
   - The equal sign in an equation <u>is used to mean</u> "the total is," "the result is," "the sum is," or "is equal to."
   - The equation above is read "two x plus 1 is equal to seven."
   - Equations can be "**<u>true</u>**" or "**<u>false</u>**" statements. The number statement $4 + 3 = 7$ is a true statement; however, $4 + 5 = 7$ would be a false statement because $4 + 5$ **does not** equal 7. Is $16 \div 4 = 4$ true or false? (True) Is $15 - 5 = 9$ true or false? (False)
   - Provide other examples, as needed.

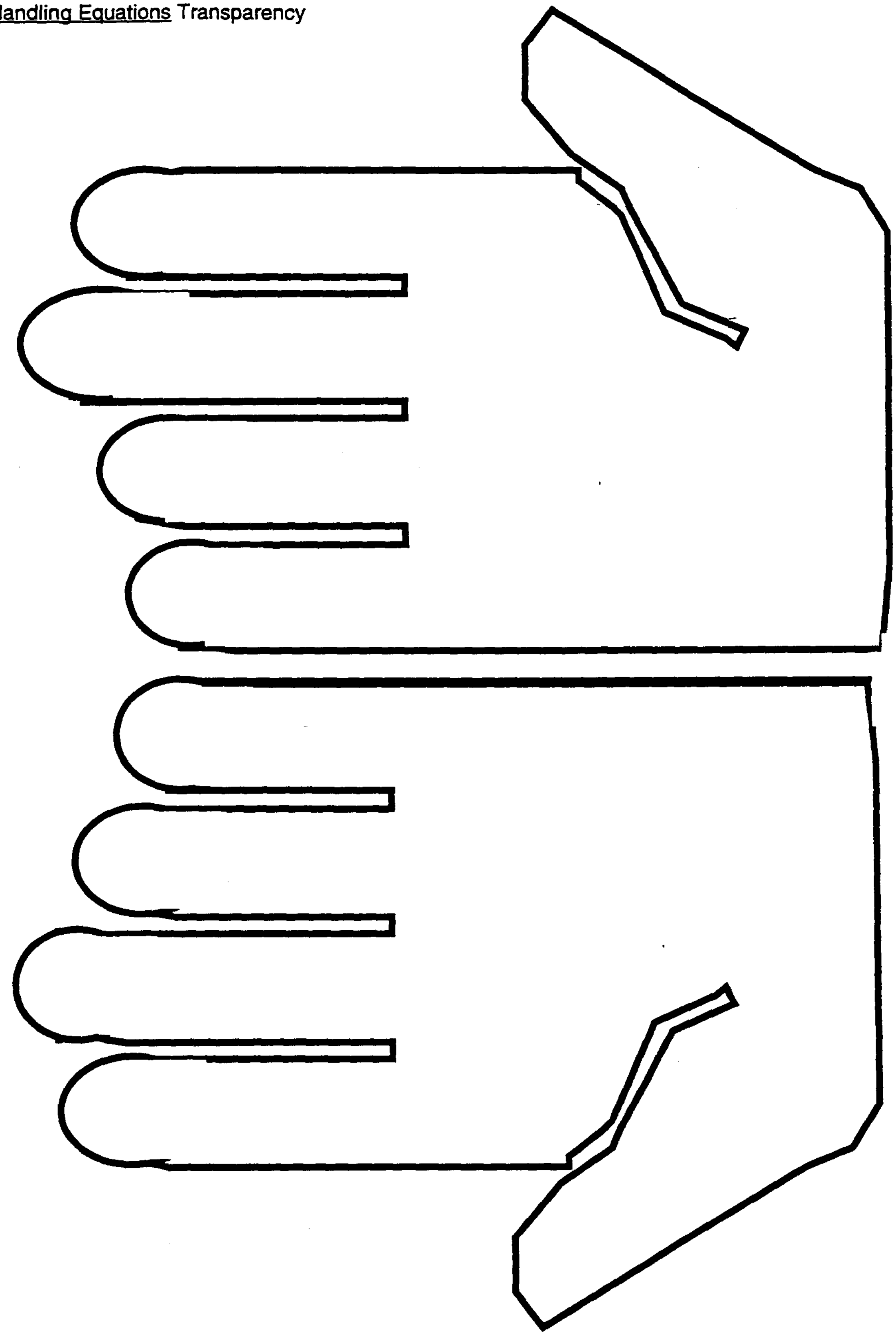
6. The number 3 is called the **solution** to the equation $2x + 1 = 7$.
   - Ask students <u>why</u> they think 3 is called the solution to this equation.
   - Check the solution by replacing 3 in the original equation. (The solution is the number which can replace the variable to make the equation "**true**.")
   - Model placing other numbers into the equation and discuss why they are not a solution to the equation. An example would be:

$2x + 1 = 7$

$2(4) + 1 = 7$ (Although this <u>is</u> "written" as an equation which <u>states</u> that the two sides are equal, 4 is **not a solution** because $2(4) + 1 = 9$ instead of 7. This statement should be written $2(4) + 1 \neq 7$.)

   - Explain to students that merely putting an equal sign between two expressions does **not** make the statement true. To be **sure** it is a solution to the equation, you should <u>always</u> do a **check**.
   - Have students complete the <u>Equations</u> activity sheet.

<u>Handling Equations</u> Transparency



Name ______________

Equations

An equation is ____

____.

Tell whether each statement below is **true** or **false**.

1)	$12 + 13 = 24$	________	9)	$24 \div 6 = 3$	________
2)	$h + h = 4h$	________	10)	$(x)(x) = 2x$	________
3)	$2(3 + 4) = 14$	________	11)	$36 + 0 = 3$	________
4)	$y^4 = y \bullet y \bullet y \bullet y \bullet y$	________	12)	$(c)(c) = c^2$	________
5)	$5 + (6 + 2) = (5 + 6) + 2$	________	13)	$a + a + a = a^3$	________
6)	$2 \bullet 3 + 2 \bullet 4 = 2\,(3 + 4)$	________	14)	$2 \times 9 = 81$	________
7)	$5 + 3 - 2 = 2 + 8 \div 2$	________	15)	$x + y = xy$	________
8)	$a(b + c) = ab + ac$	________	16)	$67 - 0 = 0$	________

Find the **solution** to each equation.

1) $x + 7 = 12$ (Think: What number plus 7 equals 12?)

$x =$ _____

2) $12 - y = 3$ (Think: What number subtracted from 12 is equal to 3?)

$y =$ _____

3) $7z = 35$ (Think: What number, multiplied times 7, equals 35?)

$z =$ _____

4) $\frac{q}{6} = 4$ (Think: What number, divided by 6, equals 4.)

$q =$ _____

5) $6 + m = 13$

$m =$ _____

6) $b - 10 = 6$

$b =$ _____

7) $14\,a = 14$

$a =$ _____

8) $\frac{15}{n} = 5$

$n =$ _____

Answer Key
Obj. 34

Equations

An equation is a **statement that two quantities are equal.**

1.	false	9.	false
2.	false	10.	false
3.	true	11.	false
4.	false	12.	true
5.	true	13.	false
6.	true	14.	false
7.	true	15.	false
8.	true	16.	false

1.	5	5.	7
2.	9	6.	16
3.	5	7.	1
4.	24	8.	3

Objective 35: Use a concrete model to write and solve equations.

Vocabulary

equation
balance
undo

Materials

number tiles
scissors
Paper Balance
Number Tiles (cut apart pieces prior to lesson)
  transparencies
  student copies

Solving Equations
Pattern Block Equations
  student copies

Language Foundation

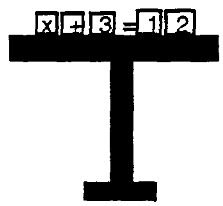
1. Review vocabulary from the previous lesson.

2. Explain that in this lesson, the prefix **-un** means to do what will reverse the act. Write these words on the overhead: dress, tie, plug, fold, pack. Ask students what will happen if you add the prefix -un to the beginning of the words. Elicit that the opposite or reversal of the act will occur.

Mathematics Component

1. • Use **transparencies** of the Paper Balance and the Number Tiles activity sheet to discuss and model the following problem:

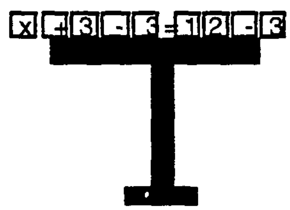
Place an X + 3 on one side of the balance and 12 on the other side as shown below.



Write:

$x+3=12$

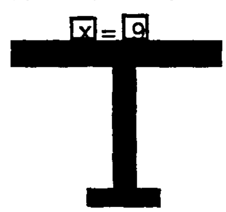
- Explain that you want to find the value of the unknown in the same way that you used the hands in the previous lesson. Remind students that the balance must stay balanced (both sides must remain equal) as they solve the problem.
- First, do the opposite by taking 3 away (subtract three) from both sides.



Write:

$x+3-3=12-3$

- Remind students that positive (+) 3 and negative (-) 3 "**combine** to form 0." (Avoid the phrase "cancel out," since this can be confusing for students.) By removing the 0 and simplifying 12-3 (substituting it with a 9), the solution becomes:



Write:

$x = 9$

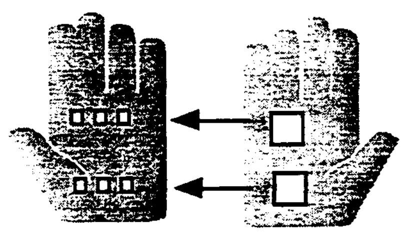
- Ask students how we can determine if 9 **is** a solution for this equation. Recreate the original equation on the transparency, **substitute** x with 9, and check to see if the statement is **true.**

2. • Distribute one copy of the Paper Balance activity sheet and one copy of the Number Tiles activity sheet to each student. Ask students to use scissors to cut apart each of the tiles on a complete Number Tiles page if this has not already been done.
- Explain to students that they will use the balance and number tiles to solve an equation. Write the equation $x + 9 = 23$ to the side on the transparency copy of the Paper Balance. Ask students how this equation can be represented on the balance.
- Model on the transparency placing $x + 9$ on one side of the balance and 23 on the other and ask students to do the same .

- Lead students through the process of undoing this equation as shown above. Begin by using the number tiles to subtract 9 from both sides. Record each step on the transparency.
- Use the number tiles to show the solution $x = 14$.
- Ask students to "check" the solution by substituting 14 for x in the original equation.
- Reinforce appropriate vocabulary (undo, combine, substitute, check, solution, true, etc.) through discussion.
- Have students remove tiles from their paper balances.

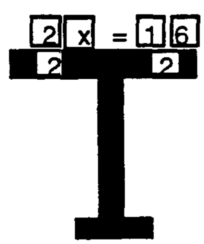
3. - Clean off the transparency and write the following equation: $2x - 1 = 15$.
   - Ask students to represent this equation on their balances and then model on transparency by placing $2x - 1$ on one side of the balance and 15 on the other.
   - Undo the subtraction by adding 1 to each side. Add tiles for + and 1 to both sides of the equation and have students do the same.
   - Discuss the fact that +1 and -1 **combine** to form 0 and model removing those tiles as students do the same.
   - Have students simplify 15 + 1 and replace it with 16 on the right side. (The equation now reads $2x = 16$.)
   - Review the procedure used in the previous lesson for determining the value of one tile when there were two tiles together in one hand. (Remind students that by dividing each hand into two separate equal piles, the value of one tile was determined.)

Example:



x = 3

   - On the balance, place a two under the 2x on one side of the equation and a 2 under the 16 on the other side, as shown below:



   - Explain that two over two "**divides** to give 1". (**Avoid** using the word "cancel" to describe this procedure, since this can confuse students.)
   - Ask students to simplify and show the new equation on their balance.
   - Model the **solution** $x = 8$ on the transparency, discussing why 1x can be written as x.)
   - **Check** the solution.

4. • Give each student a copy of the activity sheet Solving Equations.

- Ask students to represent the first equation on their balance. ($x - 6 = 13$)
- Discuss and model, using transparency. Have students discuss how we might **undo** this equation to solve for the unknown x. Lead students to understand that they need to add 6 to both sides of the equation and model on the transparency.
- Have students record this procedure under the original equation on their activity sheet.
- Ask students to simplify both sides on their balance: combine -6 and +6 to make 0, remove those tiles and then simplify 13+6 and replace those tiles with 19. The **solution** $x = 19$ should now be represented on the balance.
- Model recording this solution in the appropriate place on the activity sheet as shown.
- Have students **check** the solution by substituting 19 for the variable x.

1.	$x - 6 = 13$	Check:
	$x - 6 + 6 = 13 + 6$	$x - 6 = 13$
	$x = 19$	$19 - 6 = 13$
	$x = 19$	$13 = 13$ ✓

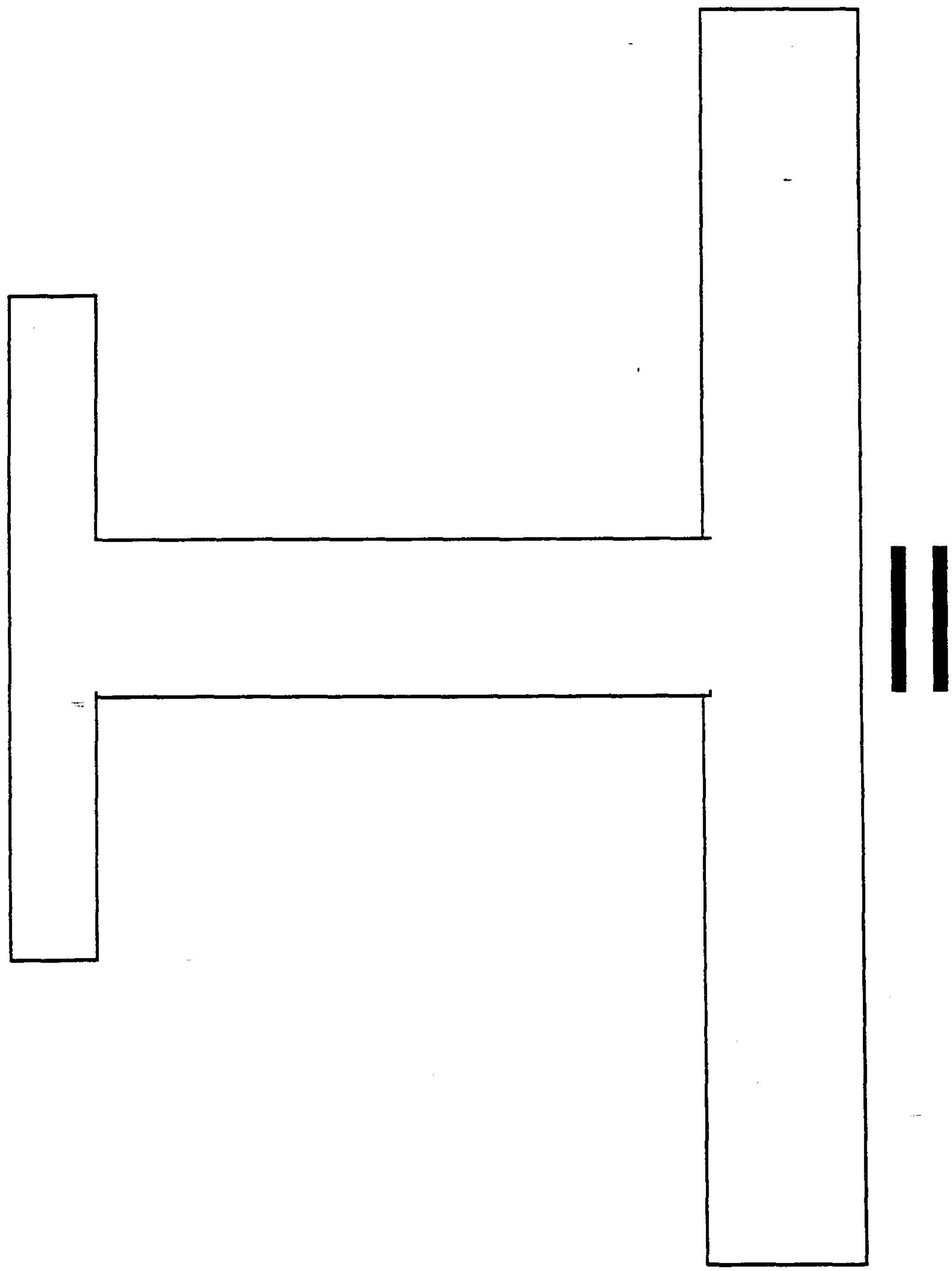
- Continue to work several problems together, as needed, and then allow students to work with a partner to complete individual activity sheets.

Note: - At the conclusion of this lesson, distribute small zip-lock bags to store student number tiles for future use. You may also want to collect the Paper Balance activity sheets.

Problem Solving Activity

- Distribute Pattern Block Equations to each student.

Paper Balance



0	1	2	3
4	5	6	7
8	9	+	+
X	X	•	•

0	1	2	3
4	5	6	7
8	9	+	+
X	X	•	−

0	1	2	3
4	5	6	7
8	9	+	+
X	X	−	−

Name ____________

Date ___________

Solving Equations

1. $x - 6 = 13$ Check:

2. $3x = 24$ Check:

3. $5 + x = 13$ Check:

4. $56 + x = 72$ Check:

5. $x - 4 = 0$ Check:

6. $4x = 36$ Check:

7. $10 + x = 17$ Check:

8. $\frac{x}{7} = 5$ Check:

9. $2x + 5 = 21$ Check:

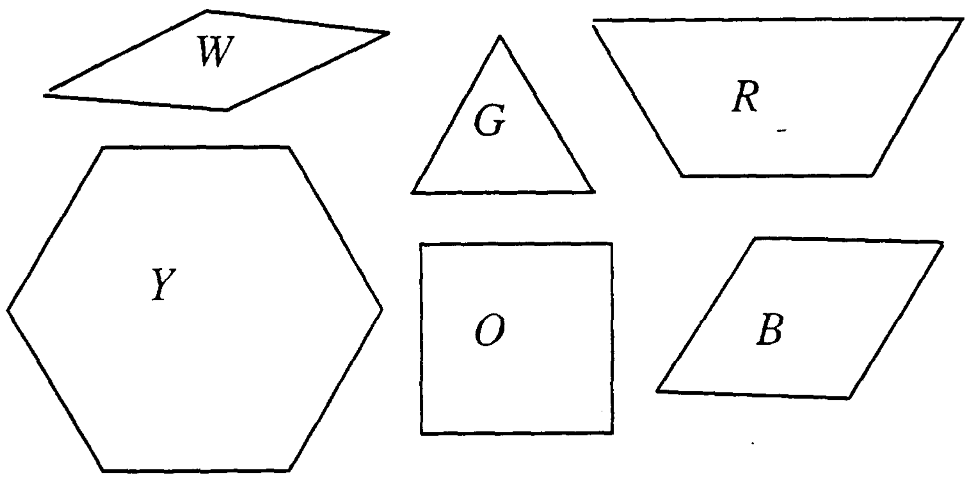
10. $3x - 6 = 18$ Check:

Name ________________ Problem Solving Activity

Date __________

Pattern Block Equations

Use pattern blocks to solve each equation. For each equation, find which pattern block has an area equivalent to x.



1. $2Y + 2x = 5R + G$

2. $2G = Y - 2x$

3. $3B + 2G = x + B$

4. $3x = Y + B + G$

5. $1R + G + B = x + 3G$

Answer Key
Obj.35

Solving Equations

1. 19	2. x=8
3. x = 8	4. x = 16
5. x = 4	6. x = 101
7. x = 7	8. x = 35
9. x = 8	10. x = 8

Problem Solving Activity

1. x = 1 blue
2. x = 1 blue
3. x = 1 yellow
4. x = 1 red
5. x = 1 red

Objective 36: Simplify and solve equations using one inverse operation.

Vocabulary

balance
equation
simplify
inverse operation
reciprocal
coefficient

Materials

<u>Solving Equations Using Inverse Operation</u>
<u>Using Inverse Operation</u>
student copies

Language Foundation

1. Review vocabulary from previous lessons.

2. Remind students that an **inverse operation** is one that will undo another operation. Addition and subtraction are inverse operations, as are multiplication and division. Show a couple of examples of this before beginning the lesson.

3) Many students will not be familiar with the word **reciprocal**. In math, a reciprocal is a number or expression by which a given number or expression is multiplied to produce one. For instance 3/5 and 5/3 are reciprocals since 3/5 x 5/3 = 1. You will need to explain and demonstrate how this works in the lesson.

Mathematics Component

1. Place the following one-step **equation** on the board or on a transparency: $x + 6 = 15$.
   - Review the definition of an equation. (A statement that two quantities are equal.)
   - Ask students if $x + 6 = 15$ is an equation and why. (Yes, because it is a statement that the total amount on the left-hand side is equal to the total amount on the right hand side.)
   - Ask what the letter "x" is called and discuss why it is used in this equation. (A variable, such as "x," is used to represent an unknown.)
   - Remind students that an equation can be true or false. Ask students if this equation is true or false and lead them to understand that it could be true or false, depending on the value of "x."

2. Refer back to the activities with the hands and the balances and ask students how we can undo to find a solution to this equation.
   - Discuss **inverse operation** and reinforce the concept that when <u>the same</u> operation is done to both sides of the equation, to keep the equation **balanced.**
   - Illustrate this concept quickly with whole numbers:

$$3+4=7 \qquad (7=7)$$
$$3+4+\mathbf{2}=7+\mathbf{2} \qquad (9=9)$$

   - Have students discuss the steps involved in solving the equation $X + 6 = 15$ (using inverse operation) while you model recording the steps.

<u>Solution</u>	<u>Check</u>
$x+6=15$	$x+6=15$
$x+6-6=15-6$	$9+6=15$
$x=9$	$15=15$ ✓

   - Ask how we could determine if 9 is a solution to this equation. Lead students to understand that they must "check" the equation by substituting 9 in place of the variable to see if the statement is true.
   - Model the correct procedure for recording a check by placing the check beside the equation.
   - Discuss why 9 is a solution to this equation. (The variable can be replaced with 9 to make a true statement.)

3. Give students a copy of the activity sheet <u>Solving Equations Using Inverse Operation.</u>
   - Use a transparency copy to model the procedure for solving equations. Begin the first problem by explaining that before solving any equation, students need to make sure that both sides have been **"simplified"** by combining like terms or applying the distributive property.

- Ask students if either side of this equation can be "simplified" and model combining like terms as students record line two on their activity sheets.
- Once both sides have been simplified, allow students to discuss the steps required to solve the equation as you model on the overhead. When completed, student activity sheets should show the solution and a check as follows:

1)

Solution	Check
$3+y+2=26+2$	$3+y+2=26+2$
$y+5=28$	$3+23+2=26+2$
$y+5-5=28-5$	$28=28$ ✓
$y=23$	

- Ask students to discuss the second problem together and decide if it needs to be simplified. (No)

- Direct the student's attention to problem 6. Read the problem out loud, "x divided by 9 equals 4". Ask students what is the inverse operation of division (multiplication). We need to multiply by the **reciprocal** on both dies of the equation. It is necessary for students to understand the following:

$\frac{x}{9} = \frac{1x}{9}$ therefore the reciprocal is $\frac{9}{1}$

	check
$\frac{x}{9}=4$	$\frac{x}{9}=4$
$\frac{(9)}{(1)}\frac{1x}{9}=\frac{4}{}\frac{(9)}{(1)}$	$\frac{36}{9}=4$
$\frac{\cancel{(9)}}{(1)}\frac{1x}{\cancel{9}}=\frac{4}{}\frac{(9)}{(1)}$	
$x=36$	$4=4$ ✓

- Complete additional examples as necessary.
- Ask students to work together in pairs to record their work in solving the equation and recording a check.
- When students have finished, allow one student to model the correct procedure using the transparency copy of the activity sheet.

2)

Solution	Check
$x-16=37$	$x-16=37$
$x-16+16=37+16$	$43-16=37$
$x=43$	$37=37$ ✓

- Work problem three together, reminding students to simplify before using **inverse operation** to solve the equation.

3) <u>Solution</u>

$$-5+3+y=12-6$$
$$-2+y=6$$
$$-2+2+y=6+2$$
$$y=8$$

<u>Check</u>

$$-5+3+y=12-6$$
$$-5+3+8=12-6$$
$$-5+11=6$$
$$6=6$$

- Have students complete the activity sheet individually, working with partners to discuss their work.
- Review solutions together when all students have finished. When looking at the solution for problem eight, ask students how many y's needed to be collected in order to simplify the first expression. ($2y + 3y = 5y$) Remind students that the constant attached to the variable is called the **coefficient**. The coefficient of the term $2y$ is 2, the coefficient of the term $3y$ is 3, and the coefficient of the term $5y$ is 5.

- When reviewing problem 10, point out to students that $-y = 30$ is **NOT** a solution to the equation because that is telling us the value of the **opposite** of y! We are looking for the value of y. Allow students to explore and discuss the question of what to do when the solution includes a negative exponent.

4. Explain that any variable which stands alone, is assumed to have a coefficient of 1.
   - Therefore, in the equation $12 - y = 42$, $-y$ actually means $-(1)y$. Since the y is multiplied by -1, we can apply inverse operation by dividing both sides by -1. The negative 1's will cancel out in problem 10 and the result is as follows:

10) <u>Solution</u>

$$12-y=42$$
$$12-12-y=42-12$$
$$-y=30$$
$$\frac{-(1)y}{-1}=\frac{30}{-1}$$
$$y=-30$$

5. Additional practice problems are provided on the activity sheet <u>Using Inverse Operation</u>. One column has like terms collected and one column does not. Some problems include the concept of multiplying by a reciprocal and **may need to be discussed in advance.**

Additional Activities

The Algebra Lab, Middle School, Lessons 14 -16

Name ________________

Solving Equations Using Inverse Operation

Solution	Check	Solution	Check

1) $3 + y + 2 = 26 + 2$

2) $x - 16 = 37$

3) $-5 + 3 + y = 12 - 6$

4) $p - 11 = 36 + 2$

5) $6w = 42$

6) $\frac{x}{9} = 4$

7) $15 + y - 24 = 32$

8) $3y + 2y = 15$

9) $34 + 2 = 2x - 3 + x$

10) $\frac{y}{8} = 4$

Name ________________
Date ________

Using Inverse Operation

Directions: Copy each equation onto notebook paper. Show all steps necessary to solve the equation and provide a written check.

1) $m + 40 = 90$

2) $56 + z = 72$

3) $h - 1/2 = 4$

4) $32 = 8r$

5) $0 = z - 7.5$

6) $10x = 10$

7) $1/2m = 16$

8) $\frac{5}{8} = \frac{q}{16}$

9) $\frac{a}{100} = 0$

10) $s - 48 = 12$

11) $10y = 10$

12) $14a - a = 26$

13) $34 = 8m + 9m$

14) $5r - r = 20$

15) $10b + 7b - 8b = 27$

16) $t + t + t = 6$

17) $-15 + 6 - 3 + m = 10$

18) $p + 8 - 6 = 36 - 5$

19) $34v + 20v - 18v = 0$

*20) $1/3k + 1/3k = 6$

Answer Key
Obj. 36

Solving Equations Using Inverse Operation

	Solution	Check		Solution	Check
1)	$3 + y + 2 = 26 + 2$ $y + 5 = 28$ $y + 5 - 5 = 28 - 5$ $y = 23$	$3 + y + 2 = 26 + 2$ $3 + 23 + 2 = 26 + 2$ $28 = 28$	2)	$x - 16 = 37$ $x - 16 + 16 = 37 + 16$ $x = 53$	$x - 16 = 37$ $53 - 16 = 37$ $37 = 37$
3)	$-5 + 3 + y = 12 - 6$ $-2 + y = 6$ $-2 + 2 + y = 6 + 2$ $y = 8$	$-5 + 3 + y = 12 - 6$ $-5 + 3 + 8 = 6$ $6 = 6$	4)	$p - 11 = 36 + 2$ $p - 11 = 38$ $p - 11 + 11 = 38 + 11$ $p = 49$	$p - 11 = 36 + 2$ $49 - 11 = 36 + 2$ $38 = 38$
5)	$6w = 42$ $\frac{6w}{6} = \frac{42}{6}$ $w = 7$	$6w = 42$ $6(7) = 42$ $42 = 42$	6)	$\frac{x}{9} = 4$ $\frac{9(x)}{9} = 4(9)$ $x = 36$	$\frac{(x)}{9} = 4$ $\frac{36}{9} = 4$ $4 = 4$
7)	$15 + y - 24 = 32$ $y - 9 = 32$ $y - 9 + 9 = 32 + 9$ $y = 41$	$15 + y - 24 = 32$ $15 + 41 - 24 = 32$ $32 = 32$	8)	$3y + 2y = 15$ $5y = 15$ $\frac{5y}{5} = \frac{15}{5}$ $y = 3$	$3y + 2y = 15$ $3(3) + 2(3) = 15$ $9 + 6 = 15$ $15 = 15$
9)	$34 + 2 = 2x - 3 + x$ $36 = 3x - 3$ $36 + 3 = 3x - 3 + 3$ $39 = 3x$ $\frac{39}{3} = \frac{3x}{3}$ $13 = x$	$34 + 2 = 2x - 3 + x$ $34 + 2 = 2(13) - 3 + 13$ $36 = 26 - 3 + 13$ $36 = 36$	10)	$\frac{y}{8} = 4$ $\frac{(8)y}{8} = 4(8)$ $y = 32$	$\frac{y}{8} = 4$ $\frac{32}{8} = 4$ $4 = 4$