

Backgrounds: Recommendations for Teaching

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Mathematics For Students with Learning Disabilities from Language-Minority Backgrounds: Recommendations for Teaching

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Abstract: This article examines characteristics and cultural and linguistic factors relevant to mathematics instruction of students with learning disabilities from language-minority backgrounds. Recommendations and strategies are introduced with attention given to balancing the influences of language, culture, and disability. Specific recommendations relate to: 1) appraising math ability; 2) selecting the language of instruction; 3) moving from concrete to abstract understanding; 4) using strategies that help students develop mathematical concepts; 5) using math for language development; and 6) taking student strengths into consideration.

Introduction

Professionals in both special education and bilingual education have worked adamantly to keep routes to learning accessible to populations whose educational needs differ from students of the mainstream. Many of the issues that have had an impact on bilingual education have also had an impact on special education. One such issue is the current reform movement aimed at improving academic achievement among all students.

In examining mathematics reform and its implications for students with disabilities, Rivera (1993) responded to issues that have been raised by researchers in the field of special education. In a review of the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989), Rivera found that the standards were written primarily for a general education audience and did not provide sufficient information about the learning needs of students with disabilities in math. She also discovered that only brief reference was made to student diversity. She called for educators to consider the diverse needs of their student population and to investigate diverse ways to teach mathematics concepts.

There are methods that have demonstrated success for special education and bilingual education. However, a major concern among bilingual educators is that the approaches generally used in special education to provide mathematics instruction are not only incompatible with the holistic approach typically used to promote natural second language acquisition, but that they may even stifle language development (Gersten & Woodward, 1994).

What, then, constitutes effective instruction for students with learning disabilities from language-minority backgrounds? Gersten and Woodward call for an effective balance between bilingual and special education methodology. They suggest providing language-minority students with engaging material, comprehensible input, and encouragement for expression of complex ideas and feelings. They further suggest that special

educators bring their task-oriented instructional practices in useful ways to meet the needs of students with disabilities from language-minority backgrounds.

In this article, a brief description is provided of basic characteristics that must be considered when addressing the individual needs of students from language-minority backgrounds who also have disabilities in mathematics. Recommendations for mathematics instruction are suggested based upon the limited research which examines the interplay of language, culture, disability and mathematics.

Students With Learning Disabilities

Briefly defined, a learning disability is a disorder in one or more of the basic psychological processes involved in understanding or using language, which may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations (Kretschmer, 1991). It must be pointed out that not all students with learning disabilities do poorly in math. On the contrary, for some students math is an area of strength.

Students with learning disabilities who are struggling in math generally have average or above average intelligence (Bley & Thornton, 1989). However, math ability may be restricted by substantial differences in the areas of attention, perception, visual-motor abilities, language processing, memory, reading / writing, and in the use of cognitive strategies. These significant differences will stand out in each language used by a bilingual student with a learning disability. If differences appear only in English, then those differences are probably due to the challenges of learning a new language.

Attentional differences may be evident in students who are highly distractible, impulsive or demonstrate extremes in perseveration. Students with attentional deficits may guess at answers or have difficulty completing their work. Students with visual and auditory perception differences may frequently lose their place, have trouble noticing size differences or have trouble hearing patterns. Visual-motor differences may become apparent as students learn to shape numbers or as they are asked to write on lined paper. Difficulties with memory may result in reversal of digits in a number, inability to retrieve basic facts or to solve multi-operational calculations. Adequate reading and writing abilities are especially critical as students work with story problems. A student who loses his/her place in the text of a story problem may exhibit problems in the areas of decoding, comprehension or sequence; and, therefore, be doubly confounded by the mathematics required to solve the problem.

Students From Language Minority Backgrounds

Mathematics is one of the subjects in which both students learning English as a second language and students with disabilities are first mainstreamed. Yet, comparatively little attention has been given to math education for language-minority students. One reason may be the perception that mathematics is a universal language, transcending language concerns (Lass, 1988). Recently, researchers in bilingual education have begun to address such issues as the complexity of the interplay between language and mathematics skills for bilingual students.

Linguistic factors must be considered during planning and instruction of mathematics. Math vocabulary is precise but not always familiar. It may be difficult, even for students who are not bilingual, to determine which meaning of 'odd' is intended in a problem (odd as in something peculiar or odd as in numbers that are not divisible by two). Special problems may exist for students with learning disabilities who are concurrently learning the English language. Cuevas and Beech (1983) noted the importance of considering

issues of language comprehension, knowledge of syntax and vocabulary, and understanding of relational terms as they apply to mathematics. Students may experience difficulty distinguishing differences and making comparisons in relationships that pertain to size, speed, space, and time. Students with learning disabilities from language-minority backgrounds are likely to encounter difficulty with language concepts and structures even in their native language. Therefore, Cuevas and Beech recommended use of native language instruction for teaching mathematics concepts before transition to English.

Cultural differences define our values and what may be perceived as a real problem by one group may be unfamiliar or unimportant to another group (Fellows, Koblitz, & Koblitz, 1994). Consider the comprehension of story problems. Within mainstream American culture, the gap between achievement in math by gender is an area of concern. In examining the work of a research project supported by the National Science Foundation, Fellows, Koblitz, and Koblitz (1994) encountered a number of units that appeared to be flawed because of cultural bias. One example was a story problem from a unit on probabilities. Although not necessarily language-based, the following is an example of how culture influences our curriculum. The motivation for this unit was baseball pennant fever. It was assumed that all children were motivated by the topic, but in reality it makes sense that boys might be more apt to have a working knowledge of the statistical aspects of baseball, especially since most leagues from Little League to the professional leagues are presently dominated by male athletes. This example might be considered minor compared to one the researchers themselves discovered in their own work. They used a story about a "Tourist Town" in which the merchants were preparing for the summer season. They presented their problem solving unit to students in Peru. The question was "What is the minimum number of ice cream stands they [the merchants] will have to build so that anyone standing at a street corner without an ice cream stand will have to walk only one block to find one?"(p.7). The story made little sense to the children. The authors had failed to consider that in a country with large unemployment, there was always an overabundance of people available to sell ice cream to tourists. Of course the children had difficulty comprehending and relating to the material.

Another difficulty that may arise with language-minority students is that of differences in algorithms. An algorithm is the procedure used for finding the solution to a mathematical problem. Most Americans learn to calculate using set algorithms taught in the schools. Students from South American or Asian countries learn algorithms that are different in sequence. The position of numbers on the written page often does not match the algorithms typically used in American schools. Therefore, when students are asked to calculate, a difference in the algorithm may be misinterpreted as lack of math ability. For language-minority students, it is more important to determine if the student knows how to obtain the correct answer and if they can explain the procedure, rather than how well the student's algorithms match those used in our schools.

Recommendations

Scott and Raborn (in press) propose that educators tap into the richness of diversity brought to the classroom by their students. By tapping into the richness of culture and language, while recognizing individual learning differences, educators can tailor curriculum and instruction to assist all students in reaching higher levels of mathematics. The following recommendations embrace the belief that diversity can be viewed as a gift.

Appraise Math Abilities

Individual needs and strengths vary between all students, including language-minority students. Bilingual students may have high ability in math and yet not be able to communicate that ability, due to lack of English proficiency or lack of communication skills in either language. Therefore, it is critical that math ability be appraised on the basis of cognitive ability and not assumed on the basis of the student's

proficiency in English, or because of a disability that affects communication skills.

According to Cawley (1985), appraisal consists of three stages: measurement, assessment, and diagnosis. The primary intent of measurement is to compare the performance of one or more individuals with that of others, primarily through norm-referenced procedures. Three examples of measurement approaches include the Wide Range Achievement Test, the K-ABC Arithmetic Test, and the Woodcock Johnson Math Cluster.

In the assessment stage, the student is evaluated across a number of mathematical strands or content areas. The purpose of assessment is to identify patterns of strengths and weaknesses so as to guide the instructor in placement decision. Two examples of assessment instruments are the Mathematics Concept Inventory and the Key MATH Diagnostic Test. Cawley stresses caution in interpreting assessment results, due to the lack of assessment instruments which deal with areas other than arithmetic computation. He stresses the need for all students to be assessed in the many areas of mathematics, including, for example, geometry and measurement. This concern is especially relevant to students with learning disabilities, whose strengths may lie in areas of mathematics other than computation.

Diagnostic procedures focus upon intraindividual performance in a single or limited number of mathematics topics. Appropriate diagnostic procedures enable teachers to identify patterns of performance in specific areas of mathematics within the capabilities of an individual student. Examples of diagnostic measures include the Comprehensive Inventory of Basic Skills (Brigance, 1984); Brigance Diagnostic Assessment of Basic Skills, Spanish Edition (Brigance, 1985); and the Diagnostic Test of Arithmetic Strategies (DTAS). The DTAS is an informal measure that allows teachers to observe the procedures children use to perform arithmetic operations.

Cawley recommends that teacher observation be integrated into appraisal of student ability and performance in math. How does the learner respond to new or novel demands in math? What algorithms does the student use and how were they learned? How do students interpret mathematical questions? How does the student relate to quantifiable material and mathematical stimuli in the environment? These types of questions provide information that is not available in quantifiable test scores, but is important in identifying needs and strengths of the learner.

Select the Language of Instruction

Determine how well bilingual students speak both languages. Many school districts assess English language proficiency and fail to obtain a measure of native language proficiency. In selecting the language of instruction, all languages to which the student is regularly exposed must be assessed to make the best decision concerning language of instruction. In addition to using an instrument to assess oral language proficiency, the instructor should observe the student in both social and academic settings. Willig and Ortiz (1991) recommended that if the student is stronger in the native language than in English, instruction should continue in the native language. Instructional use of the native language allows students to learn developmentally appropriate content without having to wait for their language abilities in English to "catch up." Students who are stronger in math skills than in English skills should be taught at their level of math ability, not at their level of English proficiency.

If it is determined that the native language will be used for instruction, then a teacher who is fluent at the academic level of that language should provide instruction. Just as the language proficiency of bilingual students can range dramatically, so can the language proficiency of bilingual teachers. If the teacher is highly proficient in the academic language of math, then the student will be more likely to learn that higher

level of their native language. A teacher who is only fluent in the social aspects of the native language may struggle to communicate precisely with mathematical terms and expressions in that language, and may not be able to take their native language students to a higher level of learning.

Proceed from Concrete to Abstract

Math can be used for connecting the language of concrete experiences to the development of abstract concepts. It is important to begin the development of concepts with concrete materials. Manipulation of concrete materials is then supported and extended through the use of language.

Often, students with learning disabilities have difficulty connecting language to actions, memories, and concepts. Because a learning disability often involves a difficulty with the processing of language, students with learning disabilities may find it especially difficult to learn mathematics when it is taught solely through language. It may be asking too much if students are presented with a request and then expected to remember a sequence based solely on verbal instructions. For students with visual strengths, non-verbal routes to understanding can reduce the role of language. Manipulation of concrete materials, carefully tied to a verbal description of visual and kinesthetic images, solidifies concepts.

Verbal labelling is a strategy that provides language for visual forms as they are manipulated and represented spatially (Kibel, 1992). An important aspect of this strategy is the concrete nature of the learning task. Say, for example, the student is learning the concept of borrowing with hundreds, tens, and ones blocks. In the first stage, the student learns to trade hundreds for tens and tens for ones, and learns to use language in describing the process. Language use may be the most difficult aspect of the initial stage of verbal labeling. Words may be unfamiliar and precision of the statement used to describe the process may be awkward. In the second stage, the student forms an internal model by describing the idea or process without the use of manipulatives or visual cues. Language use in this stage may be highly dependent upon the action. It may be difficult for the student to close his/her eyes and remember the idea and the language. At this point students can step back to the first stage and practice tying the language to the visual and kinesthetic action. Students may benefit from using the manipulatives with their eyes closed as they verbally describe the process. Eventually, the student will no longer need the cues. At this point an internal model has formed. Now, in the third stage, standard written subtraction can be introduced. Obviously, some drill is involved in verbal labeling, but it is not drill for the sake of minimizing errors in the language of instruction. In verbal labeling, drill is used as a tool to make connections and strengthen understanding of math in a meaningful context.

Use Strategies to Help Students Develop Concepts

One method that uses cognitive strategies to help students develop concepts is concept attainment. Concept attainment is much like categorization in that it helps students learn categories and requires students to make grouping decisions based upon attributes. Students compare and contrast examples that contain the attributes of the concept (positive exemplars) with examples that do not contain those attributes (negative exemplars). Joyce, Weil, and Showers (1992) pointed out that, unlike categorization, concept attainment requires students to figure out the attributes of a category that is already formed in another person's mind. These strategies provide instruction on specific concepts and can help students understand the nature of concepts (Joyce, Weil & Showers, 1992).

Concept attainment can be used with any age level and across the mathematics curriculum. The following steps can be adapted to teach vocabulary, rules, and concepts at any degree of difficulty. These examples

are more appropriate for very young children, however, the strategy can be adapted for any age group.

Step 1. The teacher begins by presenting a focus, or category, under which the concept falls. The teacher may want students to learn the definition of a circle. The teacher may say, "I'm thinking of a shape. " Step 2. The teacher then presents data in the form of words, phrases, concrete materials or pictorial representations and presents approximately equal numbers of positive and negative exemplars (typically, the data set contains 15 examples). While presenting the exemplars, the teacher may say "I have two groups of shapes here. Notice that some are under the word 'yes' and some are under the word 'no'. I want you to try to guess what shape I'm thinking of." The teacher points out the exemplars "This is a yes (referring to a circle) " or "This is a no (referring to shapes that are not circles)":

Students examine the exemplars and develop a hypothesis about the concept. They need to consider what the positive exemplars have in common and how they are different from the negative ones (see figure 1).

Step 3. Students develop a hypothesis to name the concept being presented. They are asked to describe how they arrived at their hypothesis. The teacher probes the students by asking, "Can you come up with the name of my idea?"

Step 4. Once the concept has been correctly identified, it is named. Students then provide additional exemplars of their own to test further their understanding of the concept. The teacher asks "Do you know how to see if your idea is the one I'm thinking of? I'll give you some examples and you tell me if they are a 'yes' or a 'no'. " The teacher gives examples and then the students categorize them under 'yes' or 'no'. When they appear to understand the concept, the students are then asked to provide examples for the rest of the class to categorize.

Use Math for Language Development

Students with learning disabilities from language-minority backgrounds who have deficits in receptive or expressive language skills profit from learning the language of math as they develop precise vocabulary, sequence, and comprehension skills in their native language. They need time to pose their own questions and to explore ways of answering them. It is important to give students the opportunity to talk with peers and adults so that they can experiment with and validate their own ideas.

Mathematics teaches predictable patterns and can foster a natural acquisition of social and academic language proficiency. The following excerpt is from a math lesson in a cross-categorical, self-contained Bilingual Special Education classroom. Perla, a six-year-old first grader with a learning disability, demonstrates her ability in math following a math lesson taught in Spanish. Perla has been learning addition of sums to ten in Spanish and wants her teacher to see how well she can add using Teddy Bear Counters. She also wants her teacher to see that she is learning English. Looking over her worksheet, the teacher observes that all the answers were correct. She congratulates Perla and asks her to demonstrate a problem.

Teacher: " *¡ bien, Perla! ¡ muy inteligente! A ver, enséñame.* "

(Very good, Perla! You are so smart! Let's see, show me how you did that.)

Perla: "*Four plus five equals is nine.*"

Somehow, as Perla was learning English, she had learned the "=" meant equals and she had learned that "="

meant is, so she thought "=" could be translated to mean "equals/is". Yet, when this student was asked to demonstrate in Spanish, she was able to do so with correct Spanish grammar. Activities were introduced that allowed the teacher to capture and cultivate Perla's mathematical understanding outside the structured time for math instruction. Often Perla would switch to English very naturally. The teacher continued to teach Perla math at her academic level in Spanish, and was able to pinpoint special vocabulary that was causing confusion or needed work in her academic level of English.

Take Student Strengths into Consideration

Often, it is difficult for students to recognize their own learning strengths and specific areas for improvement. Information can be obtained from diagnostic reports written after initial assessment, and from teacher tests and observation. Some students learn best when material is presented visually, others learn from kinesthetic modes. Students can learn to benefit from their strengths when those strengths are pointed out to them. It is also helpful if teachers are willing to model recognition and use their own personal strengths. The following recommendations may be helpful for teachers in providing instruction in math.

For students who are visual learners, graphic representations provide many more clues than words that are difficult to read or visualize. Students with visual strengths can benefit from drawing or examining pictures, diagrams, or graphic organizers to represent story problems. Color coded steps of lessons or worksheets which are highlighted provide additional cues to signal important information. Students using highlighter pens can learn to identify and mark important information.

Demonstration, plus permanent model, is a strategy that can be highly effective for helping children master computational mathematics (Rivera & Smith, 1987). This strategy is especially appropriate for learning math skills rather than learning math concepts. It is based on the principles of direct instruction, task analysis, and individualized instruction (Smith & Luckasson, 1995) and is especially useful in helping students to organize and remember the sequence of multi-step algorithms. Demonstration, plus permanent model, consists of three key basic steps. First, the teacher pinpoints what the student already knows about a particular math skill. Second, the teacher selects or constructs worksheets so that only those problems pinpointed at the students' level of instruction are represented. Third, the teacher demonstrates the skill to be taught and monitors the student through guided practice. This final step is key to the strategy. As the teacher demonstrates the skill, the example is highlighted and left with the student as a permanent model. As the student is directed through guided practice, that work is also highlighted, thus making it possible for the student to refer back to recently solved work as a model.

For students with kinesthetic strengths, manipulatives with tactile markings help students identify boundaries. For example, in developing manipulatives for students with visual impairments, educators at Lawrence Hall of Science invented graduated cylinders and tools for measurement that had tactile markings in addition to the visual markings typically used for measurement. Another example of materials with tactile cues are Unifix cubes, a product sold by Didax materials company, that can be snapped together and pulled apart. At the point where the cubes are snapped together is an indentation that serves as a visual and kinesthetic cue. Among the numerous materials which compliment instruction with Unifix cubes are number trays, ten's trays, and a number track. Each of these has raised edges that serve as boundaries in which a set number of connected cubes must fit. These types of boundaries provide additional tactile cues for students with kinesthetic strengths.

Of course, the student just learning English may comprehend more easily if the teacher speaks clearly and at a natural pace, facing the student so that body language can be observed by the student as an extra cue.

Teachers can check for understanding by asking students to explain directions to other students or to have them repeat instructions back to the teacher. Students stronger in their native language should be allowed to use that language, even if the teacher only speaks English. Assigning students to develop a bilingual dictionary or index card file is one way to encourage the use of math

vocabulary in the native language.

Cooperative learning is one tool that continues to provide success for language-minority students (Cohen, 1994). Cooperative groupwork in the math class can be structured so that students attain intellectual and social goals. The communication needed to learn cooperatively will not only strengthen the language, but will also strengthen understanding and retention of the mathematical concept being learned. An important component of cooperative learning is the opportunity it provides for students to take responsibility for their cooperative roles. In identifying social goals and monitoring their progress toward those goals, students develop and reinforce their understanding of class, school, and community values and behavioral expectations.

If students have difficulty with attention, instruction must be paced carefully. Although it is important to keep high expectations for learning, the quantity of assigned work may need to be carefully monitored and adjusted. Consideration must be given to the number of problems a student really needs to complete in order to demonstrate understanding and mastery of a concept. Modifications of a prepared worksheet may be needed. For example, the amount of problems assigned may be adjusted or the amount of time allowed for completion of work may be altered. Students who are highly distractible may benefit from the use of a timer. Students are often surprised at how quickly they can work when they have the visual cue of an egg timer or are presented with the auditory cue of a clicking timer.

Areas of weakness certainly cannot be ignored. Students who have difficulty writing with pencils may find that mechanical pencils produce neater end products. Students can learn to erase neatly if they are provided with the type of plastic or aluminum stencils that architects use as tools of the trade.

Summary

While educators may consider math to be universal, there are factors related to language, culture, and cognition that must be considered in math education. With careful assessment, planning, and implementation, students with diverse learning characteristics can be successful in math. A number of recommendations and strategies were presented to guide teachers in meeting diverse needs. Creativity in identifying or developing additional strategies is certainly welcome.

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