Division can be demonstrated by the same example. How many groups of -2 can be made from -8? Or,
$\frac{-8}{-2} = ?$ The answer is 4. So $\frac{-8}{-2} = 4$.

The Modeling Integer Operations Activity Sheet may be used for students to practice each operation. Students can work in pairs using two-colored counters, number lines or pictorial models to find a solution for each problem.

Additional Activities

The Algebra Lab, Middle School

Addition: Lesson 4, all activities

Subtraction: Lesson 3 and 5, all activities

Multiplication: Lesson 6, all activities

Division: Lesson 7, activities 1 and 2

Name______________________________

Modeling Integer Operations Activity Sheet

Work with a partner and use counters and/or a number line to find a solution to each problem. You may also draw a model to find a solution.

Find each sum.

1. $7 + 2$ 2. $-7 + 2$ 3. $7 + (-2)$ 4. $-7 + (-2)$

Subtract by removing items.

5. $-7 - (-5)$ 6. $-3 - 5$ 7. $7 - (-5)$ 8. $0 - 6$

Subtract by adding the opposite.

9. $-7 - (-5)$ 10. $-3 - 5$ 11. $7 - (-5)$ 12. $0 - 6$

Multiply or divide.

13. $3\,(-4)$ Three groups of negative four contains how many?

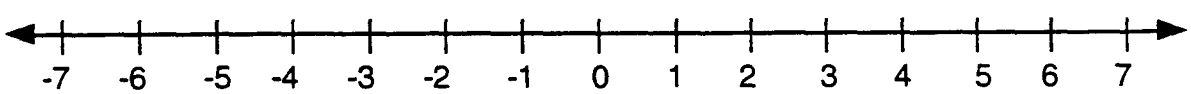
14. $6\,(-2)$

15. $-2\,(9)$

16. $\frac{-12}{3}$ Dividing -12 into three groups produces what in each group?

17. $\frac{-12}{-2}$ How many groups of -2 can be formed with 12 negative chips?

Think of a number line to find each answer:



18. $-4 + 9 =$ _____ Start at ____, move ______ spaces to the ______.
The result is __________.

19. $5 - 8 =$ _____ Start at _____, move ______spaces to the ______.
The result is __________.

20. $-2\,(3) =$ _____ Start at **zero**. Move two spaces to the left, three times.
The result is __________.

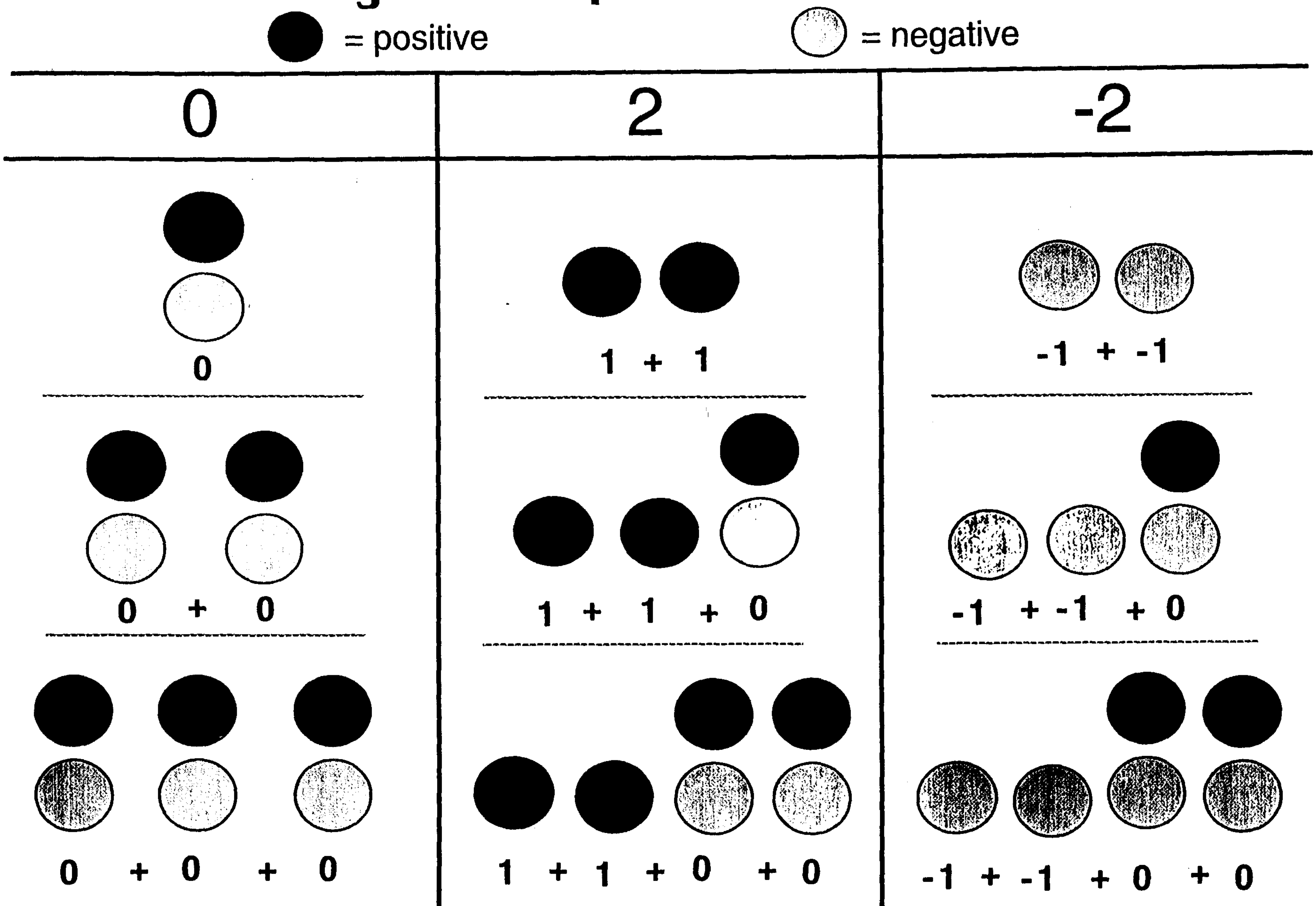
INTEGER MAT

Positive ____________ 1

Negative ____________ -1

"Zero Pair" ____________ + ____________ 0

Ways to Represent Numbers



Operations and Integers

Addition (+)	Subtraction (-)	Multiplication(•)	Division (/)

Answer Key
Obj. 19

Modeling Integer Operations Activity Sheet

1)	9	2)	-5	3)	5	4)	-9
5)	-2	6)	-8	7)	12	8)	-6
9)	-2	10)	-8	11)	12	12)	-6

13) -12

14) -12

15) -18

16) -4

17) 6

18) -4, 9, right, 5

19) 5, 8, left, -3

20) -6

Objective 20: Apply integer rules to add, subtract, multiply, and divide.

Vocabulary

add
subtract
multiply
divide
addend
combine
positive
negative
product
quotient

Materials

overhead projector
transparent counters
calculators

Applying Integer Rules Worksheet
Integer Magic Square
student copies

Language Foundation

1. Ask students to name the basic mathematical operations. (addition, subtraction, multiplication and division). List their responses on the board. Tell them that there is a word they will be seeing (write **addend** next to the list). Explain that addends are numbers used with one of the basic operations. Ask students to look at the word carefully to try to determine which operation would have addends. Illicit addition. (Underline part of the word - addend.) Explain that addends are the numbers which are added together. Write the problem 15 + 22 = 37 on the board and have the students identify the addends (15 and 22)..

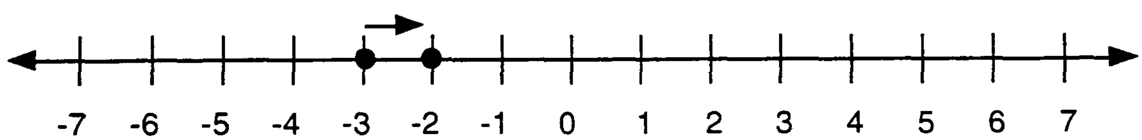
2. Review vocabulary words as needed.

Mathematics Component

Note: After working with integers using counters and number lines (previous lesson), students will now develop a set of integer rules. Using integer rules will expedite performing operations on larger numbers.

1. Begin by giving students examples of integer operations using counters and a number line. For instance, -3 + 1 = -2.

   - Start with three negative chips. 
   - Add one positive chip, which makes a zero pair. 
   - Two negative chips remain. 
   - Using a number line, model the same problem: Start at -3. Move 1 space to the right. The result is -2.



   - Ask students to think about the following problems and their answers. They can use counters or think of a number line.

3 + 1 = 4 Two positive numbers, added together, (3 positive counters added to 1 positive counter)... sum is positive.

-3 + -1 = -4 Two negative numbers, added together, (1 negative counter added to 1 negative counter)... sum is negative.

3 + -1 = 2 Positive number is combined with a negative number, (3 positive counters added to 1 negative counter)... the answer is the **difference** of the two, and it is positive. Why?

OR

1 + -6 = -5 Positive number combined with a negative number (1 positive counter added to 6 negative counters)... the answer is the difference of the two, but the sign of the answer is negative. Why?

   - Lead students through a few more examples of combining a negative and positive, so that they can write a shortened set of rules:

1) **Positive plus positive.... add... answer is positive**

2) **Negative plus negative.... add....answer is negative**

3) **Positive plus negative....subtract....sign of answer is same as that of the addend which has the larger absolute value**

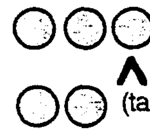
- Now try some larger numbers using a calculator. For example:

  1000 + (- 20) = 980

  20 + (-1000) = -980

  -987 + -123 = -1110

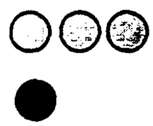
2. Developing the subtraction rule can be done by recalling the counters model which revealed that subtracting is the same as adding the opposite: For example, -3 - (-1):


(take away)

- Start with three negative chips and take away one negative chip.
- Two negative chips remain.

-3 - (-1) = -2

Now **model the opposite** which is -3 + (1)



- Start with three negative chips.
- Add one positive chip.

-3 + (1) = -2

- Explain that since one negative and one positive chip form a zero pair, the result is -2 just like in the last problem. Therefore, -3 – (-1) = -2 and -3 + 1 = -2 , so <u>subtraction is adding the opposite</u>. Help students verbalize the rule:

To subtract integers, change the subtraction problem to adding the opposite, then follow the rules for addition.

4 - (-1)	The opposite of -1 is 1. Change problem to 4 + 1. Positive plus positive;add; answer is positive 5.
-3 - (-5)	The opposite of -5 is 5; change problem to -3 + 5. Negative plus positive; subtract; 5 has bigger absolute value so answer is positive 2.
-3 - 18	The opposite of 18 is -18; change problem to -3 + -18. Negative plus negative; add; answer is negative 21.

18 - 12 The opposite of 12 is -12; change problem to 18 + (-12). Positive plus negative; subtract; 18 has bigger absolute value so answer is positive 6.

3. For multiplication and division model as needed, helping students recall that using the counters:
   - 8 groups of -2 produces -16, so that 8 x -2 = -16
   - -3 four times produces -12, so that -3 x 4 = -12.

   Also, they can recall that using the counters:
   - -24 divided into 3 groups has -8 in each group, so -24 ÷ 3 = -8.

   - To develop the set of rules for multiplication and for division, have students perform the following operations using calculators and record the results:

     2 x 6 = ____(12) Positive times positive = (positive answer)

     2 x -6 = ____(-12) Positive times negative = (negative answer)

     -2 x 6 = ___(-12) Negative times positive = (negative answer)

     -2 x -6 = ____(12) Negative times negative = (positive answer)

   - Next have the students try division using calculators and recording the results:

     36 ÷ 12 = ___(3) Positive divided by positive = (positive answer)

     36 ÷ -12 = ___(-3) Positive divided by negative = (negative answer)

     -36 ÷ 12 = ___(-3) Negative divided by positive = (negative answer)

     -36 ÷ -12 = ___(3) Negative divided by negative = (positive answer)

   - Students should notice that the rules for multiplication and division are the same:

     The product or quotient of two integers with the same sign is positive.
     The product or quotient of two integers with different signs is negative.

   - The Applying Integer Rules Worksheet can be used for student practice or as an **assessment** activity.

Additional Activities

Problem Solving

1. Explain the idea of a Magic Square
   - It is a 3X3 grid
   - A number is placed in each square so that when you add the numbers vertically and horizontally they add the same number, the magic sum
   - No digits are repeated

2. Challenge the students to create an integer Magic Square
   - Create a Magic Square with the "magic sum" of 1
   - uses both positive and negative integers
   - uses numbers between +9 and - 9
   - uses a number only once
   - distribute the Problem Solving Activity Sheet

Name______________________________

Applying Integer Rules Worksheet

Use the integer rules to find the answers to the following problems.

Rules for Addition

• Positive plus positive....	add....	answer is positive
• Negative plus negative....	add....	answer is negative
• Positive plus negative....	subtract....	sign of answer is same as that of the addend which has the larger absolute value

Rules for Subtraction

- Change subtraction to adding the opposite. Then follow the rules for addition.

Rules for Multiplication

- The product of two integers with the same sign is positive.
- The product of two integers with different signs is negative.

Rules for Division

- The quotient of two integers with the same sign is positive.
- The quotient of two integers with different signs is negative.

Complete the chart to change each subtraction problem to a related addition sentence. Then solve.

Subtraction	Think	Related Addition Sentence
+3 - (+5) =	The opposite of +5 is -5.	+3 + (-5) = -2
-4 - (-1) =	The opposite of -1 is 1.	-4 + 1 = ____
+9 - (+4) =	The opposite of +4 is ____.	+9 + ____ = ____
+5 - (-1) =	The opposite of -1 is ____.	+5 + ____ = ____
+4 - (-6) =	The opposite of -6 is ____.	____ + ____ = ____
-2 - +7 =	The opposite of +7 is ____.	____ + ____ = ____
-3 - (-5) =	The opposite of -5 is ____.	____ + ____ = ____
18 - (+3) =	The opposite of +3 is ____.	____ + ____ = ____

Find the sum or difference:

1. 14 + -38 = ___ 2. -79 + -36 = ___ 3. 99 - 127 = ___

4. -215 - (-35) =___ 5. 350 + -40 = ___ 6. -170 - 70 = ___

Find the product or quotient.

7.	12 x -5	8.	20 x -3	9.	-15 x -4
10.	-6 x -5	11.	9(-4)	12.	-21 x -3
13.	24 ÷ -6	14.	-14 ÷ -2	15.	$\frac{-250}{50}$

Write your own problems, then solve using the integer rules.

16. Rules for Addition:

Positive plus positive -- add -- answer is positive. ______ + ______ = ______

Negative plus negative -- add -- answer is negative. ______ + ______ = ______

Positive plus negative -- subtract -- sign of answer is same as that of the addend which has the larger absolute value. ______ + ______ = ______

17. Rules for Subtraction:

Change subtraction sentence to a related sentence of adding the opposite. Then follow the rules for addition. ______ + ______ = ______

18. Rules for Multiplication and Division:

The product or quotient of two integers with the same sign is positive.

______ + ______ = ______

The product or quotient of two integers with different signs is negative.

______ + ______ = ______

Name ______________________

Date _______________

Problem Solving Activity

Integer Magic Square.

Create a Magic Square which follows these rules:

- "magic sum" of 1
- uses both positive and negative integers
- uses numbers between +9 and - 9
- uses a number only once

Answer Key
Obj. 20

Applying Integer Rules Worksheet

Chart:

Subtraction	Think	Related Addition Sentence
+3 - (+5) =	The opposite of +5 is -5.	+3 + (-5) = -2
-4 - (-1) =	The opposite of -1 is 1.	-4 + 1 = -3
+9 - (+4) =	The opposite of +4 is -4.	+9 + -4 = 5
+5 - (-1) =	The opposite of -1 is +1.	+5 + +1 = +6
+4 - (-6) =	The opposite of -6 is +6.	+4 + +6 = 10
-2 - +7 =	The opposite of +7 is -7.	-2 + -7 = -9
-3 - (-5) =	The opposite of -5 is +5.	-3 + +5 = +2
18 - (+3) =	The opposite of +3 is -3.	-18 + -3 = 15

1)	-24	2)	-115	3)	-28
4)	-180	5)	310	6)	-240
7).	-60	8.)	-60	9)	60
10)	30	11)	-36	12)	63
13)	-4	14)	7	15)	-5

16-18) answers will vary

Objective 21 : Recognize, define the characteristics of, and use the associative property, commutative property, properties of zero, and inverse operations.

Vocabulary

associative property
commutative property
properties of zero
identity
inverse operations
factor
addend

Materials

Properties Activity Sheet
student copies

Language Foundation

1. **Associative** can be related to 'associate'. You associate with friends in groups.

2. Explain that **property** is something we own. It belongs to us. If you own a bike, it is your property. In math, a property is a mathematical rule that helps make it easier to solve problems. The rule never changes. When working with real numbers, there are some things that are true about how the numbers "behave". These things are called properties.

3. Relate the word **identity** to I.D. Each person has their own identity. They stay the same. When we use certain identity properties in math, the target number stays the same.

4. The word **inverse** is related to reverse. If you put your car in reverse, it goes backwards. It goes in the opposite direction. Ask students for the reverse of right (left), top (bottom), up (down). Explain that if something is inverted, it is turned upside-down. In math, operations that are opposite and undo each other are **inverse operations.**

Mathematics Component

1. **Commutative Properties of Addition and Multiplication**

Addition:

- Begin by writing the following on the board:

  1) 17 + 36 + 3 2) 26 + 18 + 4 3) 12 + 19 + 8 4) 9 + 57 + 21

- Ask students to find the sums, by working from **left to right**.

  1) 17 + 36 = 53 ... and ... 53 + 3 = <u>56</u>
  2) 26 + 18 = 44 ... and ... 44 + 4 = <u>48</u>
  3) 12 + 19 = 31 ... and ... 31 + 8 = <u>39</u>
  4) 9 + 57 = 66 ... and ... 66 + 21 = <u>87</u>

- Now ask students if they can think of a **faster** way to solve 17 + 36 + 3.
- What if we added 17 + 3? 20 + 36? Could we do the problem mentally if we added 17 to 3 and then added 36?
- What about the second problem? 26 + 18 + 4 Who can think of a faster way to solve this problem. Can we use only mental math? (26 + 4 = 30; 30 + 18 = 48)
- Discuss the last two examples. Explain that changing the order when we add sometimes enables us to do the problems mentally.
- Ask students if the answer was different or the same when we changed the order of the addition. (Same) Reinforce the idea that no matter in which order the numbers are added, the answers are the same.
- Explain that in mathematics, the **Commutative Property** allows us to change the order of the addends. As in the first example, 17 + 36 + 3 = 17 + 3 + 36. The Commutative Property of Addition states that changing the order of the terms does not change the sum.
- Tell students that one way to remember this property is to think of the word **commute.** If you commute to school by walking 3 blocks south and 4 blocks west, you have traveled 7 blocks. When you go home, you walk 4 blocks east and 3 blocks north. The distance is the same either way. (3 + 4 + 7 4 + 3 = 7) The Commutative Property is useful in addition, as we have seen, because it allows us to change the order of addends to make it easier to find the sum.

Multiplication:

- Now put the following examples on the board:

  1) 2 x 37 x 5 = ___ 2) 5 x 13 x 2 x 3 = ___

- Ask students to use paper/pencil to work these problems from left to right. Check answers together.
- Explain that working these problems from left to right is difficult. Changing the order of the numbers produces problems that can be worked mentally.

  1) 2 x 5 x 37 = 10 x 37 = <u>370</u> 2) 5 x 2 x 13 x 3 = 10 x 39 = <u>390</u>

- Explain that the **Commutative Property of Multiplication** states that you can multiply numbers in any order.

- As we can see, the operations of addition and multiplication are commutative. That is, changing the **order** of the terms does not change the sum or product.

2. **Associative Properties of Addition and Multiplication**
   - Put the following problems on the board:

     1) (36 + 10) + 5 = 2) 5 x (2 x 17) = 3) 15 + (25 + 9 + 11) =
   - Ask students what the parentheses mean. (Do this group first.)
   - Ask if it would matter if we grouped the numbers differently.
   - Ask one student to work the first problem out loud as it is shown, adding 36 + 10 first and then adding 5.
   - Ask a second student to add the 10 + 5 first and then add the 36.
   - Compare answers and point out that the answers are the same.
   - Check with students to see if numbers could be grouped differently in the other two problems and whether it would affect the answer.
   - Explain that the **Associative Property** for addition and multiplication is what allows us to change the order. It is similar to the Commutative Property.
   - The Associative Property states that changing the **grouping** of terms in an **addition** or **multiplication** problem does not change the sum or product.
   - In the first problem above, it might be easiest to add the 36 and the 10 first, as shown. Why? (Multiples of 10 make adding easier.)
   - Ask students if the grouping in the second problem above is the easiest. (Most will say no.) Why? (Multiplying 5 x 2 first would give us 10 and multiples of 10 are usually easier to multiply.)
   - Discuss the grouping of the last problem, emphasizing that there is not a wrong or right way to group them; however, some groupings may make it faster or easier to get the answers quickly.
   - Tell students that one way to remember what associative means is that students may **associate** with many different friends. You can sit between two friends and say the same thing first to one and then to the other. The result is the same no matter which friend you speak to first.

3. **Identity Properties of Addition and Multiplication**

   Addition:
   - Ask students what 5 + 0 is. (5)
   - Continue with several more addition problems involving 0.
   - Ask students what they have noticed about adding with 0. (When zero is added to any number, the sum is the same as the original number.)
   - Explain that the number zero has a special addition property. When zero is added to any number, the sum is identical, or the same as, the original number. For this reason, the number zero is given a special name and is called the **additive identity.** The original number keeps its own identity.
   - The **Identity Property of Addition** states that the sum of any number and zero is the original

number. Subtracting zero from a number does not change the number either, but remember that *subtraction is not commutative.* (24 - 0 = 24 but 0 - 24 = **-24)**

Multiplication:

- Ask students what number 6 x 1 equals. (6)
- Continue with several more multiplication problems involving 1.
- Ask what students have noticed about multiplying by 1. (When any number is multiplied by one, the product is the same as the original number.)
- Explain that the number one in multiplication has a special property. It is like like the Identity Property of Addition, but it applies to the number 1 and is used when multiplying.
- When any number is multiplied by one, the product is the same as the original number. For this reason, the number one is given a special name and is called called the **multiplicative identity**.
- Therefore, the **Identity Property of Multiplication** states that the product of any number and one is the original number.

$14 \times 1 = 14 \qquad 14 \times \frac{3}{3} = 14$ Why? (3/3 = 1)

4. **Multiplication Property of Zero**

- In multiplication, the number zero also has a special property. Ask students to suggest what they think the **Multiplication Property of Zero** states. Hint: What happens when we multiply a number times 0? (The product will always be 0.)

1) 14 x 0 = 0 2) 27 x 0 = 0 3) 1, 555, 444 x 0 = 0

5. **Review and Application to Inverse Operations**

- Before you ask the students to practice identifying and using the properties, help them recall that *addition and subtraction are* **inverse**, *or opposite, operations, as are multiplication and division.* • Investigate the Commutative, Associative and Identity and Zero Properties as they relate to inverse operations as follows:

1) We know that addition is commutative; that is, the order does not change the sum.

Examples: 7 + 4 = 11 4 + 7 = 11

Ask: Is subtraction, which is the opposite of addition, also commutative?

Examples: 7 - 4 = 3 4 - 7 = -3 (Subtraction **is not** commutative.)

2) We know that multiplication is commutative. The order does not change the product.

Examples: 5 x 4 = 20 4 x 5 = 20

Ask: Is division, which is the opposite of multiplication also commutative?

Examples: 20 ÷ 4 = 5 4 ÷ 20 = .20 or 1/5 (Division **is not** commutative.)

6. **REVIEW:**

- **Review** the **Associative Property of Addition**. Remind them to think of the word associate.

  Examine the Associative Property using subtraction and division problems:

  Examples: 49 - (27 - 14) = 36 (49 - 27) -14 = 8 (Subtraction **is not** associative.)

  24 ÷ (6 ÷ 2) = 8 (24 ÷ 6) ÷ 2 = 1.5 or 1 1/2 (Division **is not** associative.)

- **Review** the **Identity Property of Addition** which states that the sum of any number and 0 is the original number. Discuss its application as it relates to its inverse which is subtraction. (Subtracting zero from a number does not change the number either, but remember that subtraction is not commutative. 24 - 0 = 24 but 0 - 24 = **-24)**

- **Review** the **Identity Property of Multiplication** which states that the product of any number and 1 is the original number. Discuss its application as it relates to the inverse of multiplication.
  (Dividing a number by one does not change the number either.)

- **Review** the **Multiplication Property of Zero** which states that the product of any number and zero is zero. Investigate zero as it relates to division.

  Remember, 24 ÷ 0 is undefined. If 24 could be divided by zero, using inverse operations, there would be a number which could be multiplied by zero to get 24. But there is no factor which can be multiplied by zero to get 24 as a product. Therefore, division by zero is **undefined**.

  Remember, 0 ÷ 24 = 0. Zero cupcakes divided among 24 people means 0 for each!)

7. Distribute the Properties Activity Sheet so that students can practice identifying and using the properties.

Name________________________________

Properties Activity Sheet

Name the property shown by each statement:

1. 17 + 22 = 22 + 17 ________________________________

2. 12 x 46 = 46 x 12 ________________________________

3. 89 + 0 = 89 ________________________________

4. 89 x 1 = 89 ________________________________

5. 89 x 0 = 0 ________________________________

6. (8 + 4) + 5 = 8 + (4 + 5) ________________________________

7. (43 x 2) x 5 = 43 x (2 x 5) ________________________________

Use the properties to find each sum mentally.

8. 36 + 48 + 14 ______

9. 2.8 + 0 + 11.2 ______

10. 29 + 14 + 71 ______

11. 4 x 35 x 50 ______

12. (1.35)(27)(0)(13) ______

13. 274,345 x 1 ______

Complete the following statements:

14. The Commutative Property applies to the operations of ______________ and __________________. It states that the order of terms does not change the sum or product.

15. The Associative Property states that the terms can be grouped in any order in ______________problems or in ______________problems, without affecting the sum or product.

16. What number is known as the additive identity? _______ Adding this to a number does not change the number's identity.

Answer Key
Obj. 21

Properties Activity Sheet

1. Commutative Property of Addition
2. Commutative Property of Multiplication
3. Identity Property of Addition
4. Identity property of Multiplication
5. Multiplication Property of Zero
6. Associative Property of Addition
7. Associative Property of Multiplication

8.	36 + 14 + 48 = 98	9.	14
10.	29 + 71 + 14 = 114	11.	4 x 50 x 35 = 7000
12.	0	13.	274,345

14. addition, multiplication 15. addition, multiplication 16. 0

Objective 22: Recognize, define the characteristics of, and use the distributive property.

Vocabulary

properties
multiplication
distributive
addition
subtraction
handicapped

Materials

Three Crabs
Parking Garage
transparency

Distributive Property
student copies

Language Foundation

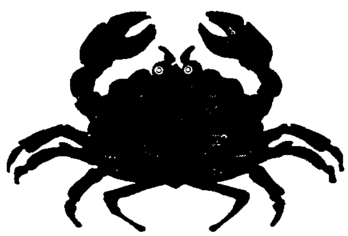
1. Review the word **property** from the previous lesson.

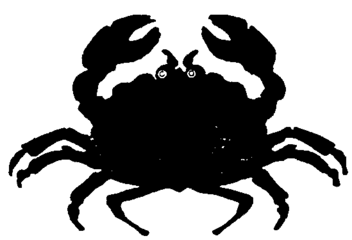
2. Explain that **distribute** means to give something (or pass something out) to each person. Tell students that the word **distributive** comes from the verb *to distribute*. Distribute a sheet of paper to each student, as an example. Ask students for other examples of items which can be distributed: books, worksheets, playing cards, cookies, etc.

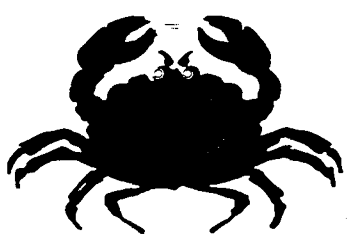
3. The word **handicapped** is used in a word problem concerning parking spaces. If students are not familiar with this word, show them the following symbol. Tell them that whenever they see this sign, the space is to be used only by persons who have a handicap, or physical disability.

Mathematics Component

1. • Begin by putting the Three Crabs transparency on the overhead, pointing out the claws and legs.
   - Tell the students you want to know how many claws and legs are on the 3 crabs altogether. Have a few students come up and show how they could find the answer. (30)
   - Show students that each crab has 2 claws and 8 legs.
   - Explain that when we count the total number of claws and legs, we can think about it two different ways.







Write on the transparency:

Each crab has **2** claws and **8** legs.
How many claws and legs all together?

(crabs claws) + (crabs legs)

1) $(3 \bullet 2) + (3 \bullet 8) = ?$
$6 + 24 = \mathbf{30}$

$\boxed{\textbf{OR}}$

crabs (claws + legs)

2) $3(2+8) = ?$
$6 + 24 = \mathbf{30}$

- Point out that both methods produce the same answer.
- Review the word **distribute** by using the following example. Say, "I will distribute some counters to each of you." Walk around the room and give each student 2 red counters and 1 yellow counter.
- Say, "How many counters did I **distribute**?"

- Write the problem two different ways:

(students red counters) + (students yellow counters)

Ex: $(15 \bullet 2) + (15 \bullet 1) = ?$

$30 + 15 = 45$

OR

students (red + yellow)

$15(2+1)$

$15(3) = 45$

- Have students find the sum of all counters distributed and compare with the answers above.
- Ask which method seems easier to do? Why? (Most students will say the second way is faster because it is easier to do mentally.)
- Explain that both problems produced the same answer because of a special rule called the **Distributive Property**. This rule states that for all numbers, *a, b, and c* .

$$a(b+c) = (ab) + (ac)$$

and

$$a(b-c) = (ab) - (ac)$$

- Go over additional examples of addition and subtraction distributive problems and compare to see if answers are the same.

4(15 - 3) ________	(4 • 15) - (4 • 3) ________	Same answers?
16(18 - 8) _______	(16 • 18) - (16 • 8) ______	Same answers?
32(16 + 4) _______	(32 • 16) + (32 • 4) ______	Same answers?
8(40 + 5) ________	(8 • 40) + (8 • 5) _____	Same answers?
2(27 - 7) _______	(2 • 27) - (2 • 7) ______	Same answers?

2. Write 9 • 12 on the board.
   - Ask if anyone can think of an easy way to find the answer using the distributive property.

- Explain that the distributive property allows us to separate 12 into two numbers. Although any two numbers whose sum is 12 would work, 10 and 2 make mental multiplication easier.
  $(9 \bullet 10) + (9 \bullet 2)$ or $9(10 + 2)$

  Note: Some students may relate to separating 12 into 10 plus 2 because in Spanish and French, some numbers are named that way. For instance, in Spanish and French, the number seventeen is named "ten and seven" and the number eighteen is named "ten and eight."

- Write 3(42) on the board.
- Ask if anyone can think of an easy way to find the answer using the distributive property. {Try to elicit (3 x 40) + (3 x 2).}
- Explain that the distributive property allows us to separate 42 into 40 + 2 and multiply each part by 3.

3. Display and read the following problem on the Parking Garage transparency.

   A parking garage has three levels. On each level, there are 80 regular parking spaces and 10 handicapped parking spaces. How many parking spaces are there altogether?

   - Ask students to help you write a problem to find the number of parking spaces altogether. Lead students to understand that two possibilities include 3(10 + 80) and (3 x 10) + (3 x 80). Write these two methods on the transparency.

4. The Distributive Property activity sheet can be used for further practice. You may want to do the first few problems together.

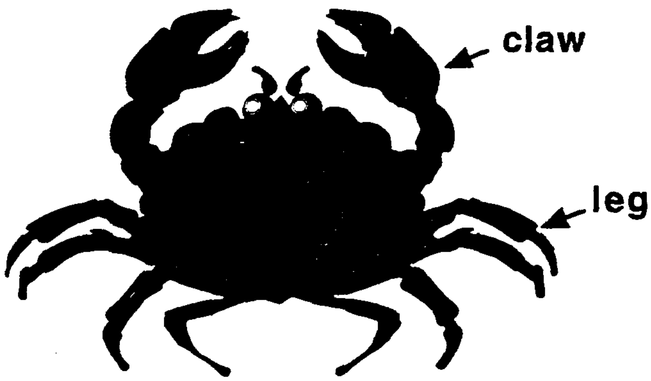
Additional Activities

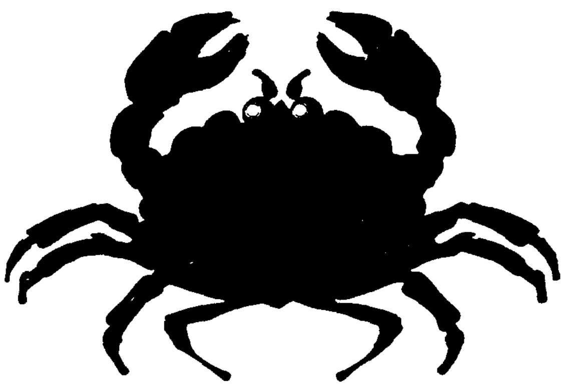
Assessment

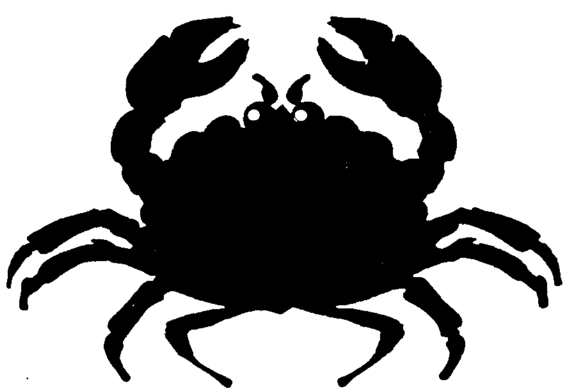
Ask students to work with a partner to create situations which can be written as distributive problems. Have students draw or write the problem. Then ask them to solve their own problems on a piece of paper. Have them exchange their word problems with another group so they can solve and check each other's work.

Three Crabs

Look at these crabs carefully. Each crab has claws and legs.
How many claws and legs do they have **altogether**?







Name________________________________

Distributive Property

Fill in each blank with the number that makes the statement true.

1. $8(43) + 8(7) = 8(43 + \_\_\_) = 8(\_\_\_) = \_\_\_\_\_$

2. $(13 \times 27) + (13 \times 13) = 13(\_\_\_\_ + \_\_\_\_) = 13(\_\_\_) = \_\_\_\_\_$

Use the distributive property to find each answer **mentally.**

3. $7(68) + 7(12) = \_\_\_\_\_\_\_\_$

4. $8(31) + 8(19) = \_\_\_\_\_\_\_\_$

5. $4(109) = 4(\_\_\_\_ + \_\_\_\_) = \_\_\_\_\_\_$

6. $2(27) = 2(\_\_\_\_ + \_\_\_\_) = \_\_\_\_\_$

Write a **distributive** problem for each situation, then **solve**.

7. Carlos bought two pairs of pants for $29.99 each and two shirts for $19.99 each. How much did he spend altogether?

____

8. Last week Soon Je ran 5 miles three times. This week she ran 5 miles four times. How many miles did she run altogether?

____

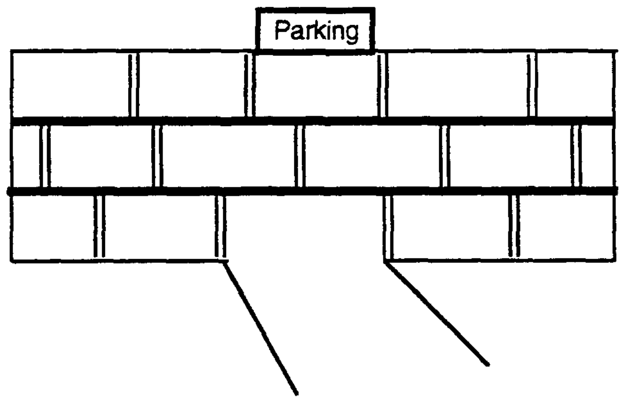
9. Maria earns $8.00 per hour. She worked 7 hours on Monday and 8 hours on Wednesday. How much did she earn altogether this week?

____

EXTENSION:

10. Last Thursday and Friday, Twan drove 7 miles to school, 4 miles to soccer practice after school, 6 miles to Tom's house, and 13 miles back home. How many miles did he drive altogether?

Parking Garage



A parking garage has 3 levels.
Each level has 80 regular parking spaces and 10 handicapped parking spaces.

How many parking spaces are there altogether?

Answer Key
Obj. 22

Chart in math lesson:

4(15 - 3) ___48___	(4 x 15) - (4 x 3) ___48___	Same answers?
16(18 - 8) ___160___	(16 x 18) - (16 x 8) ___160___	Same answers?
32(16 + 4) ___640___	(32 x 16) + (32 x 4) ___640___	Same answers?
8(40 + 5) ___360___	(8 x 40) + (8 x 5) ___360___	Same answers?
2(27 - 7) ___40___	(2 x 27) - (2 x 7) ___40___	Same answers?

Distributive Property Activity Sheet

1. 7 8(50) 400

2. 27 + 13 13(40) = 520

3. 560

4. 400

5. 100 + 9 , 436

6. 20 + 7, 54

7. 2($29.99 + $19.99) = $99.96

8. 5(3 + 4) = 35

9. 8(7 + 8) = $120

EXTENSION:

10. 2(7 + 4 + 6 + 13) = 60

Patterns and Functions

Objective 23: Identify and Extend Number Patterns

Vocabulary

. pattern
sequence
odd
even
term
arithmetic
geometric
sequence of squares

Materials

Sequences
student copies

Language Foundation

1. A **pattern** is a regular, systematic repetition; it can be numerical, visual, or behavioral. Let students give examples of visual or behavioral patterns with which they are familiar. Tell them that in this lesson, the patterns are numerical. By finding the rule for these patterns, they can predict what number will come next and next and next, as the pattern continues and repeats itself in the same order or **sequence**.

2. Review the word term. Tell students that in this lesson, each number in a sequence will be called a **term**.

Mathematics Component

1. Write the following on the board:

   1 3 5 7 9 11 . . .

   - Ask students what the next number would be. (13)
   - Ask how they knew this. (Responses may vary: the pattern is add two or the set of odd numbers.)
   - Explain that this is a number sequence because the numbers are arranged in an order and each number follows the previous number according to a rule. The rule for this sequence is add 2.
   - Write on the board the following:

     2 4 6 8 10 12 . . .

   - Ask students what the next number would be. (14) Ask how they knew this. (Responses may vary: the rule is add two or the set of even numbers.)
   - Explain that this is a number sequence because the numbers are arranged in an order and each number follows the previous number according to a rule. The rule for this sequence is add 2.
   - Ask how these two sequences can have the same rule (add 2) and be different sequences. (A sequence also depends upon the starting number. These two sequences have the same rule but different start numbers. A number sequence is defined by the rule and the starting number.
   - Explain that each number in a sequence may be called a **term**.

2. Write the following sequence on the board :

   1 2 4 8 16 32 . . .

   - Ask if this is a number sequence? (YES; because the arrangement of numbers follows a rule.)
   - Ask students what the next term would be. (64)
   - Ask what the rule is. (multiply by 2)
   - Ask what the starting number is. (1)
   - Ask students to generate the first five terms of a sequence with the same rule but a start number of 3. (3, 6, 12, 24,48, ...)

3. Write on the board the following sequence:

   1024 512 256 128 64 32 . . .

   - Ask if this is a number sequence? (YES; because the arrangement of numbers follows a rule.)
   - Ask students what the next term would be. (16)
   - Ask what the rule is. (divide by two or multiply by one-half)
   - Ask what the starting number is. (1024)
   - Ask students to generate the first five terms of a sequence with the same rule but a start number of 1. Have them write the terms as decimals and fractions. (1, .5, .25, .125, .0625, ...) and $(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots)$
   - Ask students which representation they prefer.

4. Explain that there are many different kinds of sequences:

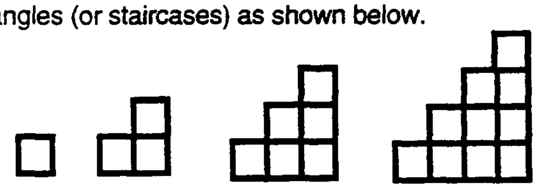
- **Arithmetic sequences** are sequences that are formed by adding (or subtracting) a constant.
- Odd numbers are one example of this type of sequence. They are an arithmetic sequence that starts with 1 and always adds 2.
- Demonstrate this sequence on the board and allow students to extend it.
- Even numbers are another example. These are an arithmetic sequence that starts with 2 and always adds 2 to each term to get the next term.
- Demonstrate this sequence on the board and allow students to extend it.

- **Geometric sequences** are sequences that are formed by multiplying (or dividing) by a constant.
- Place this sequence on the board (1, 2, 4, 8, 16, 32, . . .). Explain that it is a geometric sequence.
- Ask students what each term is multiplied or divided by to get the next term. (It starts at 1 and each term is multiplied by 2 to get the next term.)
- Ask students what kind of sequence (1024, 512, 256, 128, 64, 32, . . .) is. (Geometric)
- What is the starting term and what is the rule? (Starting term is 1024 and the rule is divide by 2.)

5. Distribute the worksheet Sequences. If students need more practice, do a few items as a class. When students are ready, have them work with a partner or independently. This worksheet could be completed for homework.

Additional Activities

1. After students complete and correct the worksheet, have them find the triangular numbers by making outlines similar to that for the sequence of squares. However, instead of making squares, they make triangles (or staircases) as shown below.



1 3 6 10 15 21

- When they have found the sequence, ask them to try to describe the pattern.

  It is quite a challenge to try to write an expression for the n^{th} term where "n" is the figure number:

$$\frac{n^2}{2} + \frac{n}{2}$$

- You may want to let students practice evaluating the expression to verify that it works for the first ten terms and to find other later (nth) terms (e.g., the 50th).

2. Explore the Fibonacci sequence which starts 1, 1 , 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .
   - Can they determine the rule? (A term is the sum of the two previous terms.)
   - Have them find out about this unique sequence by doing a little research

Name ____________

Sequences

The items below are all part of arithmetic or geometric sequences. Study them to determine the rule. Use the rule to help fill in the missing values. Then record the rule, the starting term, and whether each sequence is arithmetic (A) or geometric (G) in the appropriate columns.

Part of Sequence	Start Number	Rule	Type of Sequence
1) 1 5 25 ____ ____	____	____	____
2) 1 4 7 ____ ____	____	____	____
3) 11 15 ____ 23 ____	____	____	____
4) 4 12 36 ____ ____	____	____	____
5) 5 ____ 20 ____ 80	____	____	____
6) 1 ____ ____ 27 81	____	____	____
7) 1000 100 10 ____ ____	____	____	____
8) 100 60 36 ____ ____	____	____	____
9) ____ ____ 23 32 41	____	____	____
10) 15 14.5 14 ____ ____	____	____	____
11) 4 9 14 ____ ____	____	____	____
12) 6 ____ ____ ____ 14	____	____	____
13) 3 ____ 17 ____ 31	____	____	____

Part of sequence						Start Number	Rule	Type of Sequence
14) 1	____	____	____		-3	____	____	____
15) 5	11	17	23	29		____	____	____
16) 3	3	3	3	3		____	____	____
17) 5	10	15	20	25		____	____	____
18) 10	9	8	7	6		____	____	____
19) .1	1	10	100	1000		____	____	____
20) $\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$ (Hint: find common denominator)		____	____	____

SQUARE NUMBERS

On a separate piece of paper (graph paper), outline a square whose four sides are each 1 cm in length. Beside this square, outline a second square whose four sides each have a length of 2 cm. A third outlined square should have sides with lengths of 3 cm. Continue until you have outlined the square having sides with lengths of 10 cm. For the squares you have drawn, determine the number of small squares inside each one and record these numbers on the lines below.

1st square	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
____	____	____	____	____	____	____	____	____	____

On the following lines, rewrite each of the above amounts as some number to the second power (squared).

1st square	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
____	____	____	____	____	____	____	____	____	____

Explain on the back of this paper why this sequence is **NOT** arithmetic <u>or</u> geometric.

Answer Key
Obj. 23

<u>Sequences</u>

	Part of Sequence	Start Number	Rule	Type of Sequence
1)	1 5 25 125 625	1	x 5	G
2)	1 4 7 10 13	1	+3	A
3)	11 15 19 23 27	1	+4	A
4)	4 12 36 108 324	11	x3	G
5)	5 10 20 40 80	5	x2	G
6)	1 3 9 27 81	1	x3	G
7)	1000 100 10 1 .1	1000	÷10	G
8)	100 60 36 21.6 12.96	100	x .6	G
9)	5 14 23 32 41	5	+9	A
10)	15 14.5 14 13.5 13	15	- .5	A
11)	4 9 14 19 24	4	+5	A
12)	6 8 10 12 14	6	+2	A
13)	3 10 17 24 31	3	+7	A

Answer Key
Obj. 23

Part of sequence	Start Number	Rule	Type of Sequence
14) 1 0 -1 -2 -3	1	-1	A
15) 5 11 17 23 29	5	+6	A
16) 3 3 3 3 3	3	+ or - 1 x or ÷ 1	A or G
17) 5 10 15 20 25	5	+5	A
18) 10 9 8 7 6	10	-1	A
19) .1 1 10 100 1000	.1	x10	G
20) $\frac{1}{6}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{5}{6}$ (Hint: find common denominator)	$\frac{1}{6}$	add 1/6	A

SQUARE NUMBERS

On a separate piece of paper (graph paper), outline a square whose four sides are each 1 cm in length. Beside this square, outline a second square whose four sides each have a length of 2 cm. A third outlined square should have sides with lengths of 3 cm. Continue until you have outlined the square having sides with lengths of 10 cm. For the squares you have drawn, determine the number of small squares inside each one and record these numbers on the lines below.

1st square	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1	4	9	16	25	36	49	64	81	100

On the following lines, rewrite each of the above amounts as some number to the second power (squared).

1st square	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2

Answer Key
Obj. 23

Square Numbers (cont'd)
This sequence is not an arithmetic sequence because there is **not** one constant that you can add or subtract to get from one term to the next.

It is not a geometric sequence because there is **not** one constant that you can multiply or divide by to get from one term to the next.

(**Note**: This type of sequence which does not grow evenly each time is called a quadratic sequence. You might want to ask students why this name seems appropriate for this sequence.)

Objective 24: Represent Functions, Function Tables, and Function Rules Using Arrow Notation

Vocabulary

function
input
output
function table
function rule
arrow notation

Materials

Chart paper

Markers/crayons

Input-Output Cards - 1 card per student (master included)

Function Machine Worksheet
Function Machine Practice
student copies

Teaching Transparency 1, 2, and 3
(function machines found in lesson)

Language Foundation

1. Imagine that you are working at McDonald's. Your job, or **function**, is filling food orders for the customers. The amount of food you sell depends on the customers' orders. For example, one customer might order a hamburger and french fries, while the next customer will want a hamburger and a coke. What each customer orders is the **input**. You must fill that order according to the input. When you hand the food to the customer, let's call that the **output.** You have filled the order for the customer.

Mathematics Component

1. Ask the students if they have made popcorn at home. Have the students give the basic steps involved in making popcorn (put popcorn in a pot/popcorn popper/microwave, add heat when you put it on the stove/plug in, out come fluffy popcorn).
   - Use the popcorn popper /microwave as an analogy of a **function machine**. In a function machine, what goes in is altered and comes out in a different state.
   - Ask students what goes into the popcorn popper/microwave. (kernels) Explain that what goes into the function machine is called the **input.** The kernels are the input.
   - Have students describe what comes out of the popcorn popper/microwave after it has been popped. (fluffy popcorn) Explain that what comes out of a function machine is called the **output**. The fluffy popcorn is the output. The output is still popcorn, but it looks different because it is in a different state.
   - Review the entire analogy, using Teaching Transparency 1, to show that: popcorn kernels + heat in the popcorn machine = fluffy popped corn

Popcorn Kernels	Heat Added	Fluffy Popcorn
Input	Function	Output

   - Have students think of other real-life examples. Share or record student examples. (Use Transparency 1 to record ideas.)

Input	Function	Output

2. Use Teaching Transparency 2. Have groups guess the rule for each function machine.

Input	Output
8	5
9	6
10	7

Rule: (Subtract 3)

Input	Output
0	7
2	9
4	11

(Add 7)

Input	Output
2	6
3	9
4	12

(Multiply 3)

3. Have each student record a list of input numbers in the appropriate column on each of the Input-Output Cards. Then ask them to write a function rule on the back of each card. (For example, add 3 or multiply by 2, etc.) Have them apply the function rule to each of the input numbers to create a list of output numbers. Then ask them to form groups of 4 students and exchange cards with others in the group to guess the rule on the back of the cards.

4. Present the following input/output machine to the students. (Teaching Transparency - 2) Have students discuss in their groups: "What could the function rule be? What else could it be? Find 4 function rules that would apply."

Input	Output
6 →	6
6 →	6
6 →	6

Answer: $N+0=N$
$N-0=N$
$N \times 1=N$
$N \div 1=N$

5. Present the following function. Explain that a function if a process that can be defined as input-output or as N is the "input" and the function is the "output". Complete similar charts as a group. (Transparency-2)

N	N + 2
3 →	5
4 →	6
5 →	7

N	N x2
3	
4	
5	

N	(N x2) - 1
3	
4	
5	

6. In the small groups, have students discuss the following and find at least 2 rules that work. Discuss as a total group. (Transparency 3)

N	
2	0
4	0
7	0
11	0
15	0

(The rule could be N – N or N x 0)

7. Using the Function Machine Practice activity sheet have students work in groups or as individuals to find at least 3 input and output numbers for each of the following:

$N \div 5$

$(N - 3) \div 3$

$(2 \times N) + 5$

$N \times 1/2$

$20 - (N + 2)$

Additional Activities

Assessment

1. Have students complete the Function Machine Worksheet. Students may also record suggestions they have for other students figuring out function rules.

Input-Output Cards

Input	Output

Name ____________

Date ____________

Function Machine Worksheet

Complete the following. For problems 1 and 2, choose any numbers for N.

1.

N	N + 10

2.

N	N x 10

3.

N	N – 4
4	
	4
12	
100	

4.

N	$N^2 - 1$
3	
4	
5	
6	

5.

N	
2	200
4	
6	200
8	

6.

N	N ÷ 8
16	
	3
48	
	8

Name ________________

Date ________________

Function Machine Practice

1) $N \div 5$

N	$N \div 5$

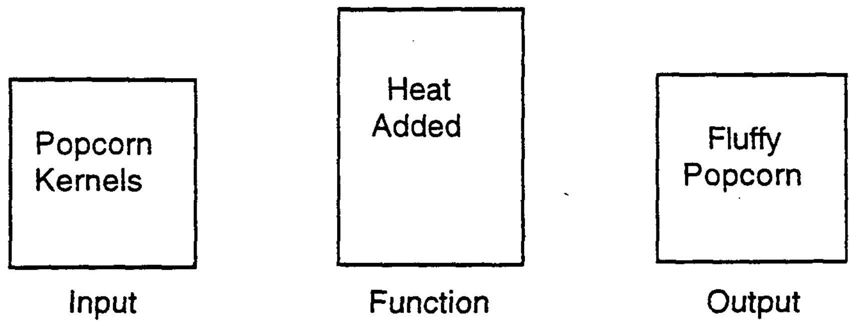
2) $(N - 3) \div 3$

3) $(2 \times N) + 5$

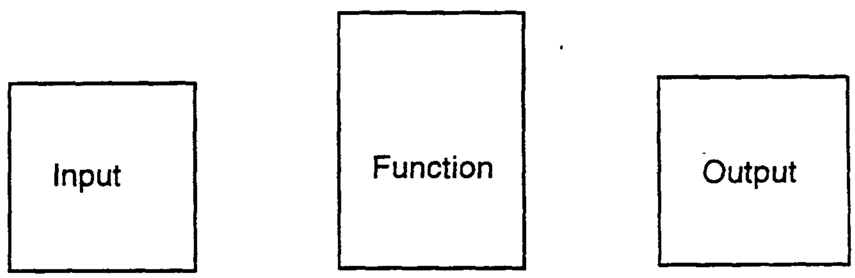
4) $\frac{N}{2}$

5) $20 - (N + 2)$

Teaching Transparency -1



(Student real-life examples)



Teaching Transparency - 2

Input	Output
8 →	5
9 →	6
10 →	7

Input	Output
0 →	7
2 →	9
4 →	11

Input	Output
2 →	6
3 →	9
4 →	12

Input	Output
6 →	6
6 →	6
6 →	6

N	N + 2
3 →	5
4 →	6
5 →	7

N	N x2
3	
4	
5	

N	(N x2) - 1
3	
4	
5	

Teaching Transparency - 3

N	
2	0
4	0
7	0
11	0
15	0

Answer Key
Obj. 24

Function Machine Worksheet

Complete the following. For problems 1 and 2, choose any numbers for N.

1. (Answer may vary)

N	N + 10
2	12
3	13
4	14
5	15

2. (Answer may vary)

N	N x 10
3	30
4	40
5	50
6	60

3.

N	N – 4
4	0
8	4
12	8
100	96

4.

N	$N^2 - 1$
3	8
4	15
5	24
6	35

5.

N	N (100)
2	200
4	400
6	600
8	800

6.

N	N ÷ 8
16	2
24	3
48	6
64	8

Answer Key
Obj. 24

Function Machine Practice

1) $N \div 5$

N	$N \div 5$
5	1
10	2
15	3

2) $(N - 3) \div 3$

9	2
12	3
15	4

3) $(2x \times N) + 5$

1	7
2	9
3	11

4) $N \times 1/2$

2	1
3	1 1/2
4	2

5) $20 - (N + 2)$

1	17
2	16
3	15

Obj. 25: Plot points on a coordinate grid.

Vocabulary

number line
coordinate
plane grid
perpendicular
origin
vertical
horizontal
axis, axes
ordered pair
quadrant
plot

Materials

graph paper for each student
ruler for each student

Transparency - Coordinate Plane

<u>Plotting Points Practice Sheet</u>
<u>Coordinate Plane Practice Sheet</u>
student copies

Language Foundation

1. Tell students the prefix 'co' means working together (cooperate) If you **coordinate** a party with a friend, you work together on the plans. Pilots or captains give their location in the air or on the sea using the **coordinates** of direction on a map. Make up an example (52^0 south, 125^0east) Use a map to show direction.Explain that coordinates in math are two numbers that work together to show location ona grid

2. Explain the various meanings of the words <u>plain</u> and <u>plane</u>. (A wide expanse of land, something that is not decorated, an airplane) In math, a **plane** is a flat surface that can go on in all directions forever. Give examples such as the blackboard, the floor, a sheet of paper, etc. Have students give further examples of a plane.

3. Show students a piece of graph paper and tell them this is a **grid**. Grids can be of any size and are formed of lines that cross horizontally and vertically. Use the example of a waffle, waffle griddle (try to get a picture or borrow a waffle maker), or a map with grid lines. Ask for other examples of grids (the layout of streets in a town, grid on Claris works drawing, etc.)

4. Show several symbols that are perpendicular. (+, T, L, F, H) Tell students these lines are **perpendicular.** Can they tell you what the lines have in common? (intersect to form 90^0 angles) Show that on a grid, the lines are always perpendicular.

5. Explain that the **origin** mans the beginning, the point at which something starts.

6. If you are standing, you are in a **vertical** position. Vertical means "up and down". Have students give examples of things that are vertical. (flag pole, lamp post, etc.)

7. **Horizontal** is the opposite of vertical. If you are laying down on the floor, you are horizontal. Explain that where the sky meets the sea is called the horizon. Horizontal means sideways. Have students give examples of things that are horizontal. (person sleeping, a picnic bench, etc.)

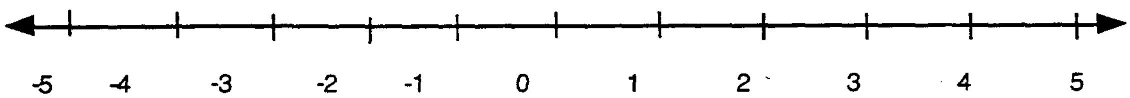
8. **Pair** means two, **ordered** means the numbers in the pair must always be in a special order. (first number, second number)

9. Tell students the prefix "quad" refers to four. Quadruplets are four babies. There are four **quadrants** on a coordinate plane.

10. When we mark a point for an ordered pair of numbers on a grid, we **plot** the point. We are getting a location for that point.

Mathematics Component

Warm-Up: Begin by asking students to recall the number **line** and how to graph points on the line. Review that the origin is zero, and that positive numbers are located to the right of the origin, with negative numbers to the left of the origin.



The number **line** is used to graph or locate points.

1. Explain that a number **plane** is used to graph or locate **pairs** of points.
   - The number plane consists of two number lines, one horizontal line and one vertical line. It is called a Coordinate Plane.
   - Use the transparency master Coordinate Plane to show students the horizontal axis and vertical axis which are formed by two perpendicular **lines**. Point out to students that since the horizontal and vertical axes are lines, they have arrows at both ends.
   - Label the **origin.** Note that the origin is a common point in both axes. or coordinate grid.
   - Explain that a coordinate plane is used to graph **ordered pairs** of numbers. The two numbers that we use to locate or plot a point on a number plane are called ordered pairs because we always graph them in a certain way (or order). 203
   - The first number of an ordered pair is graphed on the horizontal axis. This is called the **x-axis**. Label the **x-axis**.
   - Review that positive numbers are located to the right of zero, and negative numbers are on the left of zero. A number on the x-axis can be positive, negative, or zero.
   - The second number of an ordered pair is graphed on the vertical, or **y-axis**. Label the **y-axis.**
   - Show that positive numbers are located upward from zero, and negative numbers are located downward from zero.

2. Explain that ordered pairs of numbers are usually written in parentheses, separated by a comma. Write these ordered pairs on the board or overhead as examples of ordered pairs.

   (2,3) (0,5) (-2, 4) (0,0) (-2,-1) (3, -2)

   - We can use a numbered coordinate plane called a **grid** to plot the points.
   - Say, "The first number is graphed on the x-axis and it is called the x-coordinate. The second number is graphed on the y-axis, and is called the y-coordinate." (x,y)
   - Practice placing each ordered pair on a transparency copy of the Numbered Coordinate Grid. For (2,3), say, "start at the origin, then move **two to the right, up three.**" For (-2, -1) say, "start at

the origin, move **two to the left, down one**."

- Continue in this manner with additional examples.

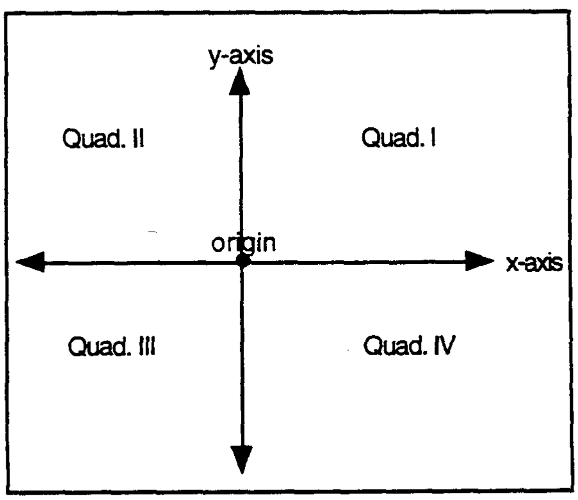
3. Refer back to the original transparency master Coordinate Plane. Point out to students that we have just plotted points using four directions (right, left,up, down) and that the coordinate plane is divided into four sections.
   - Explain that the 4 sections are referred to as **quadrants**. Refer to the transparency once again and point out the four quadrants.
   - Point out that Roman Numerals are used to name the quadrants to distinguish them from other numbers used for the ordered pairs.
   - Quadrant I is used most often in "real-life" situations such as street maps. Label Quadrant I. Show that if a point is located in Quadrant I then both coordinates are positive. The directions used in Quadrant I are "to the right and up."
   - Quadrant II is located to the **left** of Quadrant I, and we locate points in Quadrant II by moving "to the left and up." Label Quadrant II.
   - Quadrant III is down from Quadrant II, and we locate points in Quadrant III by moving "to the left and down." Label Quadrant III.
   - Quadrant IV is down from Quadrant I, and we locate points in Quadrant IV by moving "to the right and down." Label Quadrant IV.

4. Distribute Plotting Points Practice Sheet and have students practice plotting some points on coordinate grids.

5. After students have had some practice plotting points that they have been given, practice naming points that have already been placed on the coordinate plane using a transparency copy of the Plotting Points Practice Sheet.
   - Plot a point in the first quadrant and label it "P", and ask students to determine point P's coordinates. Stress that point P is located to the right, and up, from the origin. When students identify how many spaces they have moved to the right and how many spaces they have moved up, write the coordinates next to point P on the transparency [example: (2,3)].
   - Next, label the origin point "O". Ask students to determine the coordinates (0,0). Stress that it is located zero units to the right or left, and zero units up or down. The first point was located in Quadrant I (right, up), but points on the axes, including the origin, are not really located in any quadrant.
   - Next, plot a point in the second quadrant and label it "S". Ask students to determine its coordinates, then write the coordinates next to point S [example: (-4,1)], stressing four to the left, up one.
   - Repeat this procedure and label a point in Quadrant III and Quadrant IV.

6. At this point, review the following vocabulary words and concepts presented thus far, using a transparency:
   - **number line** (horizontal only), points graphed the same as x-axis on coordinate plane
   - **positive numbers** - move to the right
   - **negative numbers** - move to the left
   - **coordinate plane** (horizontal and vertical), also called **coordinate grid**
   - **horizontal axis** - positive to the right, negative to the left (x)
   - **vertical axis** - positive up, negative down (y)
   - both axes or lines are **perpendicular** and meet at the **origin**
   - **ordered pairs** - horizontal first, vertical second (x,y)
   - **Quadrants** I, II, III, IV

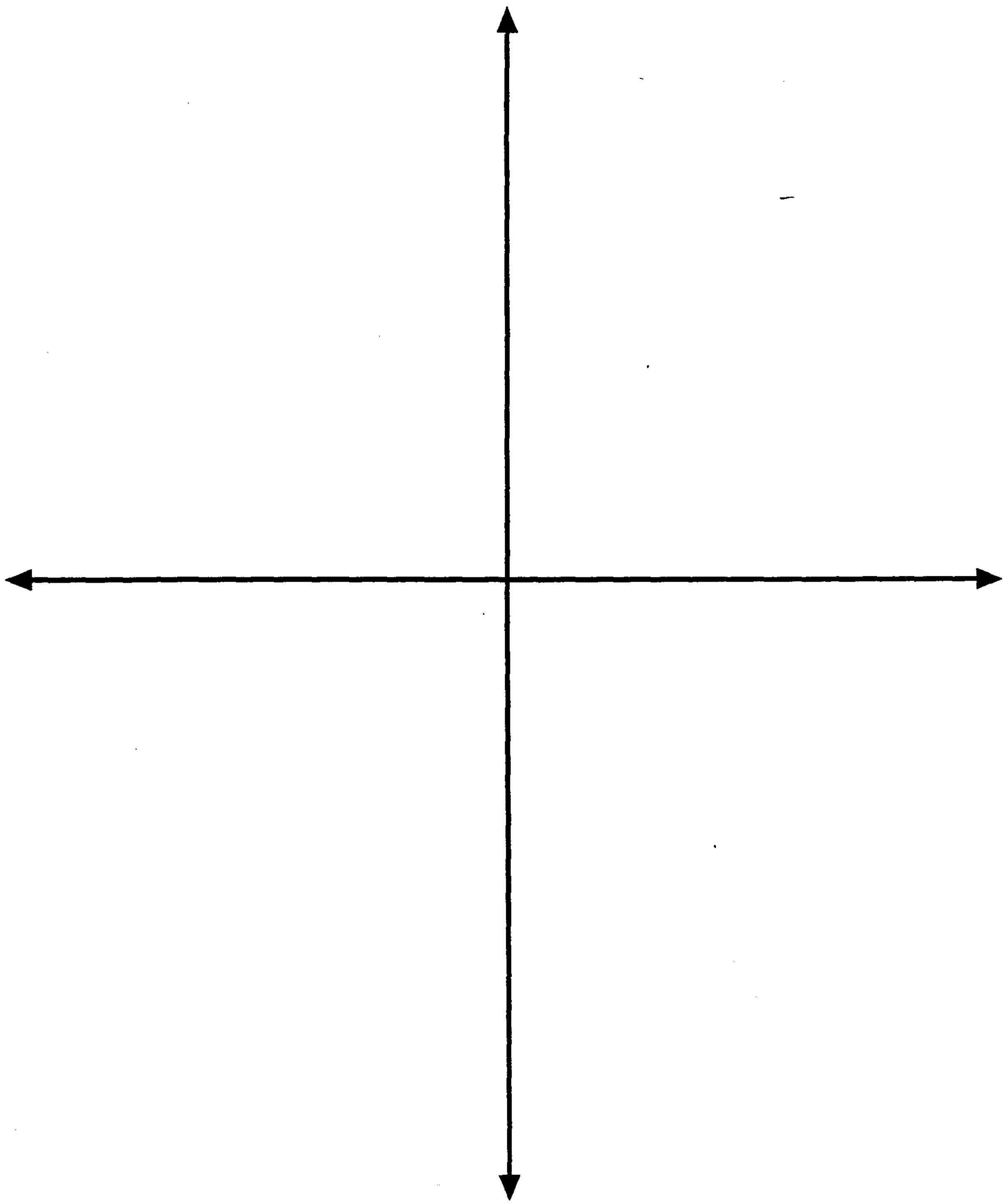
7. Distribute graph paper to student. On the graph paper ask them to draw:
   - a coordinate plane, using rulers and labeling the **x-axis** and **y-axis** (placing arrows at the ends of both axes since they represent lines)
   - label the **origin**
   - label **Quadrants I, II, III, and IV**



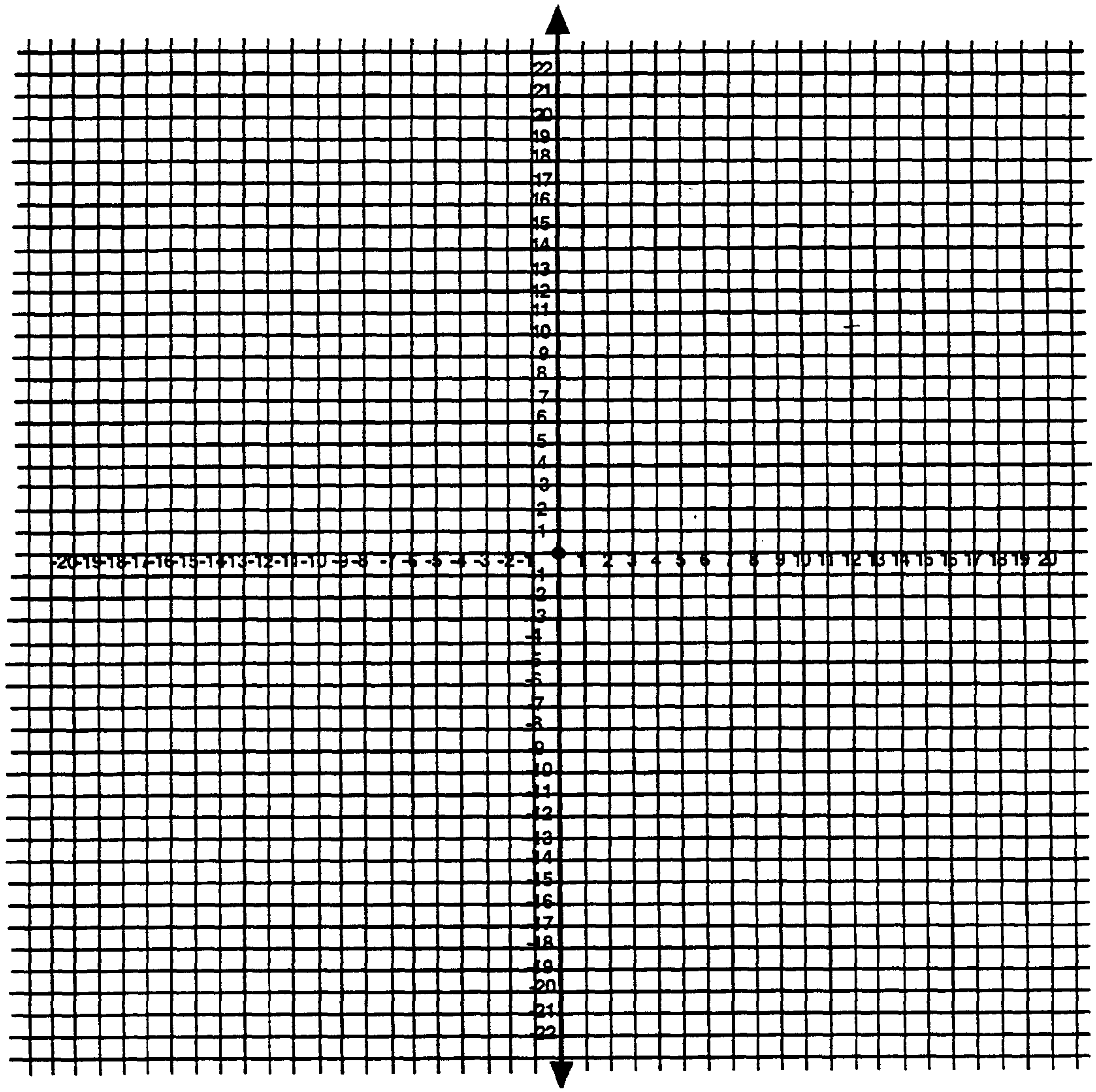
   - In Quadrant I, ask them to write "right, up."
   - In Quadrant II ask them to write , "left, up."
   - In Quadrant III have them write, "left, down."
   - Quadrant IV should be labeled "right, down."
   - Have students save their drawings for reference.

8. Distribute <u>Coordinate Plane Practice Sheet</u> for students to complete in class or at home.

Coordinate Plane



Numbered Coordinate Grid

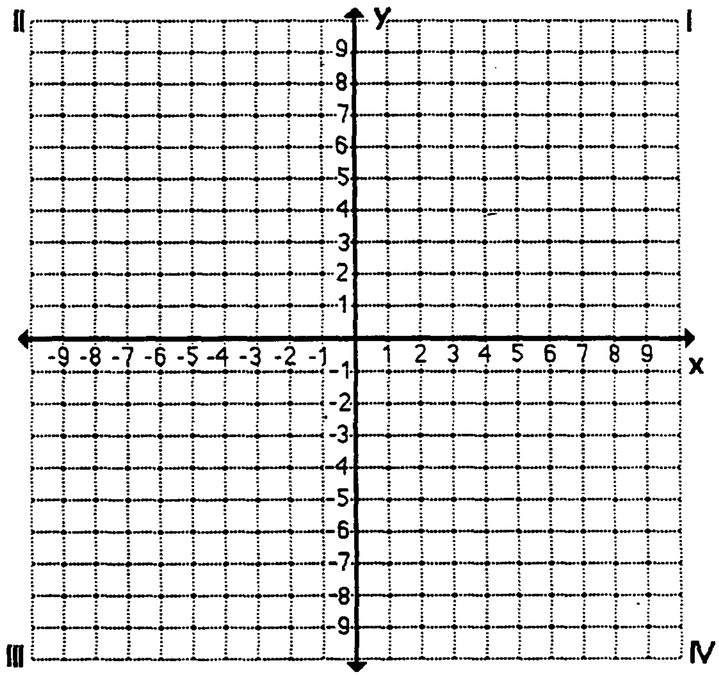


Name ______________________________
Teacher ____________________________

Plotting Points Practice Sheet

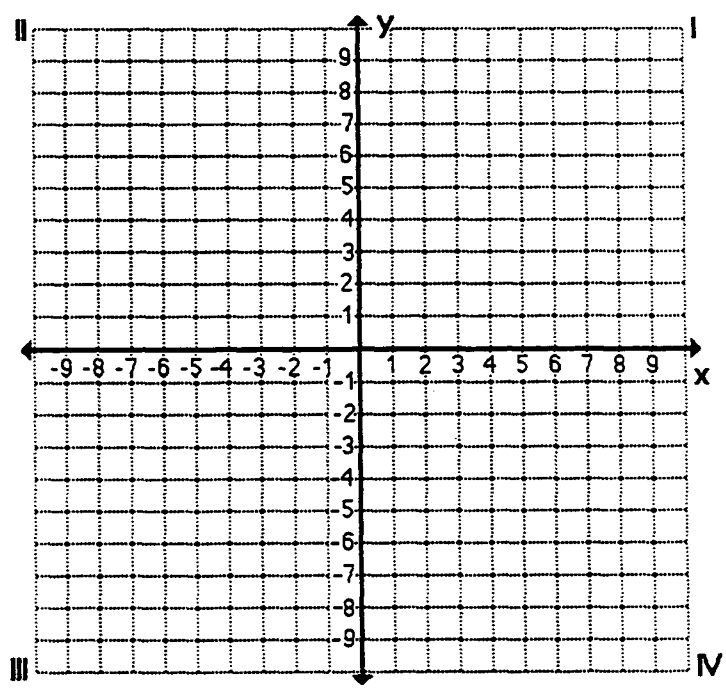
Coordinate Pairs (x, y)

x	y
8	-9
-1	1
5	7
6	-7
3	5
4	-5
2	-3
1	3
0	-1



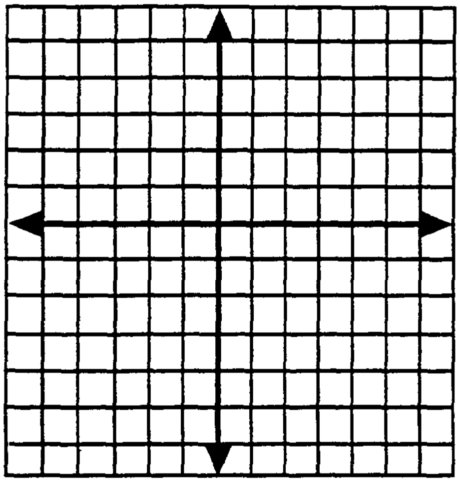
Coordinate Pairs (x, y)

x	y
1	-9
-5	-8
5	-7
1	7
-5	5
6	3
-8	2
-8	-4
0	0
8	-3



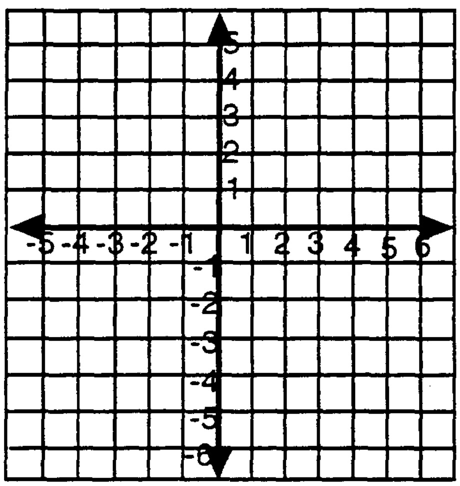
Name ____________________
Date ____________

Coordinate Plane Practice Sheet



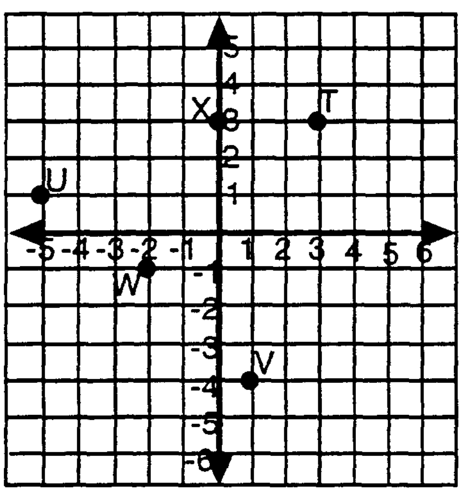
Place the following labels on the coordinate grid at the left:

x axis
y axis
Quadrant I
Quadrant II
Quadrant III
Quadrant IV
origin



Plot the following points on the coordinate grid at the left. Label each point with its corresponding letter.

A (3,5)
B (-4,2)
C (-2,-2)
D (5,-2)
E (0,0)
F (0,2)
G (-4,0)

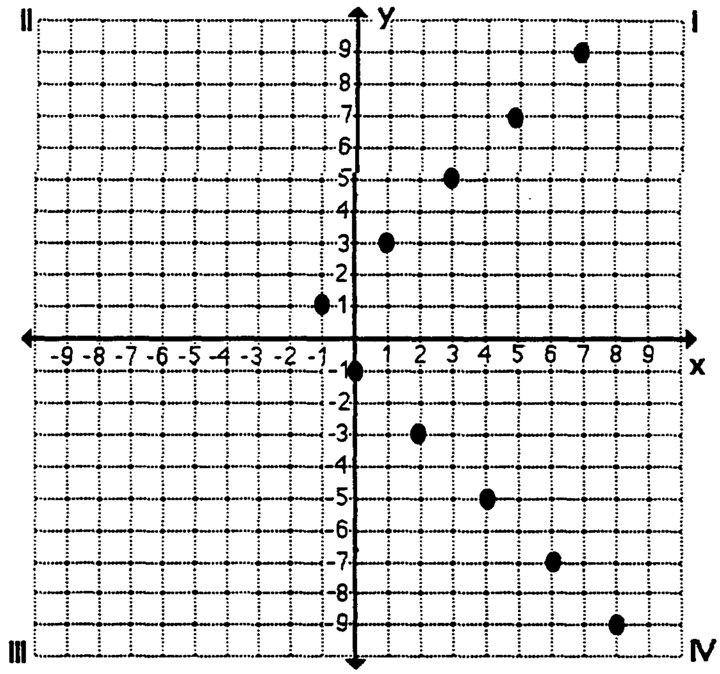


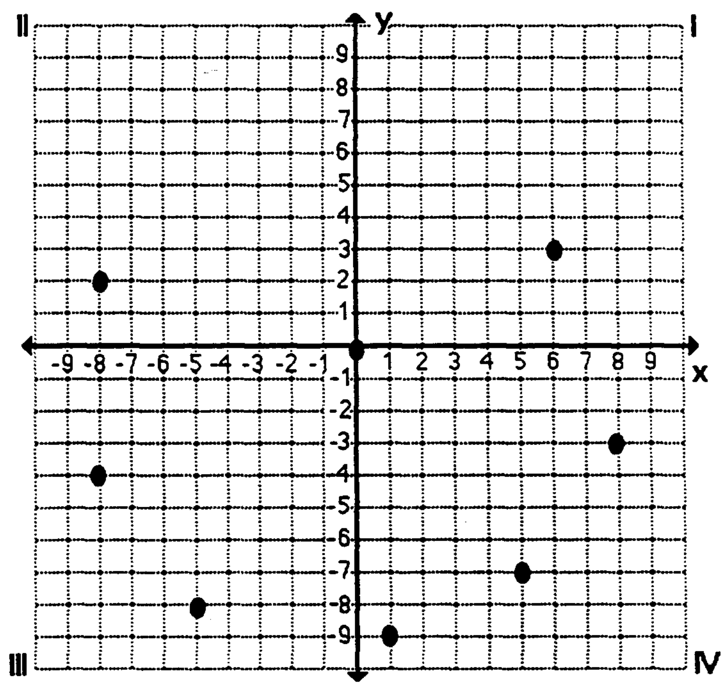
Write the ordered pair for each point that is plotted on the coordinate grid at the left.

T _____
U _____
V _____
W _____
X _____

Answer Key
Obj. 25

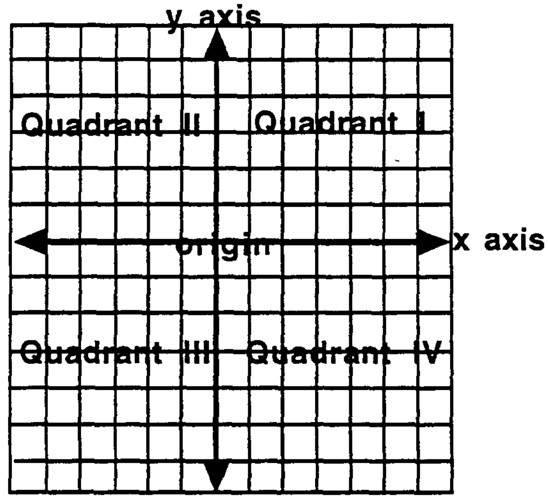
Plotting Points
Practice Sheet

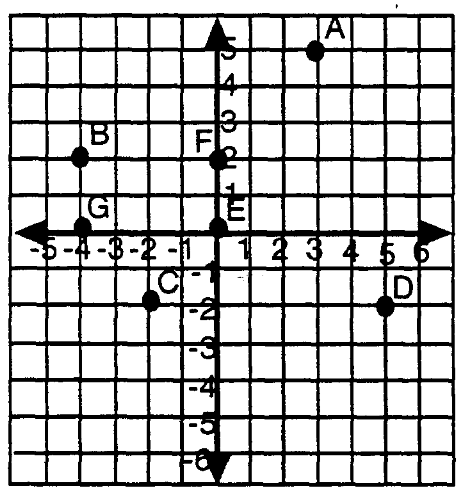




Answer Key (cont.)
Obj. 25

Coordinate Plane Practice Sheet





Write the ordered pair for each point that is plotted on the coordinate grid at the left.

T (3,3)

U (-5,1)

V (1,-4)

W (-2,-1)

X (0,3)

Objective 26: Use a coordinate graph to plot functions.

Vocabulary

function table
function rule
input
output
constant
variable
equation
graph
coordinate plane

Materials

Coordinate Grid
transparency

Graphing Functions Practice Sheet
student copies

Language Foundation

1. Discuss the meanings of the word table. (Chart, piece of furniture)
   Tell students that in math a **table** is a kind of chart that we use in statistics and graphing, not to be confused with a table that serves as furniture.

2. Explain that **graph**' can be a noun or a verb. Ask students for examples of types of graphs they have seen. (bar, circle, line, picture) Show examples of graphs from *USA Today* to show the extensive use of graphs in everyday life.

Mathematics Component

1. Introduce the lesson.
   - Tell students that **input-output tables** or **function tables** can be combined with plotting points on the **coordinate plane** to represent real situations.
   - Use the following example:

   I f Tonya babysits for a family and gets paid $3.00 per hour, we can make a function table and a coordinate graph to see how much Tonya is paid(output) depends on how many hours she works (input). Since the coordinate plane often uses x and y coordinates, we can say that Tonya's pay is represented by the **equation** $y = 3x$, where x is the number of hours worked, 3 is the hourly rate in dollars and y is the total amount earned. Connect the ideas of **function rules** explored in lesson 24 and equations.

   - Set up the following input -output or function table with the class.

<u>Input</u> (Number of hour worked)	<u>Output</u> (total amount earned)
1	3
2	6
3	9
4	12
5	15

   - From the function table, we can develop the following sets of ordered pairs and then plot these points on a coordinate plane: (1,3) (2,6) (3,9) (4,12) (5,15)
   - Use <u>Coordinate Grid</u> Transparency to draw an appropriate x and y axis. Plot the points and draw a line to represent the function.
   - Ask the students why the line is in only the first quadrant.

2. Extending the above example.
   - Change the above scenario to the following:
     What if Tonya was paid $2 for her bus fare each day, in addition to her hourly rate?
   - The equation would now be : $y = 3x + 2$ where x is the number of hours worked, 3 is the hourly rate, 2 is the bus fare, and y is the total amount earned
   - Create a function table to represent the new equation.

Input	Function Rule	Output
1	3(1) + 2	5
2	3(2) + 2	8
3	3(3) + 2	11
4	3(4) + 2	14
5	3(5) + 2	17

   - Ask students to tell you what points should be plotted: (1,5) (2,8) (3,11) (4,14) (5,17)

- Use Coordinate Grid Transparency to draw an appropriate x and y axis. Plot the points and draw a line to represent the function.
- Ask the students to predict how much Tonya would earn if she worked 3.5 hours.

3. Another example
   - Present the following example to the class:
     The Guitar Club runs a car wash every Saturday at as local gas station to raise money for a field trip. The gas station charges the club $20 each day for the use of the water and hoses. Club members was cars for $5 each. Determine the amount of profit the club can make on a given Saturday.
   - Work with the students to determine the equation which represents the function. y = 5x - 20, where 5 is the amount charged for each car, x is the number of cars washed, 20 is the amount paid to the station and y is the amount of profit or loss for that day.

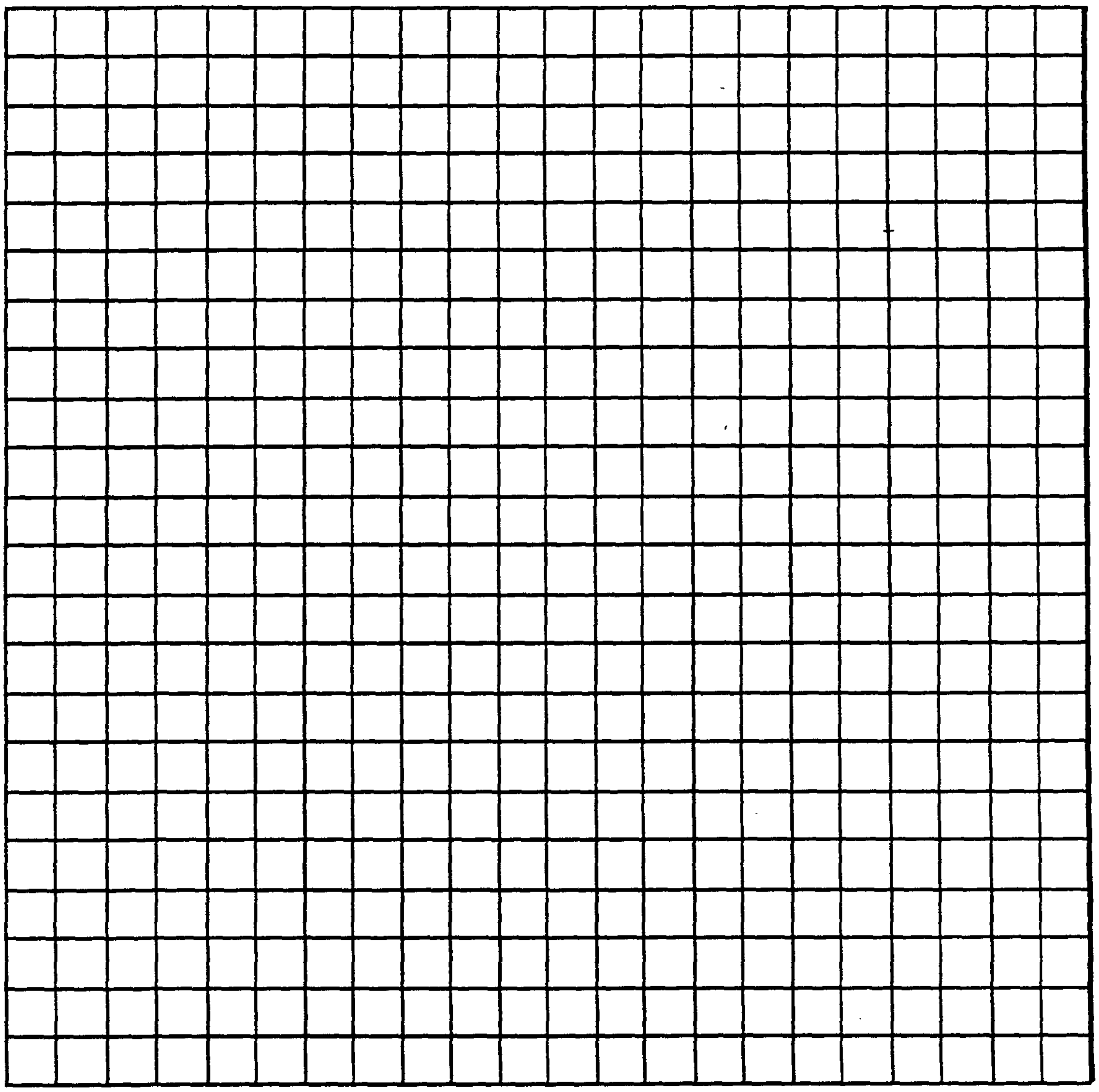
   - Ask students to work in groups to create an appropriate function table. Below is an example:

Input	Function Rule	Output
0	5(0) - 20	-20
1	5(1) - 20	-15
5	5(5) - 20	5
10	5(10) - 20	30
15	5(15) - 20	55
20	5(20) - 20	80

   - Use Coordinate Grid Transparency to draw an appropriate x and y axis. Plot the points and draw a line to represent the function.
   - Ask the students the following questions:
     •What does the point (0.-20) mean? (no customers, $20 loss)
     •What does the point (20,80) mean? (20 customers, $80 profit)
     •How many cars must be washed to "break even"? (no profit, no loss = 4 cars)
   - Ask students to think of additional examples of real life situations which can be represented by functions and graphs.

4. Individual practice.
   - Distribute Graphing Functions Practice Sheet

Coordinate Grid



Name ____________________________

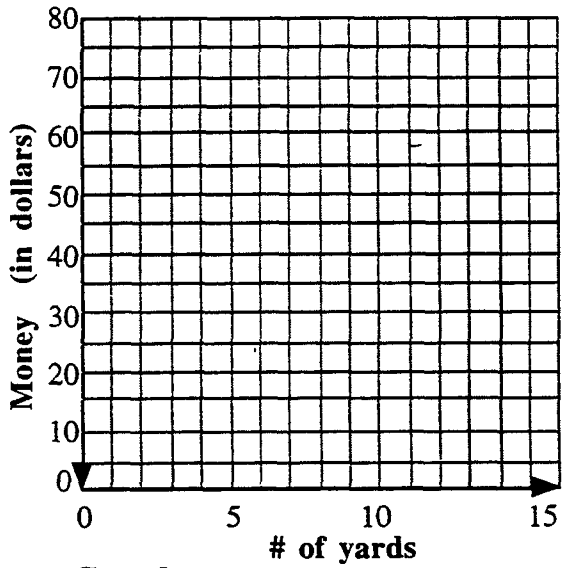
Date _______________

Graphing Functions Practice Sheet

1. Bob earns $8 for every lawn he mows. Complete the table and the graph.

Table

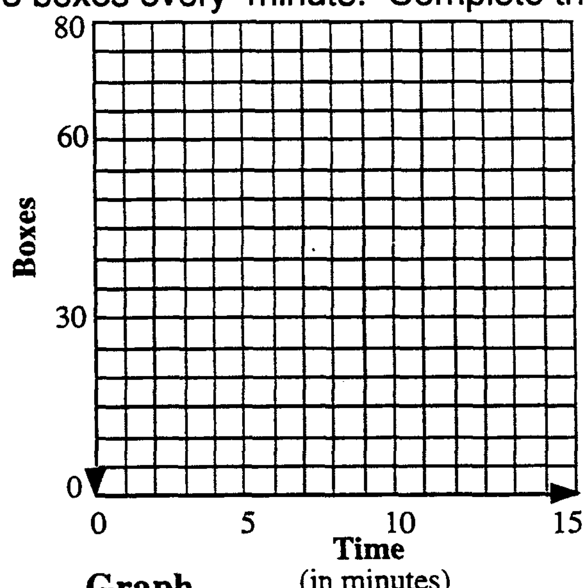
Input/Time	Function Rule	Output/ Money
1	8(1)	8
		24
5		40
8		
		80



Graph

2. A box-making machine makes 5 boxes every minute. Complete the table and the graph.

Time	Function Rule	Boxes
2		
		15
		25
		60
16		

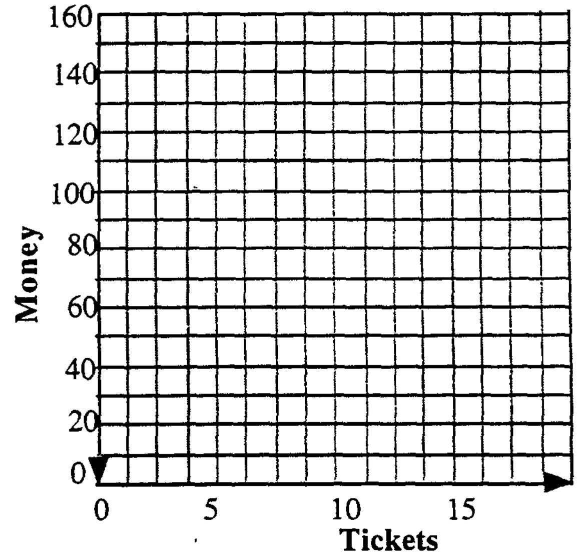


Graph

3. Cheap Tickets Company sells concert tickets for $15 a piece plus a handling charge of $5 per order. Complete the table and the graph.

Table

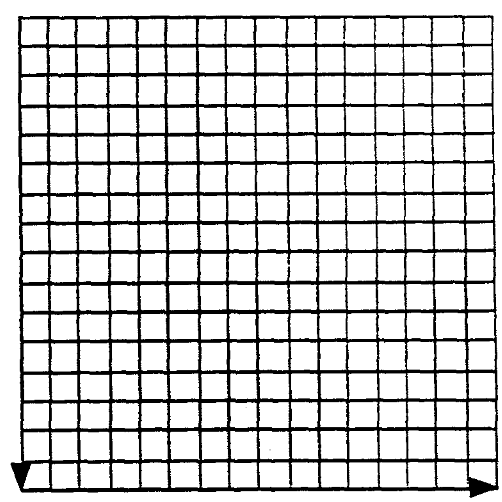
Tickets	Function Rule	Money
1	15(1) + 5	20
2		
5		
8		
10		



Graph

4. Create a table and graph to represent the following function:
Jorge can type 35 words per minute.

Table



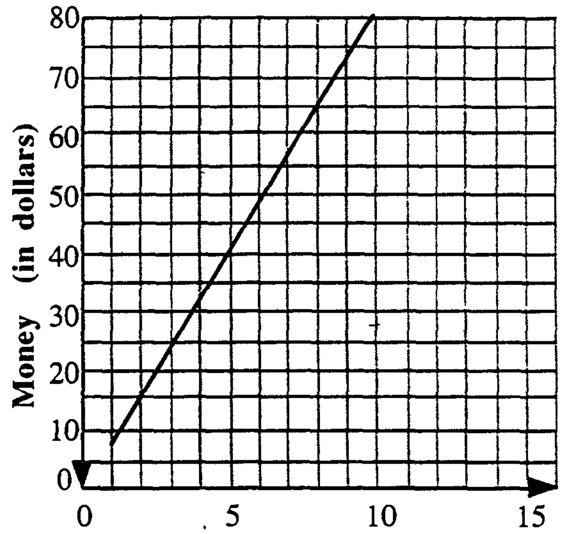
Graph

Answer Key
Obj. 26

1.

Table

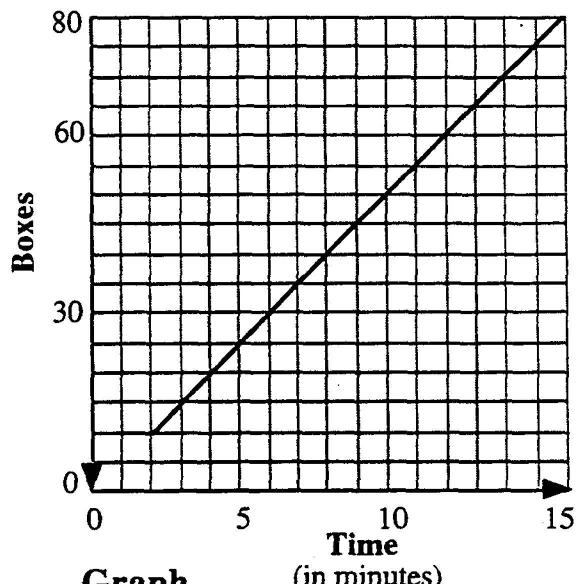
Input/Time	Function Rule	Output/ Money
1	8(1)	8
3	8(3)	24
5	8(5)	40
8	8(8)	64
10	8(10)	80



Graph

2.

Time	Function Rule	Boxes
2	2(5)	10
3	3(5)	15
5	5(5)	25
12	12(5)	60
16	16(5)	80

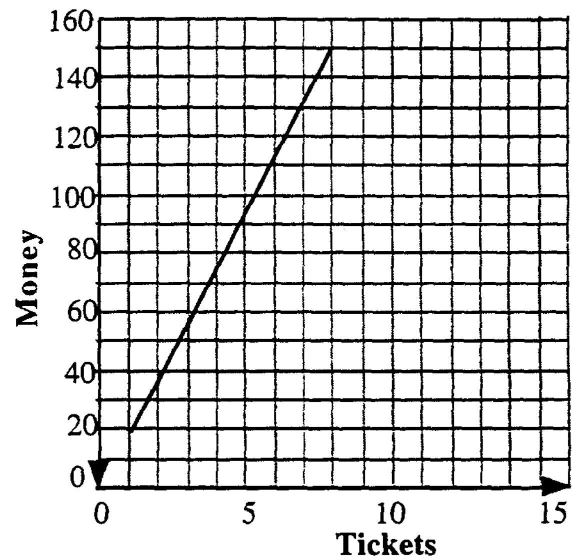


Graph

3.

Table

Tickets	Function Rule	Money
1	15(1) + 5	20
2	15(2) + 5	35
5	15(5) + 5	80
8	15(8) + 5	125
10	15(10) + 5	155

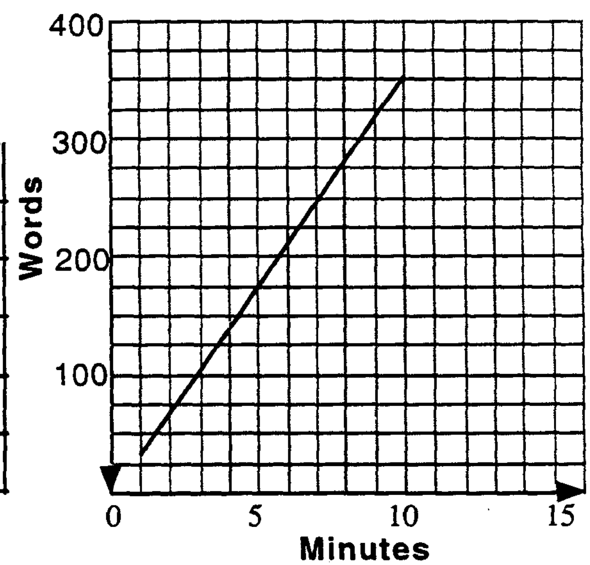


Graph

4. Answers may vary slightly

Table

Minutes	Function Rule	Words
1	35(1)	35
2	35(2)	70
5	35(5)	175
10	35(10)	350
15	35(15)	525



Graph

Ratio, Proportion and Percent