Fradd, S. H. & O. Lee (Eds.).(1998) *Creating Florida's Multilingual Global Work Force:Educational Policies and Practices for Students Learning English as a New Language*. Tallahassee, FL: Florida Department of Education.

School Mathematics for Language Enriched Pupils

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To the question "What mathematics should a language enriched pupil (LEP) learn?" the short answer is "the same that is expected of any other student." Some observers believe that, because these students are busy acquiring English language skills, they cannot or should not learn more ambitious school mathematics. Rather, these people believe, these students should limit the learning of mathematics to basic skills, such as paper-and-pencil computations and solving *very simple* problems -- what has come to be known as "check book math."

Nothing could be further from the truth. There is an ever-growing body of research literature that shows language enriched students (as is true for all students) can learn and achieve more ambitious mathematics than they are given credit for. Indeed, one reason language enriched students have had difficulty with mathematics is that current-day practices fail to support -- and in some cases, actively interfere with -- their learning.

This manuscript takes the position that the mathematics education of Florida's (and, indeed, the nation's) language enriched students needs to target more ambitious goals than is currently true. There are four reasons for this position. First, anyone leaving school knowing just checkbook mathematics will not have a checkbook worth doing mathematics in. Second, the democracy on which this country and state are founded requires that everyone can use mathematics to understand the challenges that face society. Third, the state's and larger nation's international competitiveness will be enhanced if the state can educate a work force who are scientifically literate, know two (or more) languages, and are comfortable in multiple cultural settings. Fourth, schools should be places that develop their students' knowledge and skills as much as possible. Schools should not be places where the possibilities of a person's future are truncated by institutionally set goals.

What language enriched students need are opportunities to learn mathematics. No one can learn what is not taught to them. Language enriched students are too often relegated to low-content or low-track mathematics classrooms. These students need adaptations of mathematics instruction that allow them to take part in the classroom. Many students need explanations using easy-to-understand language. The ideas can be abstract and complex, but how these ideas are explained in English needs to be clear and simplified. Language enriched students and their teachers need modifications in assessment, so that students can show what they know and teachers (as well as other observers) can determine when an incomplete answer is due to the student's making a language-based error or making a mathematical mistake. This chapter is organized around the three areas just outlined: curriculum (the content that students are expected to learn), instruction (how they are taught), and assessment (how decisions are made about what students know).

One final introductory note: There is no monolithic, "average" language enriched pupil in mathematics.

These pupils vary in their command of languages, in how they have been socialized to behave at home, in how they show respect, in their school-related and informal learning experiences, and in their cultural beliefs about going to school. Not only do language enriched students vary in how much of this or that experience they have had (for instance, in *how much* their parents may have read to them or played number games with them), they also vary in the *quality* of those experiences (for instance, in exactly *what* stories were told and read to them or games they played).

Curricular Considerations

The mathematics curriculum should be targeted toward realistic problem solving, including content that is needed in order to function in an increasingly complex society and support the development of students' reasoning about mathematics.

Problem Solving

The main reason for learning mathematics is to be able to use it to solve problems in the world of everyday life, in the sciences, and in the work place. Hence, the curriculum -- the actual content that students encounter in school -- should be targeted towards mathematical problem solving and applications.

A common assumption is that, before people can solve problems, they must have mastered the basic skills that are prerequisite to solving the problem(s) in question. In fact, children enter kindergarten and first grade with sophisticated understandings and basic skills that enable them to solve word problems for which they are believed to lack the prerequisite skills. Most entering first graders can solve the following problem by counting objects or by counting their fingers:

> Mary has fifteen toy cars in all. Seven of her cars are green and the rest are blue. How many of Mary's toy cars are blue?

The mathematics curriculum ignores these kinds of word problems that seldom, if ever, appear in mathematics textbooks. Instead, mathematical ideas are often presented in ways that confuse children. For example, many children are taught to solve word problems by looking for key words, such as "in all," which is said to mean that a child should *add* the numbers, "left" is said to mean *subtract*, and so forth. If a child added the numbers 15 and 7 to solve the problem above, her answer would, of course, be wrong.

The present-day curriculum is somewhat out of balance. Language enriched pupils seldom, if ever, encounter problems that require in-depth thinking about a topic, the creation of new ways of looking at things, and the application of the basic skills that they should be learning. Instead, younger students are often kept on a steady diet of computations that are so easy they should be encouraged to do them in their heads. Older students are often asked to compute long lists of numbers that are unlike anything an adult encounters in real life. It is hardly a wonder that students turn off to mathematics.

The mathematics curriculum for language enriched students needs to strike a new balance between mastery of basic skills and problem solving. Problems should be interesting, challenge students to think hard about mathematics, and draw from the real world so that students can see how mathematics might be relevant to their lives.

The Mathematics Needed to Function in Adult Society

Language enriched pupils need mathematics that will be relevant to their futures in an increasingly complex society. Topics that used to be reserved for college students are becoming increasingly important in school mathematics. Topics that used to be reserved for secondary school are being brought into the younger grades. And old topics are being given new emphases. New K-12 mathematics topics include discrete mathematics (such as optimizing processes), statistics (such as reading and interpreting different kinds of graphs), programming, the study of growth and decay, and the study of algorithms. Topics being brought to the younger grades include algebra (such as the study of variables) and geometry. Arithmetic is being redesigned so that students study how number facts are related to one another, algebra on the use of functions to model many different kinds of real world phenomena, and geometry on its uses in dimensional analysis, in the sciences, and in modeling the real world.

The goal of mathematics is for students to *solve problems* in areas that require discrete mathematics, statistics, and/or programming; to solve problems that have a new emphasis; and to explore and solve problems at younger ages. Students should also, *eventually*, formalize what they are learning. That is, at the beginning, students should not be expected to do things purely symbolically, but at some point, they should be expected to use symbols, equations, and graphs to show what they know and can do.

Language enriched pupils should develop their ability to use mathematics across languages and cultural settings, so that, as adults, they will be able to work in the international economy or in the U.S. military. That is, the mathematics education of these students should include problems that help them to develop proficiency in their target languages and their understandings of international settings. Depending on their levels of proficiency, multilingual students should be asked to solve problems stated in different languages that revolve around technical problems requiring mathematics for their solutions. For example, a language enriched student might need to read a business memo written in a language other than English and decide whether the facts and figures in that memo are realistic, represent a good offer, or are based on questionable information. A student might be asked to develop a plan to help a business enter a particular international marketplace by using real data from that area. Such ambitious projects need not comprise students' entire mathematics programs, but they could be included to create a more balanced program that integrates different skills and applications with realistic problems.

Not only should the mathematics curriculum include problems and applications that show how mathematics is relevant in international settings, but also the program for language enriched pupils should develop, explicitly, the technical language skills needed to use mathematics to discuss important ideas. Beyond using mathematical terminology appropriately, students need to learn how to read technical materials that include mathematical text and terminology, how to write clearly and precisely using mathematical ideas and terms, and how to make clear presentations of all sorts ranging from technical to business oriented contexts.

Some people believe that children who lack proficiency in English should not be asked to solve mathematics word problems. However, first- and second-grade language enriched children in U.S. schools can solve many word problems that their English-speaking peers can solve. These children use many of the same strategies that their peers use. What is more, problems that are difficult for language enriched children are also difficult for monolingual children. Children's choices of strategy are often the same across populations. In other words, language enriched children are mathematically similar to their monolingual peers. Some may not be as advanced as others, but they all have the same potential tools and ways of thinking. These results strongly suggest that educators should not lower their curricular goals for students simply because they are acquiring English as another language.

How People Reason in Mathematics

For an increasingly broad range of mathematical topics, research is revealing that people have developed intuitions that help them understand and solve problems. Often people's invented solutions look nothing like what is taught in school. If anything, their inventions show greater sophistication than what is taught in the book. With proper instruction, students' invented strategies can provide a solid foundation for the memorization of basic facts and for the development of computational algorithms. For example, when adults are asked to rank-order the fractions 2/7, 1/4, 2/5, 2/9, 1/3 from smallest to largest, they use many different kinds of strategies. They will often:

- Convert the fractions to decimals (using a calculator, of course) and rank-order the decimals: .22... (2/9), .25 (1/4), .28... (2/7), .33... (1/3), and .40 (2/5);

- Convert the fractions to 2/7, 2/8 (=1/4), 2/5, 2/9, 2/6 (=1/3), and rank order these new fractions by either thinking that:

    a. 2 divided into 5 parts (as in 2/5) is larger than 2 divided into 6 parts (as in 1/3), so that the more parts these fractions are divided into, the smaller they are. Hence, the ordering from smallest to largest is 2/9, 2/8 (or 1/4), 2/7, 2/6 (or 1/3), and 2/5; and
    b. If I have 2 pizzas, I get a larger piece when I have to share it among 5 people than if I have to share it among 6, 7, 8, or 9 people.
- Recognize that:

    a. 1/3 is larger than 1/4;
    b. 1/8 is larger than 1/9, so 2/8 (or 1/4) is larger than 2/9, and hence, 2/9 is the smallest fraction in the list;
    c. 2/5 (= .40) is larger than 1/3 (=.333...); and
    d. 2/7 is between 1/4 and 1/3. Hence, the ordering from largest to smallest is 2/5, 1/3, 2/7, 1/4, 2/9.

While adults have such strategies, only one of the many thousand adults who have tried this problem has ever suggested that the original fractions be converted to fractions with common denominators -- the method that is commonly taught in school. Luckily for adults, in spite of years of being taught otherwise, they retain some common sense when confronting a mathematical problem. Children, on the other hand, sometimes experience a disconnect between what common sense suggests they do and what their books or their teachers tell them to do.

Most people can see how ideas are related to one another. For instance, they understand that division is what happens when things are cut into equal-sized groups. Such relationships help people to solve real-world problems. Unfortunately, textbooks treat mathematics in such a way that students lose sight of how the big ideas are related to one another. For example, division is commonly taught to fourth graders through a rather tortured series of lessons involving single- and multi-digit divisors into single and multi-digit dividends without and then with remainders. Yet, by creating equal-sized groups of objects, first graders can solve many division problems (including those with remainders) that stump their older siblings.

School mathematics should begin with how students reason and think about mathematics. Then, it should build on and develop that reasoning to the sophisticated end points that are the goals of the curriculum. One reason that so many students fail in today's mathematics programs is that the curriculum fails to take account of what they already know.

Mathematics Instruction

Students should be active and responsible for their learning of mathematics. For instance, teachers should invest considerable class time -- even an entire class session if necessary -- having students explore a single complex mathematics problem and sharing their solutions with one another and the rest of the class. Students should have spirited conversations among themselves or with their teacher on their thinking about the problem or in diagnosing the flaws in one another's thinking. A wrong answer, in other words, can be an opportunity for discussion and clarification of the lesson's intent. The focus on wrong answers should not be limited to merely deciding who is the better or more able student.

The research on student learning clearly shows that students need to spend time thinking about what they are learning and relating new information to what they already know. Class discussion should help students focus on the most important features of the lesson. The many exercises that students are assigned as homework do not really support learning and understanding because:

- If a student already understands the lesson, more than a few exercises for the student to check his or her understanding become an exercise in tedium, and

- If a student does not understand the lesson, then the homework does little more than reinforce errors and increase frustration.

Students learn best by doing and by seeking clarification when they make mistakes or do not fully understand. The expectation that students will sit silently through a mathematics lecture, with little or no opportunity to try out the ideas and to clear up their misunderstandings at the point where they try out the ideas, is counterproductive. Only a few students can understand the mathematics under such conditions, and then it is only because they exert additional effort -- usually after class -- to develop that understanding on their own.

The mathematics classroom, as described above, should be adapted to include language enriched students as fully as possible. Students should be encouraged to use whatever language they are most comfortable with; other language enriched students can judge whether their answers are right and make sense. Students who have a stronger command of both languages can translate original answers into English so that everyone in the class can understand how a particular solution is derived. In such a setting, the student who is doing the translating profits twice. First, she or he must pay close attention to the mathematical ideas presented in the non-English language. Second, the student has to develop mathematical terminology and precision in both languages so that people can understand the translation. In the world of international commerce, both skills -- the ability to listen and to communicate across languages -- are highly valued.

Explanations and the language used in presenting mathematical problems often need to be simplified for language enriched students. First graders learning English understand word problems more easily when they contain simple sentences, active voice, present tense, and unnecessary adjectives stripped away. For instance, the problem "Mary has 15 cars. 8 are green. The rest are blue. How many are blue?" is easier to understand than "Mary had 15 toy cars. Eight of her cars were green and the rest were blue. How many of Mary's toy cars were blue?"

Some language enriched students may need someone to help them read mathematical text and to explain the gist of what has been said. They should be taught to study worked-out examples, to look words up in the glossary, and to ask questions that indicate what they do not understand.

Teachers need to check that their language enriched students understand from what other students are talking about. Sometimes, teachers will slow down conversations with a request that students explain themselves a bit more slowly.

If the mathematics is interesting, language enriched students will often persist in trying to solve a problem and to develop sophisticated explanations. That their English is, at times, less than fluent should not exclude them from opportunities to provide explanations.

Mathematics Assessment

Mathematics assessment serves two fundamentally different purposes. Assessment first serves classroom functions. Mathematics teachers assess students to pace the instruction, to assign grades, to group students based on ability, and to track students' mathematical development over time. In order to fulfill these purposes, the assessments used in teachers' classrooms need to match, rather closely, the mathematics content that their students have *actually* covered or will cover.

A second purpose for assessment is non-instructional. Students may be assessed to evaluate the effectiveness of a particular curriculum, to monitor the functioning of the educational system locally (as in the case of district or state assessments) or even nationally (as in the case of the National Assessment of Educational Progress), to hold students, their teachers, or their schools accountable (as in the case of student testing to obtain a diploma or to enter post-secondary education or the armed forces), or to compare students to one another (as in the case of international comparisons). When geared for non-instructional purposes, mathematics assessments can be targeted to a set of outcomes around which a political consensus has developed. Regardless what students have covered in their mathematics classes, they *should* know the content found on these tests.

Classroom Assessment
Classroom assessment should be used to diagnose and to remedy student errors and misunderstandings, in addition to its other uses of pacing instruction and grading student performance.

Classroom-based assessment is often focused on what students cannot do. While that information is helpful, teachers and students need more detailed information on what students actually understand and *can* do in mathematics. Otherwise, it is very difficult -- if not impossible -- to plan instruction. Students who seldom have the opportunity to demonstrate what they know become demoralized as their teachers make wrong-headed judgments on where to begin and how to develop instruction.

A group of secondary urban students were discussing their homework. One girl explained, in clear Spanish, how she had solved the problem: "The sum of two angles is 90 degrees. One angle is twice the other. What are the angles?" Her solution was short, to the point, and almost elegant: "One angle is X; the other is 2X because it is twice the first. So, I know that 3X equals 90. X equals 30, and 2X is 60 degrees" [my translation of her explanation in Spanish]. Another visitor to this classroom asked her to repeat her explanation in English. Beyond her many grammatical errors, this girl made substantive mathematical mistakes. In the absence of her earlier explanation in Spanish, one could only conclude that she had gotten the right answer with help from someone else.

This example illustrates the importance of distinguishing between language proficiency and mathematical errors in classroom assessment. Classroom-based assessments need to be closely matched to the mathematics curriculum that is in place, thereby increasing the likelihood that it taps into what a student actually knows.

Mathematics tests should be administered to ensure that the test provides the information that it was designed to provide. If a problem asks students to write persuasive arguments that use technical terminology appropriately, then students should be allowed to refer to a dictionary or technical glossary since, in the real life, most people who write such arguments have ready access to such materials. Recently, mathematics tests have encouraged, and in some cases assumed, student use of pocket calculators. As a result, many mathematics assessments now use messy numbers instead of numbers that are easily divisible by one another or that sum to nice totals. Finally, mathematics tests for language enriched students might require that some answers be provided in one of the target languages if the intent of the program is to develop students' technical knowledge and skills in both English and other languages.

Teachers need specialized training on what to look for and how to score student work on their tests. That training should include information on how right answers can provide evidence of student knowledge and, under what conditions, wrong answers might provide evidence that the student was actually trying to solve the problem in a way that is more sophisticated than the thinking of a "normal" student who got the answer right. For example, the student who says that 6+7=13 because 6+6=12 and one more make 13 has provided a more sophisticated answer than the student who gives the same answer because he or she counted 6 blocks and then counted additional 7 blocks.

When scorers encounter work produced by students whose proficiency in English and in another language vary from the native speaker of that language, then the teacher needs to understand when errors or low-level work are due to student failures in the mathematical content, students' lack of familiarity with the context that was provided, or their lack of fluency with the language in question. Oftentimes, students' work looks wrong because of misspellings, use of inappropriate technical language, bad grammar, convoluted sentences, or some other problems with language. Teachers need experience with many examples of such work so that they can discriminate among the many possible reasons why something might be wrong.

Non-Classroom Based Assessments

Non-classroom assessments need to balance among competing ideas of what every student should know. Hence, they represent a political compromise. While classroom-based assessments should contain opportunities for teachers to incorporate informal observations of how their students perform and reason, non-classroom assessments do not have such a luxury. Non-classroom assessments need to be as clear and self-contained as possible.

Non-classroom assessments should include items that balance among basic skills, short applications, and longer open-ended problems. When a student's performance on an assessment has clear consequences -- for instance, the student receives a diploma, the school's results are published in a newspaper, teachers receive positive or negative feedback about their teaching -- teachers feel a strong pressure to ensure that students learn the content in that assessment. Hence, mathematics tests that are overloaded with basic skills will constrain what students learn. On the other hand, mathematics tests that are balanced in what they ask students to do encourage teachers to ensure that the students have the opportunity to learn a wide range of mathematical topics. For example, students could be asked to present a reasoned argument for why they agreed or disagreed with a statement, such as in the following:

> A high school student decided there was no reason to worry about getting into college since two local colleges accepted half the students who graduated from his high school. As a result, this

student believed that acceptance to one or the other local college would be a certainty.

High schoolers' responses reflected a lack of understanding of this problem. Fewer than 6% gave a well-reasoned answer explaining how the two colleges probably did not accept everyone from the local high school. Since fewer than half of all high school students go to college, one should ask: *why* would anyone expect better performance on a test item that was irrelevant to over half of the students taking the test? However, students can learn to reason and effectively respond to similar types of questions that relate to their own interests and experiences.

While most test developers would delete a problem like this, it may not be possible (or even desirable) to eliminate all open-ended problems with even a hint of bias and still develop a mathematics test that could provide a balance among different kinds of basic skills and applications. Instead, multiple versions of the same problem might provide better information about students taking the test. For instance, an alternative problem would pose the same question about a student trying to get tickets for one of two rock concerts that would be oversubscribed. At concert A, 60% of the people who apply to get tickets do, in fact, get them; and at concert B, 50% of the applicants get tickets. Hence, person X assumes that a ticket is a sure thing and makes plans for Saturday night. The problem would be the same: Give a reasoned response as to why one agrees or disagrees with person X's belief about the possibility of activities for Saturday night. It seems relatively an easy thing to generate additional versions of the above problem that entail different settings and ask students to chose from among a group of problems.

Problems should be written in easy-to-understand English. Unless necessary to convey an important idea, text should be written in relatively short sentences, active voice, and present tense. While technical language should not be avoided, its use should be relevant to the problem settings.

Test instructions should be explicit and clear. If, in order to score high on an open-ended question, a student must do more than just provide an answer, exactly what is required should be included in the problem. It may not be enough to write: "give a mathematical justification for X." Many teachers mean different things when they say something like: "justify your answer." Somewhere in the test, students should see what counts as a "mathematical justification."

Test administration can be adapted or changed to ensure the gathering of valid information, that is, the conditions under which tests are administered increase people's confidence that the test results do, in fact, tell us what we want to know about the students' knowledge and skills. In addition, test administration should be adapted to ensure that specific students have a realistic opportunity to show what they can do. The most commonly used adaptations are:

- Allowing students extra time to complete a test.
- Encouraging them to use calculators, computers, or other technical instruments.
- Providing students with specially trained readers or translators to explain the test's directions.
- Allowing students to dictate their answers to someone (specially trained) who can write the answers on the student's behalf.
- Providing answers in another language that are then translated into English (or are scored by someone who is literate in that language).

- Allowing students to use their books or notes during the examination.
- Other individually-negotiated methods that test administrators and students agree would provide for a fair and valid assessment of what the students know.

For students who are not fluent in English, the provision of test readers and translators, dual-language dictionaries, calculating aides, and extra time are among the most-often used adaptations. When the administration of a mathematics test varies, the test administrator should note the adaptations provided to individual students. Whenever someone helps a student to take a test, by reading or translating or some other means, that individual should be carefully trained to ensure the help does not accidentally give the answers. When a task is open-ended and rather complex, the likelihood of someone accidentally giving away the complete answer is lessened than when the task is a simple, multiple-choice question. On the other hand, the more complex a problem becomes, the more likely it would seem that the person who is helping will accidentally give away parts of the answer since that individual would be helping with many different parts of the question. Hence, the person who is helping should receive special preparation and review the test, item by item, to determine what help is and is not appropriate.

In some cases, a test that is usually group-administered might have to be administered individually or in small groups to some students. For instance, an immigrant student who knows a lot of mathematics might have been in the United States too short of a time to read and understand non-computational problems. In this case, someone might have to read and translate the problems, individually, to that student. Alternatively, the student might be provided with a tape recording that translates the problems in question.

Concluding Comments

The central problem addressed in this chapter is how to include language enriched pupils in a school's mathematics program. Though inclusion is a complex issue, the associated problems are solvable and achievable in Florida.

In order to be successful language enriched pupils need more than the technical solutions outlined in this chapter. They need business people, educators, and interested stakeholders to agree that all students are worth the time and energy that will be required to ensure that they are included, *meaningfully*, in their schools' mathematics programs. Unfortunately, many people seem willing to do something about the "problem" of language enriched students, provided that solving the problem does not cost too much, does not require careful thought, can be done easily, or has been solved elsewhere. Under such circumstances, efforts to improve schools' mathematics programs may become more powerful ways of depriving Florida's language enriched pupils from the educational opportunities they require.

This nation's language enriched students deserve the best that can be offered. First of all, they deserve it as a simple matter of dignity. They are America's children; they deserve what is appropriate for all of our children. Second, language enriched pupils merit the best because they provide this country with an incredible set of linguistic and cultural resources that could be used to help the state and nation become more fully a part of the international marketplace -- not just the economic marketplace, but also the marketplace of ideas. If, through their deeds, Americans show that they believe in opportunity for all by actually providing that opportunity, the belief in democratic opportunity will be exportable to all places on the globe. Finally, Americans need to invest in this nation's language enriched children in order to ensure that the democratic prospect remains vibrant into the next century. As Mr. Jefferson noted, education is the anvil on which we forge the nation's democracy. For its citizens to participate fully in its varied aspects, they must be well educated enough to take seriously the problems of the democracy and to exercise their responsibilities in an enlightened manner. Mathematical literacy is central in all three of the above.

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