# Developing Target Second Language Skills Through Problem-Solving Activities in Mathematics[1]

Luis Radford
*Laurentian University*
Joan Netten
*Memorial University*
Georges Duquette
*Laurentian University*

This article provides classroom teachers with clear directions and specific classroom strategies which will enable students to develop target second language skills through the teaching of mathematics. Students learn a number of problem-solving strategies and they apply these strategies to specific, real context situations using arithmetic and algebraic formulas. Students are encouraged to co-operate and communicate in finding solutions to the mathematical problems they face and thus develop communications skills related to the problem-solving activities.

## Introduction

The purpose of this article is to provide teachers with clear directions and specific classroom strategies which will enable students to develop target second language skills through the teaching of mathematics, while at the same time learn the necessary mathematics concepts.

In the past, both mathematics and second language teachers used to teach these subjects separately in formal classroom settings. Considerable emphasis was placed on memorization of paradigms. Today we understand that to be effective in developing competence in both mathematics and a second language, emphasis must be placed on process rather than product-oriented teaching strategies. In addition, current teaching strategies for both areas require the successful application of the subject specific knowledge to authentic situations.

In the last few decades, problem-solving has been considered an important element in teaching mathematics (Radford, 1996a, 1996b). Students must be able to apply arithmetic and algebraic formulas to real situations. In second language learning, research has indicated that to develop competence in a target language, the language must be used in natural, real-life situations (Fishman, l989, 1968; Krashen, 1987; Leblanc, 1990; Mollica, 1996). These goals impact on the teaching strategies used in both subject areas.

Furthermore, the learning of mathematics and of a second language are by nature complimentary activities. Second language learning is in itself a problem-solving activity. Students are constantly required to solve problems about the way the target language works in order to be able to understand and produce messages. This experience enables students who develop competence in a second language to demonstrate enhanced problem-solving skills (Duquette, 1995).

Another similarity in the cognitive processes involved in the learning of mathematics and of a second language is that of the formulation and testing of hypotheses. In the learning of a second language, students create hypotheses about the way in which the target language works from the input data to which they are exposed. These hypotheses are tested when learners produce utterances in the second language. The reactions to their target language production constitute feedback which enables them to revise and refine their hypotheses about the way in which the language works (Lightbown & Spada, 1994).

In the learning of mathematics, hypothetical reasoning is important because it is one of the bases in the construction of formulas as models of concrete or mathematical situations (Radford, 1996c). Indeed, in order to describe the relationship between two or more variables, the students have to form a hypothesis about the variables and then test it.

In the bilingual or immersion classroom, there is another important connection between the learning of mathematics and of the target language. Academic achievement in immersion classrooms is correlated with competence in the target language (Genesee, 1987; Netten & Spain, 1983). In addition, some research has suggested that the learning of mathematics, in particular, is more related to target language achievement in the immersion classroom than is the case for monolingual classrooms (Netten & Spain, 1980). Thus in order for the learning of mathematics to be most effective, the learning of the related target language must also be effective.

Research has shown that learning a second language occurs in a remarkably similar fashion to that of first language learning (Hawkins & Towell, 1992). The brain appears, to a certain extent, to be programmed to learn language (Pinker, 1994). Linguistic features are extrapolated subconsciously from the target language input while the learner is attending to the message being conveyed. This process is the basis for the implementation of bilingual programs where students learn the target language by studying academic content in the second language. The learning of mathematics, for example, becomes the message on which the student concentrates while subconsciously assimilating data and making hypotheses about the lexical, morphological, and syntactical properties of the target language. This subconscious processing of target language data enables students to learn, so to speak, much more than can be taught.

In order for this process to operate as effectively as possible, target language use must be as authentic as possible. There must be real communicative intent for the interchange to be valuable as a second language learning task. The teaching of mathematics in the target language may ensure this intent, as it is a requirement that students learn mathematics. Furthermore, for the second language learning processes to operate effectively, the student must be intellectually involved in the communicative activity. Therefore, the mathematics content must be presented in such a way as to involve the student in understanding the processes rather than in memorizing formulaic solutions. Motivational or psychological involvement in the communicative activity is also important, as the limbic aspects of the brain must be engaged for effective second language learning to occur (Paradis, 1994). Marginal involvement appears to reduce linguistic processing. Social interaction is also an important factor to second language learning (Germain, 1993). Consequently, activities which encourage discussion and interaction among learners are an essential part of immersion pedagogy. Teachers should encourage students to manipulate the target language in order to test their knowledge and hypotheses about the linguistic aspects of the second language (Lentz, Lyster, Netten, & Tardif, 1994). Teachers must encourage students to negotiate both target language meaning and form with each other and the teacher in order to improve their understanding and control of the target language (Ellis, 1986; Lyster, 1994). The use of pair and group work is appropriate to achieve these ends (Artzt & Newman, 1990).

Research has also shown that the learning tasks undertaken by students must possess certain characteristics in order to be most effective. Problem-solving activities are the best type of group work to ensure language manipulation as well as intellectual and motivational involvement (Long and Porter, 1985; Paulston, 1995). Thus, engaging students in interesting activities in which the focus is on sharing experiences, communicating, understanding, and internalizing the mathematics content responds directly to all the criteria mentioned above.

Lastly, in a second language learning context where the emphasis is on communication rather than on analysis of the target language, a hierarchical sequence of language learning tasks needs to be developed which is not directly related to the linguistic difficulty of the target language. Thus learning tasks must be developed in such a way as to enable students to move from relatively simple linguistic requirements to more complex ones. The use of a hierarchical sequence is also recommended in the teaching of problem-solving skills in mathematics, guiding students from a trial and error method where numbers are chosen in an unsystematic way to an approach that requires abstract or hypothetical reasoning. This progression integrates both a linguistic and a cognitive hierarchy which can enhance learning in both areas.

## Helping Students to Construct and Link Problem-Solving Models

In order to give an idea of how our approach can be implemented in the classroom, we shall now present an example. For our discussion, we will use the following problem[2] .

> "Jane Kimble, a grocer, checked her supply of milk and counted 80 containers of milk. Some were 2 L cartons and others were 3 L bags. Altogether there was a total of 220 L of milk. How many of the containers were 3 L bags?"
>
> (Target problem)

This example belongs to a wide group of problems that we can write using algebra with two unknowns in the form of a linear system of equations: $x + y = a; bx + cy = d$. However, the methods of algebra with two unknowns are introduced usually in grade 10. How can we help students in the intermediate grades (i.e., 7, 8, and 9) to construct an algebraic problem-solving model without using two unknowns? In what follows we shall suggest a possible didactic path which shows how we can help students in grades 7, 8, or 9 to construct a sequence of problem solving models moving from a model based on concrete thinking to another one based on symbolic thinking. This process requires four stages of development which we will outline below.

The following diagram illustrates the path that we will follow in our examples. One moves from trial and error thinking to symbolic thinking. Each level develops problem-solving models that are used in the next level, but in a more complex way.

Our starting point is that, often, when students face a new problem they do not have an existing model or resolution to fall back on. Generally, the construction of a new mathematics model which would allow them to solve the problem proves to be a very difficult task. The main idea of our approach is that it is preferable to advance towards the process of problem-solving gradually. Instead of constructing a problem-solving procedure for the target problem, it would be better to first solve other simple problems related to it. The problem-solving procedures for the simplified problems will then allow the students to evolve gradually towards a more complex problem-solving method which will, in turn, solve the target problem.

At the level of language and communication, it is important that the problem be provided in the target language in such a way that every student understands the given problem. The target language input that is provided to students needs to be comprehensible for the subconscious processing of language data to operate effectively (Krashen,1987). The terms of reference, names and expressions to describe the procedures should be gradually introduced, used and reused by the students in relation to the target problem to assist them in gaining automatic control of these target language items. This is another reason for making sure that students progress from the simple to the more complex.

We shall begin by considering a *simplified* mathematical version of the target problem.

> “Jane Kimble, a grocer, checked her supply of milk and counted 30 containers of milk. Some were 2 L cartons and others were 3 L bags. Altogether there was a total of 83 L of milk. How many of the containers were 3 L bags?”
>
> (Simplified problem)

Figure 1. Changes associated with the models (the processes used in each problem-solving model are described below).

| Type of model | Changes from one model to the next |
|---|---|
| Algebraic-symbolic model | |
| ^ | The student faces conceptual changes when moving from arithmetic abstraction to an algebraic symbolic abstraction |
| Arithmetic model | |
| ^ | Abstract frontier (manipulatives are left behind) |
| Manipulative-based model | |
| ^ | The student faces conceptual changes in the thought process when moving into a more organized way of thinking |
| Trial and error model | |

## The Trial and Error Method

The trial and error method is a simple method which has the advantage of requiring knowledge of only simple arithmetic concepts. It has the disadvantage that, to solve other similar problems, it can take a long time to find the answer, depending on the complexity of the numbers involved. In this method we simply repeat the same procedure with different quantities until we obtain the correct answer. Neither abstract thought nor hypothetical reasoning plays a role. At this stage, students have the opportunity to become familiar with the target language lexicon used in the problem. The trial and error process may be conducted in small groups or pairs, giving all students an opportunity to use these terms repeatedly for the purposes of achieving problem resolution. Because of the relative simplicity of the problem, discussion among students can be sustained, and students have the opportunity to manipulate the target language.

## A Manipulative-Based Method

When using the manipulative-based method, we begin by making a supposition or a hypothesis. For example, suppose the number of bags is the same as the number of cartons. We then make calculations to test our hypothesis, and generate new data which allows us to revise it. Unlike the trial and error method, we are now using hypothetical reasoning; however, with the use of manipulative aids, the students have great power in making this transition as manipulatives assist concept formation. Thus we place 15 cartons or plastic boxes containing 2 objects each on a table in front of each group of students. Beside this table there is another table on which we place 15 bags, each containing 3 objects. At this point we ask the students to calculate the total number of litres; they will find the answer to be 75 litres. Instead of replacing the number of cartons and the number of bags, as done in the trial and error method, we will think in terms of how much we failed in our starting assumption. We have 75 litres, but should have 83 litres; we missed the correct answer by 8 litres. This means that we must add some bags or cartons. Helped by manipulatives, students can see that if they add 1 bag, they must remove a carton in order to keep the number of containers equal to 30. Changing some data in the original problem gives students practice in using the new arithmetic-manipulative-based model as well as manipulating the key words and structures associated with these concepts. Small groups or pairs of students should then be encouraged to develop other similar problems, using manipulatives, in order to internalize both the mathematical concepts and the target language data.

## An Arithmetic-Abstract Problem-Solving Model

An arithmetic-abstract problem solving model continues to use hypothetical reasoning, but it is based on using only arithmetic concepts without using manipulatives, and thus moves the student into abstract cognitive processes for solving mathematical problems. In order to motivate students to use this method, we need to confront the students with a problem whose solution, using the manipulative method is not easy. Therefore, we change the data in the problem in such a way that quantities are so large that it becomes tedious to solve via manipulatives. This brings us back to our target problem. We should now encourage students to solve the problem by manipulating ideas instead of concrete objects.

Suppose that the number of milk bags is the same as the number of milk cartons. We then have 40 of each. Calculate the number of litres. In the cartons, we have $40 \times 2 = 80$ litres; in the bags we have $40 \times 3 = 120$ litres. We then find that we have $80 + 120 = 200$ litres of milk, but the problem requires a total of 220 litres. We are 20 litres short of the total. So our assumption that we have the same number of bags and cartons is wrong. We can then conclude that we need more bags than cartons. If we add one milk bag, we have to remove one milk carton (in order to keep the number of cartons and bags equal to 80), and we gain one litre of milk. But we need 20 litres, so we have to remove 20 cartons and replace them with 20 bags. So it remains, $40 - 20 = 20$ cartons and $40 + 20 = 60$ bags of milk.

This method of problem-solving needs to be internalized by the students. Students in groups or pairs can then be asked to propose other problems changing some of the data in the last problem[3]. The teacher and/or students can also propose other problems of the form $x + y = a; bx + cy = d$. They can also discuss and construct problems of the type $x \pm y = a; bx \pm cy = d$. The students will realize that the new problem-solving model has the advantage of allowing them to approach a relatively wide family of problems in a direct way; no trial and error is required, nor do they require manipulatives. The development of other examples of the same type of problem enables students to internalize the related target language items, as well as assists them in making more automatic the cognitive processes associated with their use.

## An Algebraic Problem-Solving Model

Instead of beginning with a numerical solution (dividing the number of containers between cartons and bags and then deducing the number we have to add to the first and remove from the second) we can suppose (hypothetical reasoning) that we know this number already. Let $x$ be this number. The exact number of cartons is not 40 (half of 80) but $40 - x$. In the same way the exact number of bags is not 40, but $40 + x$. Given that a carton holds 2 litres, and a bag holds 3 litres, we can then express the total number of litres as $2(40 - x) + 3(40 + x)$, where this quantity must equal 220 litres. We get the equation: $2(40 - x) + 3(40 + x) = 220$. It is to be noted that this is one type of equation usually taught in grade 9 in Ontario.

This algebraic problem-solving model needs to be internalized by students. In order to achieve this, the students can propose other similar problems to be solved by the new model, as was done in the previous problem-solving models.

## Concluding Remarks for Teaching

The main idea of our approach consists in trying to construct a hierarchical progressive sequence of learning activities for mathematics and second language acquisition. In mathematics, the sequence of problem-solving models, $M1 \rightarrow M2 \rightarrow \ldots \rightarrow Mn$, starts from a simple model, *M1*, and arrives at a more complex model, *Mn*. The choice of the models that precede the final model, *Mn*, depends on the *Mn* model itself and on the knowledge of the students. We aimed at a model capable of solving the grocer's problem using the algebraic knowledge presented in the intermediate grades. Therefore, we should use algebra with only one unknown. In order to accomplish this task, we must use a progression in which the next model is constructed in the hierarchical sequence, based on the previous model. We can note at this point that the grocer's problem can be solved by other methods. One does not need to divide the number of containers in half. One can select any $x$ value between 0 and the number of containers, that is 80. Although other methods could solve the grocer's problem, the method of dividing the number of containers in two naturally leads to the algebraic problem-solving model at which we aimed. That is why the "dividing in two method" plays such a central role in our didactic sequence.

The previous principles can be applied to many situations. The success of our approach in a mathematics class will depend upon the teacher's ability to choose appropriate simplification of the problem and to obtain a suitable hierarchical sequence of linked models. It will be particularly necessary that the use of concrete models based on manipulatives be coherent with the abstract problem-solving models.

For second language learning, it is imperative that a coherent sequence of hierarchical activities be developed in order that a progressive sequence may naturally ensue in the linguistic demands in the target language made upon the students. In addition, it is important that activities be so organized that students work together in groups to interact and manipulate the target language. The integration of the cognitive processes of comprehending and internalizing the mathematical principles and encoding this knowledge in the target language will create a situation in which both mathematics and the second language are learned more effectively. It may also be hypothesized that if skills in formulating and testing hypotheses, as well as problem-solving, are enhanced for mathematics, the subconscious use of these techniques at the same time in second language acquisition may also improve these skills for target language learning.

## Conclusion

New mathematics materials for use in the second language classroom are currently being produced. In particular, these materials place examples of mathematics problem-solving in situational contexts which encourage student interaction and participation. One such example of this type of material is the unit entitled "L'affaire des biscuits" (the business of crackers) developed in Canada and published by Chenelière-McGraw Hill (1996). In this unit, students are required to place themselves in the position of an entrepreneur who is setting up a bakery shop. Numerous problems require the students to calculate the cost of producing different types of confections as well as the related aspect of sale price in order to cover costs and make a profit.

Besides considering problem-solving as a central axe in the curriculum of mathematics that leads to an active interaction and participation of the students in the classroom, communication has been increasingly recognized as a key point in the learning of mathematics (e.g., Provincial Standards of Mathematics of the Ontario Ministry of Education and Training, 1993). This requires us to change our perception of the teaching of mathematics and to adopt a more vivid perspective in which communication among students and between students and teachers acquires a greater importance. By the same token, within this new perspective, mathematics, we believe, appears as an interesting tool with which to develop second language skills. Indeed, given that students are encouraged to communicate and to co-operate in the finding of solutions to the problems embedded in the classroom activities, they are forced to use the second language throughout the communication process. Doing so, the learning of a second language, as well as the learning of mathematics itself, acquires a new, practical, and challenging sense.

In closing, we see that the process of developing skills in mathematics provides teachers with an excellent opportunity to stimulate and encourage the development of skills in the target second language. It is becoming increasingly clear that language teachers should collaborate more closely on such projects with other subject matter teachers, especially at the secondary level where subject areas are compartmentalized in separate departments. Studies that focus on findings of such collaborative projects should prove to be insightful in helping us understand how language for communicative purposes is intricately linked with the development of other skills.

1. This article is supported by two Canadian research grants. The first is a grant from *“ Le Fond de recherche de l’Université Laurentienne (FRUL)”*. The second comes from the Department of Education of the Government of Newfoundland and Labrador.

2. This problem was taken from “*Ontario Provincial Mathematics Benchmarks*” (Ministry of Education and Training, Validation Draft, September, 1992, p. 39). *Inquiry and Problem-Solving Domain*, Grade 7 to Grade 9. The problem is supposed to be solved by all the students.

3. An interesting problem is the following: “Jane Kimble checks her milk inventory. There are 80 containers of milk: Some are 2 L cartons and the others are 3 L bags. There is a total of 195 L of milk. What, then, must be the number of 3 L bags?” Let’s suppose, as in the previous problem, that we have 40 milk bags and 40 milk cartons. We find that we have 200 litres instead of 195 litres. Thus, now we have 5 litres *more*, so we have to replace bags by cartons.

## References

Artzt, A. F., & Newman, C. M. (1990). *How to use cooperative learning in the mathematical class*. Virginia: National Council of Teachers on Mathematics.

Duquette, G. (1995). *Second language practice: Classroom strategies for developing communicative competence.* Clevedon, England: Multilingual Matters.

Ellis, R. (1986). *Understanding second language acquisition.* Oxford: Oxford University Press.

Fishman, J. (1989). *Language and ethnicity in minority sociolinguistic perspective.* Clevedon, England: Multilingual Matters.

Fishman, J. (1968). *Readings in the sociology of language.* The Hague: Mouton & Co.

Genesee, F. (1987). *Learning through two languages.* Rowley, MA: Newbury House.

Germain, C. (1993). *Le point sur la didactique des langues (2e édition).* Anjou, Québec: Centre éducatif et culturel.

Hawkins, R. & Towell, R. (1992). Second language acquisition research and the acquisition of French. *French Language Studies, 2, 97-121*.

Krashen, S. D. (1987). *Principles and practices in second language acquisition.* Englewood Cliffs: Prentice Hall.

Leblanc, R. (1990). *National core French study: A synthesis.* Ottawa: CASLT.

Lentz, F., Lyster, R., Netten, J., & Tardif, C. (1994). Vers une pédagogie d'immersions. *Le journal d'immersion, 18(1),* 15-27.

Lightbown, P., & Spada, N. (1994). *How languages are learned*. Oxford: Oxford University Press.

Long, M. H., & Porter, P. A. (1985). Group work, interlanguage talk and second language acquisition. *TESOL Quarterly, 19,* 207-28.

Lyster, R. (1994). La négociation de la forme: stratégie analytique en classe d'immersion. *The Canadian Modern Language Review/La Revue Canadienne des langues vivantes, 50 (3)*, 1-20.

Mollica, A. (Ed.). (1996). *Teaching languages.* Welland: Éditions Soleil Publications.

Netten, J., & Spain, W.H. (1983). *An evaluation of the late immersion project in bilingual education, 1981-82: A report prepared for the Avalon Consolidated School Board.* Avalon Consolidated School Board.

Netten, J., & Spain, W.H. (1980). *An evaluation study of the Avalon Consolidated School Board late immersion project in bilingual education: 1981-82*. St. John's, Newfoundland: Institute of Educational Research and Development, Memorial University of Newfoundland and the Avalon Consolidated School Board.

Ontario Ministry of Education and Training. (1992). *Ontario provincial mathematics benchmarks.* Validation Draft. Toronto, ON: Queen's Printer.

Ontario Ministry of Education and Training. (1993). *Provincial standards: Mathematics - Grades 3, 6, and 9*. Toronto, ON: Queen's Printer.

Paradis, M. (1994). Neurolinguistic aspects of implicit and explicit memory: Implications for bilingualism and SLA. In R. Ellis (Ed.), *Implicit and explicit learning of languages* (pp. 393-419). London, UK: Academic Press.

Paulston, C.B. (1995). Magic or chaos: Taskbased group work. In G. Duquette's (Ed.), *Second language practice: Classroom strategies for developing communicative competence.* Clevedon, England: Multilingual Matters.

Pinker, S. (1994). *The language instinct.* New York: William Morrow and Company.

Radford, L. (1996a). La résolution de problèmes: Comprendre puis résoudre? *Bulletin AMQ, 36 (3),* 19-30.

Radford, L. (1996b). La résolution de problèmes dans la classe de mathématiques. *Revue du Nouvel Ontario, 18,* 11-34.

Radford, L. (1996c). Some reflections on teaching algebra through generalization. In N. Bednarz, C. Kieran & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 107-111). Dordrecht/Boston/London: Kluwer.